The price competition simulation at the trading market in the presence of electronic and traditional trade

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Abstract. In the present work, an attempt has been made to apply economic and mathematical methods for the simulation of electronic trading market operation based on price competition between e-trade companies and traditional trade enterprises. The developed price competition model based on the concept of symmetric product differentiation. The results obtained in the present investigation demonstrate that in a mixed strategy, firms sell products at different prices, depending on the price strategy or the volume release strategy. The company that sets the volume, sells more, but at a lower price than its competitor which sets prices. The influence of strategic output exceeds price influence. Thus, the company that sets prices, falls into an unfavorable situation and receives lower profits compared with its competitor with the strategy for the volume of production. The company that has decided to introduce electronic trading technology initially will bear losses.

1 Introduction

Modern world economic conditions, economy globalization, acceleration of market development processes, information technologies, sociopolitical factors demand from the trading enterprises new approaches to consumer demand and supply formation, the development of adequate methodological solutions and tools in the field of management of the trade activity, especially it concerns new forms of trading, such as e-trade [1]. Companies today are working in a turbulent environment facing continuous change because of hyper-competition, changing demands of customers, regulatory changes and technological advancement [2].

E-trade, as compared to traditional business, has substantial advantages. In particular, the use of new electronic communication channels significantly reduces costs related to organization and support business infrastructure, and the possibilities of e-commerce allow re-designing business strategy at any moment. The functions of modern e-trade market mechanisms are not limited by a small number of fields, such as, for example, automated reservation systems in tourism, financial sector operations and electronic supermarkets in the retail sector, the range of today's e-trade markets is far larger according to the range of applications. New products and services and innovative trade mechanisms have appeared on e-trade markets: communications that facilitate news autsourcing, ratings, forecasts, services and the implementation of innovative ideas have been developed. As a result, e-trade has become a very profitable form of relationship with the buyer who is developing, not seeking to replace it with other forms of trade contacts and connections. Economic properties and peculiarities that have emerged in the process of

becoming e-trade have not only provided it with the possibility of a competitive global existence in the world of modern global business, but also created the prerequisites for quite optimistic forecasts of its future [3].

New features of computer and information technology affect both the production and distribution of goods and services. E-trade allows firms and companies to sell their products without the use of traditional sales channels [4]. The use of electronic trading changes both the production process and the sales process in two main directions [5, 6]: 1) electronic trading reduces the time between production and sales, as flexible technologies allow firms to create goods and services in accordance with their demand; 2) restrictions on production are decreasing, as new technologies allow for almost unlimited duplication of informational products with extremely low costs.

Effective management of e-trade development, as well as the processes of the economy informatization as a whole, is impossible without a full and comprehensive economic and mathematical research of the whole complex of problems, including, on the one hand, the activities of enterprises in the field of electronic commerce, and, on the other, the use of information technology in enterprises and organizations of all branches of economy [7-9]. Due to the wider introduction of electronic trading technologies, the scientific development of methods of applying economic and mathematical methods in the research of the state and prospects of electronic trading development has significantly intensified and, most importantly, has increased their demand for practical work. The application of economic and mathematical methods to solve many specific problems can increase the efficiency

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of economic entities that actively use electronic trading in their activities. In general, we are talking about a toolkit developing that can be used to analyze the complexities of e-commerce, and which will be the basis for developing effective mechanisms for effective governance and decision-making [10-14].

In the presented research, an attempt has been made to apply economic and mathematical methods for the model development of electronic trading market operation based on price competition mechanisms between electronic trading actors. Suppliers who have already adopted electronic technology, act as competitors for the price, because they may not link themselves with the volume of output. Other firms that continue to use the usual technology of production and sales should increase or decrease their production capacity before starting production. Thus, they compete in terms of output. Considering the various reasons for competition in price and quantity while moving towards the electronic trading and trading firms set of strategies in the oligopolistic market, it is worth mentioning the following provisions. A trading company can choose a strategy for output, if it needs to make a managerial decision on the volume of production before or after the production commencement, in which case the company must make irreversible investments. The price strategy and the product-oriented strategy can be interpreted as extreme cases of an elastic or inelastic output function and depend on the different angles of inclination of the marginal cost function [15]. While the extremely low costs lead to price strategies, product launch strategies meet the high marginal costs associated with inflexible technologies.

