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Renaissance: Series of problems as *varietas*

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Abstract. In the present paper I wish to discuss three texts published by two mathematical authors, Oronce Fine and his student Jean Borrel. These three texts are typical of the time and are examples of the transformation occurring in the *genres* of mathematical books. This had direct consequences on the ways and the purpose of presenting series of problems followed by further mathematical authors. Oronce Fine's contribution has been to present commercial problems as Euclidean problems on proportions, as well as to replace the university *quadrivium* by four practical disciplines: practical arithmetic, practical geometry, cosmography, sundials. The two books by his disciple Borrel reflect the view on mathematics promoted by Oronce Fine: we can recognize mathematics in every aspect of the world. Borrel is also very concerned with distinctions in the human world: variety in the *Opera geometrica* and in the *Logistica* corresponds to redefinition of professional roles according to Fine's program. In particular, Borrel wants to stress the role of a new category of mathematicians, specialized in the practical disciplines Fine taught at the Collège Royal, the *g*eometers. They dealt with practical problems by using classical humanistic education and practical mathematics, exactly what jurists did: Through the crucial rhetorical notion of *varietas* he is able to illustrate, in the *Opera geometrica,* the multifarious mathematical competence required for jurists. Among geometers, Borrel distinguishes a group of people, the logisticians, dealing in particular with the computational side. The readership of his *Logistica* was also, by and large, constituted by jurists. In fact many jurists were logisticians or needed competence in this art, and many logisticians had a training in law. The texts by the two authors examined here show a use of series of problems as *varieties* at two levels: the level of the presentation of examples for a rule, mostly in the main part of the text, and the level of a list of cases at the end on the book.

Résumé. La Renaissance : les séries de problèmes comme *variétés*. Dans cet article je souhaite discuter trois textes publiés par deux auteurs mathématiques, Oronce Fine et Jean Borrel. Ces trois textes sont typiques et de l'époque et de la transformation des *genres* des livres de mathématiques. La contribution d'Oronce Fine a été de présenter les problèmes commerciaux comme des problèmes sur les proportions Euclidiennes et de remplacer le *quadrivium* de l'université (arithmétique, géométrie, musique, astronomie) par quatre disciplines pratiques : arithmétique pratique, géométrie pratique, cosmographie, cadrans solaires. Les deux ouvrages de Jean Borrel représentent la vision des mathématiques de Fine : nous pouvons reconnaître le mathématiques dans tous les aspects du monde. Borrel est aussi très concerné par les distinctions dans le monde social : la variété dans *Opera geometrica* et dans *Logistica* corresponds à la redéfinition des rôles professionnels selon le programme de Fine. En particulier, Borrel veut souligner le rôle d'une nouvelle sorte de mathématiciens spécialisés dans les disciplines pratiques enseignées par Fine au Collège royal, les géomètres. Ils traitaient les problèmes pratiques en faisant appel à la formation humaniste classique et aux mathématiques pratiques, exactement ce que faisaient les juristes qui s'occupaient de mathématiques. Par la notion rhétorique cruciale de *varietas* dans son ouvrage *Opera geometrica*, Borrel arrive à montrer les compétences multiples demandées aux juristes. Parmi les géomètres, Borrel inclut aussi les logisticiens, concernés particulièrement par les calculs. Le lectorat de sa *Logistica* était aussi en bonne partie constitué par des juristes, puisque beaucoup d'entre eux étaient logisticiens ou avaient des connaissances dans cet art, et plusieurs logisticiens avaient une formation juridique. Les textes des deux auteurs étudiés ici montrent un usage de la série de problèmes en tant que *variétés* à deux niveaux : celui de la présentation d'exemples d'une règle, notamment dans le corps principal du texte, et le niveau d'une liste de cas à la fin du livre.

1. Introduction

Series of problems are an aspect of a text. Being aware of the competence in rhetoric of sixteenth-century mathematicians[1], I looked for a rhetorical way to conceive series of problems. My claim is that in the sixteenth-century the key to interpreting series of problems in mathematical texts is variety, that is *varietas*[2]. So, as announced in the general introduction, the theme "series of problems" will be dealt with in the present paper as well as in Serra's and Coquard's contributions in a specific aspect, the rhetorical notion of variety. In the most general way, it can be understood as the range of diversity within a context. In the case of mathematics, variety is a rhetorical tool meant to test validity, that is to convince the writer as much as the reader: if a mathematical tool or theory solves problems belonging to different branches of knowledge, it can be said to be useful in a variety of fields in nature. A series of problems can be usefully conceived as a variety in this sense.