2 Results and discussion

2.1 Model

Prices play an essential role in any market and understanding how they are fixed is a fundamental part of the Economic Science. However, complex problems such as social networks or the launching of new digital platforms can set new challenges in understanding how those prices are fixed [16].

Oligopolistic markets are known to be associated with a high degree of price and output rigidity. This is due to mutual interdependencies among firms in the market with regard to price and production [17]. An oligopoly represents a market where power is concentrated among a small number of firms. The exact number of firms is not important; what matters is that a few firms produce most of the market's output. The barriers to entry for an oligopolistic market are high as a result of the scale of the incumbent firms and the competitive advantages that are derived from that scale. Moreover, unlike perfect competition, monopoly, and monopolistic competition, it is most useful to study an oligopoly in terms of the interdependence and rivalry among its firms. Given that the primary characteristic of any oligopoly is the interdependence and rivalry among its firms, any firm in an oligopoly that ignores the

critical nature of its interdependence with its competition places its share of the market and its capacity for profits at risk [17].

Let us consider a market in which part of firms moved to e-trading technology, while others use traditional, that is, there is a market with firms competing for the price and volume of manufactured products. As a base one we apply the concept of symmetric differentiation of goods [18]. In the assumption of profit maximization [19], we will construct the general curves of responding firms that choose a pricing strategy or a strategy focused on the issue for determining the equilibrium, and consider the impact of switching to e-trading to choose a strategy firm, in particular: how the change of technology will affect the own production of the company, its competitors, market efficiency and investment.

Let's construct a model that uses the concept of symmetric product differentiation. In this case, N - is the number of firms using linearhomogeneous technology that creates individual and constant C_i – expences – for the production of a limited variety of symmetrically differentiated product x_i, that is sold at a price p_i. The functions of demand for products of the company are the typical consumer with linear quadratic utility [20]:

$$u(x_1,...,x_N) = \sum_{i=1}^{N} x_i - \frac{1}{2}\left(\sum_{i=1}^{N} x_i^2 + b\sum_{i=1}^{N}\sum_{i \neq j} x_i x_j\right) - \sum_{i=1}^{N} p_i x_i \quad (1)$$

where the inverse functions of demand:

$$p_i = 1 - x_i - b\sum_{i \neq j} x_j \quad (2)$$

Parametric variable b evaluates the degree of substitution between any two products. If $b=1$, the products are complete counterparts, while all firms make different products if $b=0$. We assume that n is the number of firms ($i=1, ... , n$) that follow the strategies in terms of production volume, that is specify the volume of output, while $\{N\text{-}n\}$ is the number of firms ($i= n + 1, ... , N$) that follow the pricing strategy. The distribution of external and internal prices and volumes of output, leads to such demand for the company j, which sets the volume of production, and the company k, which sets the price, respectively:

$$p_j = 1 - b\sum_{i=1,i \neq j}^{n} x_i - b\sum_{i=n+1}^{N} x_i - x_j = 1 - b\sum_{i=1}^{n} x_i - b\sum_{i=n+1}^{N} x_i - (1-b)x_j \quad (3)$$

$$p_k = 1 - b\sum_{i=1}^{n} x_i - b\sum_{i=n+1,i \neq k}^{N} x_i - x_k = 1 - b\sum_{i=1}^{n} x_i - b\sum_{i=n+1}^{N} x_i - (1-b)x_k \quad (4)$$

From the equation (4), we obtain the direct demand function for production:

$$x_k = \frac{1 - b\sum_{i=1}^{n} x_i - b\sum_{i=n+1}^{N} x_i - p_k}{1-b} \quad (5)$$

By summing up the (*N-n*) demand function of firms that use price strategies and making the corresponding transformations, we obtain the total output of products made by firms which set the price:

$$\sum_{i=n+1}^{N} x_i = \frac{(N-n)\left(1-b\sum_{i=1}^{n}x_i\right)-\sum_{i=n+1}^{N}p_i}{1+b(N-n-1)} \quad (6)$$

Substituting the formula (6) into the equations (3) and (5) and marking the choice of price $\sum_{i=1+n}^{N}p_i$ and output volume $\sum_{i=1}^{n}x_i$ through R and X, respectively, we obtain the functions of demand in this form:

$$p_j = \frac{1-b}{1+b(N-n-1)} - (1-b)x_j - \frac{b(1-b)X}{1+b(N-n-1)} + \frac{bP}{1+b(N-n-1)} \quad (7)$$

$$x_k = \frac{1}{1+b(N-n-1)} - \frac{p_k}{1-b} - \frac{bX}{1+b(N-n-1)} + \frac{bP}{(1+b(N-n-1))(1-b)} \quad (8)$$

Thus we obtained a system of two equations that can be solved on the basis of balance state conditions [20, 21]. The market activity of firms j with strategy on the volume of production, so firms k with the pricing strategy designed to maximize profits Pr [20]:

$$\max Pr_j(x_j, P, X) = p_j(x_j, P, X) - C_j x_j \quad (9)$$

$$\max Pr_k(p_k, P, X) = p_k x_k(p_k, P, X) - C_k x_k(p_k, P, X) \quad (10)$$

That is, the company j is looking for the opportunity to maximize its own profits by choosing the volume of output x_j, taking for it the total output of products manufactured by competitors ($X-x_j$). While company k determines the influence of the decision on its own price on aggregate P, assuming instead of it the established X and the established aggregate prices of competitors ($P-p_k$).

Solving the equations (7) and (8), the substitution function [22] $\mu_i(X, P)$ for company j and for company k will look like:

$$x_j = \mu_i(X,P) = \frac{1-b-b(1-b)X+bP-(1-b+b(N-n))C_j}{(1-b)(2-b+2b(N-n))} \quad (11)$$

$$p_k = \mu_k(X,P) = \frac{1-b-b(1-b)X+bP-(1-2b+b(N-n))C_k}{2-3b+2b(N-n)} \quad (12)$$

Unlike the reaction function, μ_i does not describe the optimal response of the market participant (x_i, p_i) to the strategic choice of its competitor (that is $X-x_j$ or $P-p_k$, respectively), but describes the reaction to the total X or P, which include its own strategic level.

Using the fact that in the [23] the general reaction corresponds to the aggregate strategic choice, herewith $\sum_{i=1}^{n}\mu_i(X,P) = X$, and $\sum_{i=n+1}^{N}\mu_k(X,P) = P$, we can find a strategic issue and a total strategic price:

$$X = \frac{(1-b)n+bnP-(1-b+b(N-n))\sum_{i=1}^{n}C_j}{(1-b)(2-b+b(2N-n))} \quad (13)$$

$$P = \frac{(1-b)(N-n)(1-bX)}{2-3b+b(N-n)} + \frac{(1-2b+b(N-n))\sum_{i=n+1}^{N}C_k}{2-3b+b(N-n)} \quad (14)$$

2.2 Simulation

The decision on the total output of products X depends on aggregate prices and turnover. Accordingly, equations (13) and (14) can be regarded as collective reaction functions. Figure 1 shows the calculated by the formulas (13) and (14), depending on the reaction of firms that set the price and volume of output. It should be noted that in order to describe the mechanisms of the e-trade markets operation, in this case, the calculations are performed by dividing all market participants into the corresponding number of market participants with a start-up on the production volume and price strategy. Thick (black) lines describe the behavior of market participants with two firms with a strategy on the production volume and three firms with a pricing strategy at zero marginal costs and $b=0.5$.
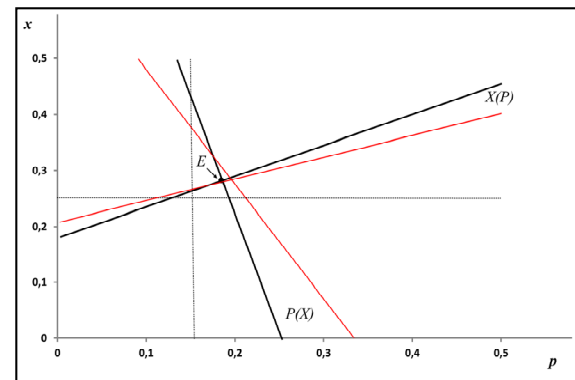