In the present paper I wish to discuss three texts published by two mathematical authors of the sixteenth-century. The authors are Oronce Fine (1494–1555) and his student Jean Borrel (1492–circa 1570). These three texts are typical of the time and are examples of the transformation occurring at the time in the *genres* of mathematical books. This had direct consequences on the ways and the purpose of presenting series of problems.

2. *Varietas*

Varietas is a rhetorical feature described in most particularly in the *Rhetorica ad Herennium*, a work attributed to Cicero during the Renaissance.The first occurrence of *varietas* is *varietas rerum*: according to the author a good *narratio* concerning people should have a nice style, different feelings and changing of facts (*rerum varietas*) such as change of fortune, unexpected accident, sudden happiness, happy end of the events.

In the following occurrences in the book *Ad Herennium*, *varietas* is meant to contribute to the ornament of the oration in the sense of its dignity: the orator should keep the attention of the listeners by changing style. Later, in his *Institutiones Oratoriae*, Quintilian developed the idea of integrating different styles in the same discourse. As Carruthers shows in her article on the subject (2009), the classical rhetorical meaning of *varietas* takes a turn with Christianity, when it becomes synonymous of integration of different styles of belief. She uncovers the developments of this idea in the Middle Ages. It seems to me that this analysis should be continued for the Renaissance, a period in which the *Rhetorica ad Herennium* was particularly appreciated and largely published. Quintilian's *Institutiones Oratoriae*

[1] I have given elsewhere many examples of the interaction between mathematics and rhetoric in the writing of sixteenth-century mathematical books.

[2] There is a vast bibliography on the notion of *varietas* in the Middle Ages and Renaissance. Among recent works, see (Carruthers 2009; Courcelles 2001; Föcking and Huss 2003).

had also a great reception in the Renaissance: to mention only Gryphe, the great humanist printer in Lyon, he had printed it already in 1534, and there are still copies dated 1537, 1548 and 1551. Jean Borrel had access to it before publishing his *Opera geometrica*. Among the many passages in Quintilian of interest to our present concern we can mention the following:

> "Of the best authors we should take the multiplicity of words, the variety of figures and the order of composition, for the mind must be directed towards the example of all their virtues. For there is no doubt that a great part of any art is in imitation. To find has been and is the most important, similarly what has been well found should be followed."[3]

Here we see that the very idea of imitation implies that the imitator works starting from excellent examples by reproducing their inventions. However, their inventions make sense in their complexity: in the *verborum copia* and in the *figurarum varietas* (as well as in the *ratio componendi*), that is in their variety of themes and styles. In the same sense in which it appears important to imitate the richness of vocabulary (*verborum copia* that is, literally, abundance of words), it is also important to imitate the diversity of the rhetorical figures (*figurarum varietas*). What appears as a sound rhetorical principle to improve the efficacy of a speech was adopted as a guideline to produce a longer text, in particular sixteenth-century books.

In conclusion, in ancient rhetoric, *varietas* meant first of all a change in language or style, but through the centuries it became more connected to the idea of multiplicity, in its positive as well as negative sense, not only in words, but also in things. For, multiplicity is a feature of creation, an expression of God, but also, for the fragile human nature, a distraction from God. The Renaissance tended to stress the positive aspects of variety as *copia*, that is abundance. While abundance is accessible to human nature, infinity is not. Discourse, text should represent infinity by abundance. In sixteenth century Europe, with the new perceptions of space and time through geographic and astronomic discoveries, this was a sensible matter, far from being reserved to a few theologians or mystics.

We should also mention that in Vitruvius it got the sense of multiplicity to be mastered by *ordo*, another rhetorical notion: this is relevant because Vitruvius was very much studied in the Renaissance, and most particularly by humanist mathematicians we are concerned with.

In mathematics nowadays a series of problems is often used in the attempt to capture, through a diversity of topics, a sense of the generality of a mathematical tool. If successful, it becomes a concrete proof of the generality of that tool. While diversity and variety is not a distinctive feature of sixteenth-century mathematical books and series of mathematical problems, we can say, as I hope to show, that the diversities appearing in these examples share the description of being a search for variety.

3. Oronce Fine's *Protomathesis*, Euclid's proportions for merchants and practitioners

Fine, the first professor of mathematics at the *Collège Royal*, published his *Protomathesis* in 1532. The first part, devoted to *Arithmetica Practica*, deals with *algorismus* arithmetic, that is the four operations and roots extraction on whole numbers, fractions, and sexagesimal fractions. The second is devoted to geometry with a stress on practical geometry, elementary trigonometry and surveying. The third included cosmography, in this case geometrical description of the Earth and elementary astronomy. The fourth book was devoted to sundials astronomical instruments. The four disciplines of the *quadrivium* were therefore transformed into their practical aspect, the theoretical was absorbed or eliminated, and

[3] Ex optimis auctoribus et verborum sumenda copia est et varietas figurarum et componendi ratio, tum ad exemplum virtutum omnium mens dirigenda. Neque enim dubitari potest quin artis pars magna contineatur imitatione. Nam ut invenire primum fuit estque praecipuum, sic ea, quae bene inventa sunt, utile sequi. (*Instit. Orat*. X, 2.)

music was replaced by construction of sundials and clocks. In this way Fine realized the project of the Collège Royal of providing teachings not offered at the university.

Let us look more closely at an example of variety in Fine's mathematical text: here, variety is a striking mixture of mathematical styles, topics and *genres*. In Chapter 3 of the fourth book of Arithmetic, with the title *De aurea quatuor proportionalium numerorum regula*, Fine introduces the topic by writing that in Proposition 19 of book IX of the *Elements* Euclid gives this rule, popularly called the rule of three. But this proposition needs the condition that the first number divides the other two, and Fine does not stress this point at first. Rather, he focuses on stating the value of this rule, such that "one can hardly find a problem not solved thanks to it" (*folio* 43). Then he posits the condition: given four numbers such that the first and the second are in the same ratio than the third and the fourth, if any of them is unknown, it is easy to find it. He takes four letters A, B, C, D, says that he takes four numbers such that the ratio of A to B is the same as the ratio of C to D. He supposes that at first D is unknown, then writes:"If you want to know it, multiply a mean with the other, in this case, B by C, and divide the product by the first, that is A, and you will get the fourth proportional." However, writes Fine, the first and the third number, as well as the second and the fourth should be presented in a way that it should be "convenient" to each other *re atque nomine*, that is in substance and name. We are in the theory of proportions.

Fine then moves to a numerical example. "Let C be 10, the question should be formed in the following way. If 8 give <plural in the text>12, how much will give 10, similar to 8? Multiplying 12 by 10 we shall get 120, which divided by 8 will give 15, which are convenient to 12 in substance and name, 10 to 15 is in the same ratio as 8 to 12 for both numbers have a ratio of 2/3."

Particularly striking for us is that at this point Fine introduces a practical example: "Therefore if 8 arms of a given piece of fabric are worth 12 francs, 10 arms of the same fabric will be worth 15 francs."

This is followed by another practical example: "Now, if in 8 hours a certain wheel makes 12 turns, in 10 hours the same wheel makes 15 circuits."

This presentation is surprising first of all for what appears to us as a change in the subject, given that the previous example was purely numerical. This is also a change with respect to previous works. Juan Martinez Siliceo's *Arithmetica,* a book known to Fine who gave a version of it in 1519, had reserved a limited space to examples taken from commerce, only twenty "questions" involving the rule of three were included at the very end of the book: the examples in the main text are strictly numerical and there is no mention of commercial concerns in the course of the main presentation. In this collection addressed to the *Collège Royal* we find, instead, a problem on the pay of a soldier, a few problems of society, some barter problems (selling or buying different objects at the same price). There are also two problems of inheritance, solved by the rule of three, as in the example we shall see below from Borrel's text.

I think that the presence of commercial calculation in the *Protomathesis* can tell much about Fine's conception of mathematical utility in the world, perhaps more than his better known liminary statements of platonical style. Here we see that utility at work, and its inclusion in a teaching context.

This inclusion in the very middle of the text indicates clearly that proposition 19 of book IX of Euclid's *Elements* is considered as an application of rules of proportions, but also definitely identified with the rule of three coming from the world of merchants. Here, more than a representation of variety we have an instance in which the mathematical author has chosen to give the sense of a mathematical tool directly from the *variety of experience* in the world. Here this experience was that of the world of merchants, largely mathematized in terms of commercial arithmetic and metrology, but also in terms of surveying and chronology.

In conclusion, Oronce Fine transformed mathematics not only by focusing on practical mathematics at the university level, but also by introducing practical problems in the very context of Euclidean mathematics and by interpreting Euclidean mathematics in terms of practical problems.

4. Jean Borrel's *Opera geometrica*: A collection of problems and the question of its genre

Jean Borrel, a monk and Oronce Fine's former student, published in 1554 a volume under the name of Buteo: *Buteonis Delphinatici Opera geometrica* (*Geometrical works by Buteo from Dauphiné*). It is a collection of fifteen booklets, or chapters, each of them dealing with a single problem coming from classical tradition. It is a collection of problems.