Fig. 1. Dependence of the market participants general reaction with different trading strategies.

The intersection of the lines (point E), in our case, indicates a mixed equilibrium. Thin (red) lines indicate the behavior of market participants $n=3$ and $N-n=2$. Dotted lines on the graph show the results for firms with market participants with the same strategies, they are designed for comparison with mixed strategies. Figure 1 illustrates the relationship between the strategic aggregate price and aggregate issue: if prices rise, then the firms, which set prices, implement a less aggressive strategy, and market participants with a strategy for the volume of production will react to an increase in output. As X grows in P, it becomes a strategic complement to the aggregate price of P. On the other hand, the firms, which set prices, will lower their prices if the firms, which set the output, act more aggressively. Consequently, the firms' prices with price strategy is a strategic supplement to the cumulative output X.

For the system of equations (13) and (14) there is an appropriate analytical solution. It is seen that both of the response functions are linear with respect to P i X, that is, there is a single solution of this system of equations:

$$P^* = \frac{1}{(1-b)z}\left((N-n)(1-b)(2-b+2b(N-n))+\alpha\sum_{i=1}^{n}C_i+\beta\sum_{i=n+1}^{N}C_i\right) \quad (15)$$

$$X^* = \frac{1}{(1-b)z}\left(n(1-b)(2+b(2N-2n-1))+\gamma\sum_{i=1}^{n}C_i+\delta\sum_{i=n+1}^{N}C_i\right) \quad (16)$$

where

$z=(1-b)(4+b(6N-4n-4))+b^2(2N(N-n)-N-1)$,
$\alpha=b(N-n)+b^2(N-n-1)$,
$\beta=(1-b)(2+b(4m-3n-3))+b^2((2N-n)(N-n)-(N+1))$,
$\gamma=(1-b)(2+3b(N-n-1))+b^2((N-n)^2-(N-n))$,
$\delta=bn+b^2(N-n-2)$.

Values α, β, δ, γ, and z are positive for any $n<N$, $n \in N$, and $0<b<1$.

Substituting (15) and (16) in (11) and (12), we obtain the balance state [23, 24] of the price and output for company j, which uses the strategy for the volume of output, and the company k, which sets prices:

$$x_j = \frac{(1-b)\left[4+8b(N-n-1)+b^2(2(N-n)-1)(2(N-n)-3)\right]}{z(1-b)(2-b+2b(N-n))} \quad (17)$$
$$+\frac{\varepsilon\sum_{i=1}^{n}C_i+(\varepsilon-b^3)\sum_{i=n+1}^{N}C_i-\eta C_j}{z(1-b)(2-b+2b(N-n))}$$

$$p_j-C_j = \frac{(1+b(N-n)(1-b))\left[4+8b(N-n-1)+b^2(2(N-n)-1)(2(N-n)-3)\right]}{z(1+b)(N-n-1)(2-b+2b(N-n))} \quad (18)$$
$$+\frac{(1+b(N-n))\left[\varepsilon\sum_{i=1}^{n}C_i+(\varepsilon-b^3)\sum_{i=n+1}^{N}C_i-\eta C_j\right]}{z(1+b)(N-n-1)(2-b+2b(N-n))}$$