The first nine problems are of miscellaneous origin and content, concerning mathematical questions from the Bible or from various classical sources, mostly in literature, but also in history and mathematics. Instead, the second part of the *Opera*, consisting of six problems, is devoted to questions coming from the *Corpus Iuris civilis* (the corpus of civil law in the Roman Empire).The title of the book is *Opera geometrica*, but its genre is far for being obvious, both in terms of mathematical topics involved and in terms of the structure of the chapters. It is another example of variety.

We have first *De Arca Noe, cuius formae, capacitatisque fuerit libellus (On Noah's Arc and on which shape and capacity it was).* While the first reference is obviously to the Bible, the problem is solved by quoting authors from classical as well as from the Christian tradition, so Cicero and Vitruvius are next to Origen and Hugh of St Victor.

The second chapter is *De sublicio ponte Caesaris (The Sublicius bridge in Caesar);* here the authority is Caesar, *De Bello gallico (The Gallic War)*, but Borrel responds also to the recent interpretation of the same problem by Cardano.

The third chapter is *Confutatio quadraturae circuli ab Orontio Finaeo factae (Refutation of the squaring of the circle by Oronce Fine).* Here Borrel addresses his former teacher. At stake is an interpretation of Archimedes in terms of abstract mathematical entities as opposed to concrete numerical approximation.

The fourth chapter is *Ad locum Quintiliani geometricum explanatio (Explanation of a geometrical passage in Quintilian).* Here Borrel mentions and comments on a famous passage of the *Institutiones Oratoriae* dealing with a mathematical problem and criticises the ambiguities of Quintilian's text concerning isoperimetric figures. The passage belongs to Quintilian's first book, where he shows by examples the interest of a mathematical culture for the orator. In fact, Quintilian is the ancient authority who has most explicitly insisted on the need for a mathematical education and proficiency among the competences of a good orator. Given that Renaissance jurists were trained to identify the ideal of their profession with the classical figure of the orator, from Cicero on, they took Quintilian's words for an instigation to develop mathematical studies in their curriculum. Therefore, this chapter of Borrel's *Opera* should be seen in connection with the one devoted to Quintilian in the second part of the *Opera geometrica,* bearing the title *Geometriae cognitionem Iureconsulto necessariam* (The knowledge of geometry is necessary to the jurist). The entire *Opera geometrica,* and especially the second part, explicitly addressed to jurists. It is in fact a proof by examples of Quintilian's idea.

In *Ad problema cubi duplicandi (The problem of the duplication of the cube)* Borrel mentions Archimedes, but also his contemporary Stifel.

De fluentis aquae mensura (On measuring water streams). This chapter has as its main reference Julius Frontinus (40-103), the general who was also famous for his work *De aquaeductu*, who in turn was very aware of Vitruvius work on the topic.

Emendatio figurationis organi a Columella descripti (Correction of the figure of the instrument described by Columella). This chapter concerns the figure of an instrument presented in Columella's *De re rustica* (*On Agriculture*, the main treatise on the topic in the roman in the roman empire)[4].

[4] An edition theoretically accessible to Borrel was Columella *De re rustica libri XII. Euisdem de Arboris liber, separatus ab aliis.* Lyon, Sébastien Gryphe, 1541.

The following chapter is *De libra et statera (On scales and steelyards).* Here the classical reference is Aristotle's *Mechanics.*

Finally we have *De precio margaritarum (On the price of pearls).* The question is also a classical one. Plinius and the contemporary classicist Budé (*De Asse*) are mentioned next to Pacioli and Etienne de la Roche. Interestingly, the problem is solved with the approach of the abacus schools.

The second part of Borrel's *Opera geometrica* bears the title *In iure civili*. The first chapter is *De fluviaticis insulis secundum Ius civile dividendis. Ubi confutatur Tyberias Bartoli (How to divide rivers. Where the work "Tiberias" by Bartolus is refuted).* This chapter is one of the largest and more dense of quotations from authorities, especially juridical, such as Bartolus (1313–1357). The outcome is a way to put a grid pattern on the shape of the island, and to calculate on that basis.

The second chapter is *De divisione fructus arboris in confinio natae (On division of the fruits of a tree born on the border)*. A classical problem of juridical architecture which has been treated by Justinian's *Digest*, mentioned in the text.

The third chapter is on *Geometriae cognitionem Iureconsulto necessariam (Knowledge of geometry is necessary for the jurist.* It contains a problem from Bartolus.

The fourth chapter is *De lege Papiniana, divortio (On the law by Papinianus on divorce).* Here the authority is Ulpianus (170-228), facing the first problem of division of patrimony.

The fifth chapter is *Ad legem Iuliana, si ita scriptum (On the law according to Salvius Julianus' "If it is so written")*. Here Borrel deals with a passage in Julianus' *Digesta*[5] concerning a classical problem for mathematicians and jurists: inheritance. We shall come back to this problem, which is present also in Borrel's main other book, *Logistica.*

The sixth and last chapter is *Ad legem Africani, Qui quadringenta. In secundo capite Qui quadringenta (On the law according to Sextus Caecilius Africanus' "someone who left four hundred")*[6]. Here the question is a legacy under a certain condition.

5. A mathematical book for jurists

Let us understand better the expected audience for this work: jurists. The idea that mathematics is important for jurisprudence is ancient. The two main realms in which this fact was traditionally evident were architectural jurisprudence, including design of buildings and definition of territories by surveying on the one hand and inheritance jurisprudence, including legacies and testaments on the other. The book is in fact devoted to these two realms.

Jean Borrel, as we have seen, was an abbot who had been a student in Paris and had obviously an excellent juridical culture. In particular he knew quite well the history of Roman Law and many of its commentators. Mathematical passages of the juridical classics had received attention in the literature, but not as an independent topic. This work is probably the first devoted entirely to provide jurists with enough of a competence in mathematics to master the mathematical passages of the Classics explicitly concerning them. Finally, it motivated jurists to acquire firmer mathematical skills.

The series of problems collected under the title *Opera geometrica* is quite different from the ones transmitted by the mathematical traditions, for it does not contain either geometrical or arithmetical problems *per se* or related to a mathematical – liberal arts – context, but only in connection with classical literary texts. Likewise, the topics are not related to arts and crafts or commerce either: there are no commercial problems transmitted by the abacus schools, nor the problems transmitted as practical geometry *per se;* when these problems or machines appear, they do so as a part of a classical text.

[5] Julianus was born around 110 and died around 170.

[6] Sextus Caecilius Africanus was a pupil of Julianus. He died also around 170.

Concerning the structure of the *Opera geometrica*, there is no progression in the difficulty of problems, so it seems that there is no general technique to learn and that it cannot be defined as a series of problems, but only as a collection of problems. As a consequence, there is no comparison to be made between the various problems, for they are different by all means. Rather, the structure is given by what seems to be a random order of problems. The only explicit indication about the organization of content is the distinction between the first and the second group of problems, the second stressing a more direct connection to Civil Law and bearing the title *In iure civili* (*Concerning Civil Law*). What all the problems have in common, as we have seen, is the fact of being classical problems, appearing in classical books. Another element in common is that Borrel corrects classical and non mathematical authorities on mathematical points, with some harshness at times. The first purpose here seems to be to show by examples that a sounder knowledge of mathematics is useful in reading ancient sources: this is in fact, at the time, the most common justification for the need of mathematics in school training. At a deeper level, this work deals twice with Quintilian's text: *Ad locum Quintiliani geometricum explanatio* and *Geometriae cognitionem Iureconsulto necessariam.* As mentioned above, Quintilian's long passage on mathematical education as necessary for jurists was a *topos* in the Renaissance. By working on his text Borrel emphasises this thesis, so, the book is supposed to accompany a humanist scholar in the reading of classical sources, but most particularly it is supposed to give instances of the use of mathematics in law, to accompany the jurist in his practice. At best, this book should encourage jurists to study mathematics, having in mind the variety of the cases they have to deal with and the possibilities of mathematics in solving many different problems.

We tend to think that sciences were conceived of as built around trees in a systematic way according to Aristotelian categories or to Ramus's dichotomies: in fact, this is not the case: Borrel's *Opera Geometrica*, as many other books of the time, is built around problems in a non-hierarchical way. There is no progression such as from simple to complex or from concrete to abstract etc.; this work belongs to a *genre* of books which took the problems posed by classical sources as constituting the very skeleton of the new science. This was a sixteenth-century way of privileging classical sources in the shaping of the new scientific agenda. According to this rhetorical strategy, problems were the elements to start with in the construction of a book. This corresponded to the fact that sixteenth-century professors structured teaching around questions, and most particularly questions arising from classical texts. Similarly, a scholar, at various levels of expertise and testing, had to accept the challenges posed by classical texts. There were two main means to get a proper training at school and in textbooks. First, a student was supposed to write in notebooks and to memorize the commonplaces, that is the basic data coming from classical heritage: they are passages chosen because of their value as "rhetorical turns of phrase, dialectical arguments, factual information" (Blair 1992, 541). Second, in books the main tenets of each discipline were presented as structured by questions. In many cases, the practical technique learned at school influenced by dialectical circles was the reduction of texts into their logical *question and answer* structure, whatever the topic dealt with. *Protomathesis* by Fine, for instance, uses this technique, even though not uniformly. By this rhetorical device, old problems and old solutions could be easily provided together with new ones. In other cases, as in Borrel's *Opera geometrica*, the questions are the classical passages themselves. Again, contemporary tools and techniques were mixed with the ancient way of presenting problems and solutions.