$$x_k = \frac{(1-b)\left[4+8b(N-n-1)+b^2(2(N-n)-1)(2(N-n)-3)\right]}{z(2-3b+2b(N-n))} \quad (19)$$
$$+\frac{\varepsilon\sum_{i=1}^{n}C_i+(\varepsilon-b^3)\sum_{i=n+1}^{N}C_i-\eta C_k}{z(1-b)(2-b+2b(N-n))}$$

$$p_k-C_k = \frac{(1+b(N-n-2)(1-b))\left[4+8b(N-n-1)+b^2(2(N-n)-1)(2(N-n)-3)\right]}{z(1-b)(1+b(N-n-1))(2-3b+2b(N-n))} \quad (20)$$
$$+\frac{(1+b(N-n-2))\left[\varepsilon\sum_{i=1}^{n}C_i+(\varepsilon-b^3)\sum_{i=n+1}^{N}C_i-\eta C_k\right]}{z(1-b)(1+b(N-n-1))(2-3b+2b(N-n))}$$

where $\varepsilon=(1-b)[2b+b^2(4N-4n-3)]+b^3[2(N-n)^2-(N-n)]$, $\eta=(1-b)[4+b(10N-8n-8)+b^2(4(2N-n)(N-n)-8(N-n)-3(N-1))]-b^3[N(2N-n)^2-(N-n))-(N-n)]$. The coefficients ε and η are positive for all admissible N, n, and b.

The analysis of the formulas (17-20) shows that the output and increase in production costs will decrease, with the increase in the number of companies in the market (see increase z). The increase in the cost of firm's own production has the same effect, while the rising costs of competitors lead to the opposite: the own increase and the issue in this case are increasing.

Using the obtained results, we can approach to the research of the strategic impact of electronic trading on market functioning. The use of economic and mathematical methods to describe the mechanisms of development and electronic trading markets operation is based on the point of view that the main factors and trends of past periods persist and during the periods of development of the investigated area of economic activity, that is, there is a possibility to reasonably take into account the direction of future changes not only qualitatively but quantitatively.

The e-trading markets operation is implemented in accordance with the laws of a market economy, so when attempting to assess the future reaction of market participants to one or another process that occurs within the framework of the electronic trading markets operation, causal relationships should be considered in the form of rules, regularities and generally accepted mechanisms of decision-making in the processes of market operation. It is clear that in this case there is a certain inertia of social and economic systems.

To simplify the analysis, assume that production has no expenses, regardless of which technology is used, that is: $C_j=C_k=0$. Thus, any impact of costs associated with other technologies is ignored. In fact, technological innovation can change the structure of the company's costs, but reducing costs by changing technology is not the goal of this study.

Consider the market situation of the market participant with the strategy for the volume of production and compare it with the position of the company using electronic trading technology, and act as a company that sets the price.

The question arises: which of the firms setting the price or volume of output, appears in a profitable strategic situation. First of all, we will analyze the consequences of technological changes, that is, the transformation of the company into a strategy for the volume of issue in the market participant with the price strategy, in particular, determine whether it is profitable to introduce electronic trading technology from the company from a strategic point of view? Thus, the company must take into account the impact of its own technological innovation on market participants. After technological changes, there is in one traditional supplier less on the market, but another company is added to the electronic trading market.
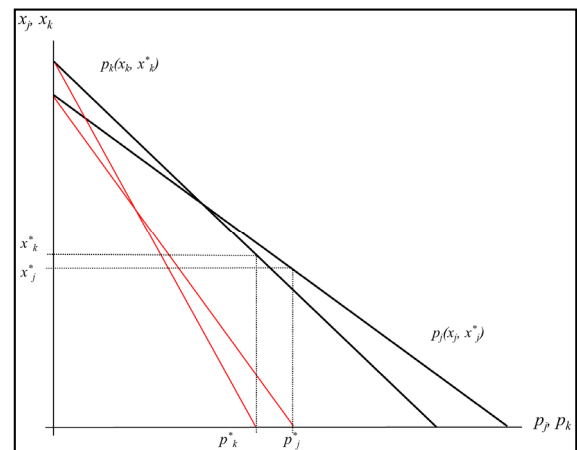

Fig. 2. Differences in demand between company j, which uses the strategy for the volume of production, and company k with the price strategy.