The mixture of old and new elements had already been typical of commentaries, from Antiquity to the Middle Ages. But mixture in itself was not necessarily well considered in rhetoric. In fact, It was opposed to variety. So, for commentaries in the Renaissance period, it was important to include all the relevant authors, but putting them with their distinct features in a perspective, in a meaningful whole. We should pay a special attention to the way in which, in these texts, mixture was transformed and assumed as illustration of diversities within a whole.

6. *Varietas* and professions. Series of mathematical problems and professional roles: Logisticians and jurists

Quintilian is the authority who, by his *ars oratoria*. has filtered the reading of other authorities. His impact on the Renaissance was enormous. He contributed to the very idea of *encyclopedia* in the first century, forging an ideal of the orator as a well rounded person, knowledgeable in most fields as the goal of education. This is the ideal adopted by humanists, from Guarino on. Variety, in this context, is mostly the necessity to provide a large enough palette of general culture.

But Quintilian goes further. Given that words (*verba*) should correspond to *res* in order to make a sensible *oratio*, *encyclopaedia* should not be seen as a mere ornament of the *oratio*, but rather as the foundation of a sound discourse. Not only orators should be well educated people, with a vast culture and a sense of responsibility, but they should be capable of having a sense of the order of society, that is of the different roles of professions. This was explicitly seen as part of the task of the good humanist, who needed to receive at least a basic education in the arts and crafts[7]. In practice, this meant to include in textbooks some basic notions and descriptions of the arts and their technical vocabulary. Furthermore, the art was presented in the context of other activities and professions. This is important when we see that Quintilian is also the authority most cited in the *Opera geometrica.* Following the teachings of Quintilian, Borrel's book suggests a kind of variety of the series of problems: this variety is defined by the profession of *jureconsultus,* the humanist jurist. A humanist jurist reads and learns many books: in some of them there are passages including mathematical problems. Furthermore, a jurist needs to solve many practical problems including calculation, mathematical notions and mathematical devices. This is the variety needed as a mathematical culture for a jurist at the time and addressed by Borrel's book.

Borrel in fact mentions explicitly the question of the social status of mathematicians.

People who compute are not all of the same kind. Here the question of variety of problems is posited the other way around, according to the people concerned: the variety of problems for jurists concerns architecture, borders, divisions of patrimony; the variety of problems for calculators is different, depending on their professional and social role. If they make use of Euclidean logistics, tables and algebra, all contents which can be associated to a higher social status than simple accountants, they can be considered logisticians. Before dealing with a problem of inheritance, Borrel writes:

> "One could show by means of an endless amount of examples, both on inheritances and on the ways of dividing inheritances, that it is impossible to give execution to a legal dispositions without geometry. Furthermore, in judgments many things happen that need the help of a geometer in what concerns calculations. For his role is to deal with numbers and figure, which are mixed in Euclid's *Elements*. However the discipline of *numerandi* (counting) subsists without geometry, whereas the opposite is not true. Modestinus calls *calculatores* (calculators) those who own a similar distinct knowledge, whereas the people called them *rationarii* (accountants). Ulpianus calls them *tabularios* (table calculators). But I say that the difference between a calculator and a table calculator is that a calculator constructs his *computationes* (computations) by *calculi* (pebbles, abacus, calculations), whereas the table calculator *supputat* (makes his calculations) using a table, which has been constructed by Pythagoras with much more science and competence. But people do not always keep this linguistic precision. Sometimes table calculators are also called *logistici.*" *(Opera geometrica,* p. 139).

Borrel distinguishes, in our terms, between accountants and mathematicians, the latter being called geometers. *Logistica* is an art of calculating found in Euclid and including Pythagoras' tables, its practitioners are called logisticians, who should not be confused with simple calculators or accountants. Borrel claims that Euclid's *Elements* contain not only geometry proper, but also logistics and even algebra. Geometers are supposed to know all of mathematics, Logisticians devote their efforts to

[7] See my (Cifoletti 2001).

practical mathematics and are supposed to know how to calculate in decimal and sexagesimal systems, that is in human affairs, in practical geometry (survey, machines) and in practical astronomy (ephemerides, calendars, chronology). This explains why the title of this book is *Opera geometrica*: in our terms, this would be *"Mathematical works"*, for most of the problems are geometrical in the sense of dealing with space, but some are arithmetical. To mention an example concerning our argument, in the chapter devoted to the law by Julianus on inheritance, Borrel, after presenting the classical solution, adds:

> "You can also procede differently, in a logistic way, according to the rule which is called rule of proportions."

Borrel will have the opportunity to deal with these distinctions in his book called *Logistica*, published in 1559.

7. Series of problems in *Logistica*: examples in the text, list of cases at the end

In 1559 Jean Borrel published his *Logistica*. The purpose was to write a book of arithmetic and algebra, but with the explicit intention of showing that algebra was deducible from Euclid's *Elements*. The title is justified in the introduction. He writes:

> "The art of computing (*ars computandi*) for human affairs (*usibus humanis*) is not only convenient, but necessary. This, I believe, nobody with a healthy mind will deny. This art was practiced among Latin people with pebbles, which are called *calculi*. From this comes the word *calculum* for computing *(computare)*, and for reasoning (*rationibus*) calculations (*calculi*) and the name *calculator*. But it was generally mental calculation (*computatio*), in which people signified and understood mostly by hand or by finger gesture. To the point that somebody who was a perfect orator but made mistakes in sums by this finger computation was considered ignorant. But for many centuries this way of counting had been considered obsolete, and numeration by letter signs has been cultivated from its primitive beginnings, first by the Greek, who developed also all the other arts. The Greek called this art Logistica. This name means *ratiocinatio* in Latin. Among its authors I find Archimedes (...) and Eutocius."

Many aspect of this introduction could be stressed. For instance, finger counting is discussed, in accordance to Fibonacci and his tradition, but Borrel mentions it in connection with orators, with a reference to Quintilian's *Institutiones oratoriae*, again to the passage concerning the importance of mathematics for the orator.

Another classical authority we encountered already in the *Opera geometrica* is Vitruvius. The term *ratiocinatio* present in the introduction of the *Logistica* is an important notion in Vitruvius, where it means the technical report describing the architectural work just built and the criterion used to build an architectural work, between the architect's project and the concrete situation (book I, 1,1). Just below, Vitruvius repeats that each part has two activities strongly related: the *opus* and the *ratiocinatio eius*. Another authority is mentioned in this introduction: Archimedes is mentioned for the *Arenarius*, while Eutocius for his commentary of Archimedes' *De dimensione circuli.* In both cases, Archimedes is acknowledged as an authority on approximate measurements and the method of exhaustion. *Logistica* is associated to the most eminent classics of measurement and approximation. Therefore, practical arithmetic and practical geometry are associated. Given that measurement and approximation are the ways in which mathematics was used in architecture[8], we have another indication of the fact that Borrel considered *logistica* as a discipline of calculation in human affairs, connected to practical geometry, as

[8] See Albrecht Dürer *Géométrie* (Peiffer 1995).

the latter based on Euclid's *Elements* and addressed to cultivated people, such as jurists and architects. At the beginning of the fourth book of the *Logistica*, Borrel writes:

> "In the previous books we have thrown so to speak the foundations, we now come to the most beautiful part of the work, and that which gives fruits thanks to the exercise of reasoning. Here I present not only logistical questions in numbers, that is those concerning arithmetical problems, but I also apply logistical problems to various things, concerning not only the use in daily life but also the meditation of the mind, or both. For, the uses of rules present themselves hidden in their contexts, or in the nature of the thing, or in the art; consequently, *there is nothing better or more useful, in order to teach them, than by the multiple variety of the investigation*[9]. There is also a great amount of particular traditions, and each opens with its own proper enigmas. On this point, however, I shall not follow the route usually taken by Logisticians (calculators), who fill books of a multitude of questions, often applying the same sort of commercial rule as if it was different. And they arrive even to teach diligently how to fraud in merchandises, impostures and different sorts of usury. I shall not follow this abuse, but I shall show the sorts of invention which are necessary and in which there is a point of more subtle sagacity, a product of the art, by means of different questions instead of by means of many questions. For, I teach to Logisticians, not to merchants. The Logistician, after having practiced in these questions, will make use of his art very easily, in whatever field to which he will devote himself. Insofar as anybody is valiant by nature or by application, he will find here matter in which he will have to stretch his mind's muscles and cleverly entertain his skills."

In this rich passage Borrel makes clear that this book is for Logisticians, that is the cultivated practitioners: they could be master of arts, if by the arts of the *quadrivium* he meant, as his teacher Fine, the practical side of mathematical disciplines. They could also be cosmographers, people calculating horoscopes, almanacs and calendars, possibly in religious orders as in the case of Borrel. Finally medical doctors or jurists could also be mathematical practitioners in this sense, doctors because of horoscopes and jurists because of architecture and inheritances. Practical mathematics had become a realm of activity for professionals in various disciplines, and Borrel had produced two books in Latin which could promote this change.

But this passage also raises another point, the question of the *varietas* of the series of problems proposed in the *Logistica*. Borrel, like many other authors of arithmetical and algebraic books, recommends his own work for having found the ideal number of problems presented with their solutions. If there are too few, the series doesn't show the potential of the mathematical tools, if there are too many, they create confusion. Leonardo Fibonacci first, Luca Pacioli later, had produced two "summae", their purpose was to include as much as possible, they transmitted an impressive amount of problems presented as examples in connection with the mathematical tool or rule in the main text, but also as a proper series at the end of the book. In the first three books of Borrel's *Logistica* there are no exemplary problems, in the third book only single examples for the rules of solutions of second degree equations. But in book IV, after this powerful introduction, Borrel included 92 questions in the same book, dealing mostly with commercial problems such as salary and the price of wheat.

Question 60 is the problem of inheritance by Julianus already presented in the *Opera geometrica*. But Borrel modified the treatment of this problem by taking his readership into account. In *Opera geometrica* the question had been presented as it appears in Julianus' comment to the Code, then Julianus' text was given *in extenso*. This was followed by a few examples, and finally various cases were developed and examined. Instead, in the *Logistica*, only the basic case is given, after a short version of Julianus' passage. The problem is reduced to its mathematics, that is in terms of fractions to be given to a son, a widow and a daughter. It is solved in general and then a numerical example is given, but only one, and without presenting various possibilities. Notice that to have an inheritance problem in an algebra book is not a novelty but was not generally common. Al-Khwārizmī had some, Fibonacci had none.

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[9] Non aliter melius, aut utilius doceri potest, quam ipsa vestigationis varietate multiplici. (*Logistica*, p. 197) The italics are mine.

The *Logistica* has also book V, which consists of a series of 67 problems. The topics dealt with are, as in book IV, mostly commercial, but not exclusively. This book shows a variety of problems solved by algebra, with a conscious attempt to reduce them to the right amount and to make them gradual.

8. Conclusion

We have accepted the challenge of looking at series of problems in sixteenth-century mathematical books and chosen Oronce Fine and his disciple Jean Borrel as the authors.

Oronce Fine's contribution has been to present commercial problems as Euclidean problems on proportions, as well as to replace the university *quadrivium* by four practical disciplines, including clock building. The two books by his disciple Borrel reflect the view on mathematics promoted by Oronce Fine: we can recognize mathematics in every aspect of the world.

Furthermore, Borrel is very concerned with distinctions in the human world. In particular, he wants to stress the role of a new category of mathematicians, specialized in the practical disciplines he taught at the Collège Royal. They are called geometers and they deal with the four liberal arts of the *Protomathesis.* Among them there are the logisticians, who deal in particular with the practical or computational side. Through *varietas* he is able to illustrate the multifarious mathematical competence required for jurists as well as the validity of his methods (notation and presentation) in algebra, where algebra should be understood as a learned discipline deriving from Euclid. Variety in the *Opera geometrica* and in the *Logistica* corresponds to redefinition of professional roles according to Borrel's program.

The texts by the two authors examined here show a use series of problems as varieties at two levels: the level of the presentation of examples for a rule, mostly in the main part of the text, and the level of a list of cases at the end on the book. Variety is important at both levels, as this appears also in Serra's and Coquard's contributions. As we anticipated in the introduction indeed, in our research group we have some examples from earlier and later collections of mathematical problems. It will be therefore possible to compare our cases to two more mathematicians, Gemma Frisius and Simon Stevin. Gemma Frisius is a contemporary of Oronce Fine, cosmographer and connected to the King. Furthermore, Gemma Frisius is, as much as Oronce Fine, a great promoter of mathematical arts: his workshop was the most renowned for maps and globes. Not surprisingly, the series of problems he includes in his arithmetic is also the illustration of the *varietas* of fields in which arithmetic can be used. Some problems are classical, some others are from surveying, Frisius' main field, others are specific to cosmography, the art that developed a mathematics art connecting the Earth with other planets.

In Stevin we find one of the most mature statements of the *varietas* of uses of mathematics in nature. Variety of this use is not only described at length and great erudition by Stevin, but also exemplified in many activities he devoted himself to, and in the books he wrote on the corresponding subjects. This will appear clear in Coquard's article on Stevin, where the stress on the universal tool of dialectic should not make us forget that such a tool can be universal only because there is a universe, a connected whole to be treated by a common tool. Stevin is the mathematician who got closer to the ideal expressed by Viète "To solve any problem". Thereby, he completed the program expressed by Fine in the passage we mentioned at the beginning, the rule of three is such that almost no problem cannot be solved by it.

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