In a mixed strategy [25], market prices set by market participants with a price strategy are higher than the prices of market participants with a strategy for output at equal low (zero) marginal costs. Figure 2 shows the difference in the demand of two types of market participants.

Analytical calculations presented in Figure 2, was made on the basis of formulas (17-20); in the calculations, the emphasis is was made on the company's transition from traditional technology to the price strategy with electronic trading technology. In addition, the number of companies with a strategy for the production volume contains one less market participant, compared with competing firms which set prices. For this reason, the demand function is more elastic for companies with a strategy for volume output. If this company acts more aggressively, it can capture all the demand from the firm, moving to electronic trading with the price strategy. As a result, the marginal profit from lowering the price will be higher for companies with a strategy for output, and they will sell products at lower prices rather than their competitors with the price strategy. For confirming this conclusion, let us find the relation between equations (20) and (18) with the same marginal cost $C_j=C_k=0$, which shows the relationship between the prices of goods of two firms operating on the market with different trading strategies:

$$\frac{p_k}{p_j}=\frac{(1+b(N-n-1))(2-b+2b(N-n))}{(1+b(N-n))(2-3b+2b(N-n))} \tag{21}$$

The right-hand side of the ratio has the form $(AB-b)/(AB-2Ab)$, where $A=1+b(N-n)$, $B=2-b+2b(N-n)$, $b>0$. The numerator exceeds the denominator, since $B<2A$. Hence $p_k>p_j$, that is, market participants with a strategy for the release sell products at lower prices than firms with a price strategy.

Now compare the marginal revenue of firms with market participants with different strategies. Using equations (9), (10), (21) and arguments of equations (17-20) we obtain:

$$Pr_j=\frac{(1-b)(1+b(N-n))}{(1+b(N-n-1))}x_j^2 \tag{22}$$

$$Pr_k=\frac{(1+b)(N-n-2)}{(1-b)(1+b(N-n-1))}(p_k-c_k)^2 \tag{23}$$

$$\frac{Pr_k}{Pr_j}=\frac{(1+b(N-n-2))(2-b+2b(N-n))^2}{(1+b(N-n))(2-3b+2b(N-n))^2} \tag{24}$$

The ratio (24) has the form: $((A-2b)B^2)/(A(B-2b))^2$. These data indicate that the denominator is greater than the numerator, if $2Ab-B(2A-B)>0$. This condition is always satisfied with positive A, B, and b, since $B<2A$ and $2A-B=b$. Thus, market participants with a strategy focused on the volume of production receive more profit compared with firms with the price strategy: $Pr_j>Pr_k$.

The conducted study of the e-trading market operation leads to the following results:

1) In a mixed strategy, firms sell products at different prices, depending on the price strategy or the production volume strategy. The company that sets the production volume sells more, but at a lower price than its competitor which sets prices;

2) While analyzing the enterprise economic activity it is necessary to take into account that the strategic output influence exceeds the pricing influence. Thus, the company that sets prices falls into an unfavorable situation and receives lower profits compared to its competitor with the strategy for the volume of production.

It is also worth mentioning that the company with a price strategy receives lower profits, but sets higher prices than a competitor with a strategy for the volume of production, so its volume of sales is lower.

3 Conclusion

On the basis of results obtained in the present investigation it seems justified to conclude that firm, which has decided to introduce an e-trade technology it will initially incur losses. It is necessary to consider that changing their own technologies affect the overall market structure: number of firms, which set prices, increases to (N–n+1), at a time when the number of firms with a production volume strategy in the market is reduced to (n–1). However, the influence of strategy changes on other companies that set prices and on the operation of e-trading market as a whole is not entirely clear, the result of close substitutes ($b>2/3$), at the same time, sales of competitors with the production volume strategy in this case are being reduced. Through a feedback effect of enhanced aggressiveness of firms which set the prices – there is a significant impact on the participants, which establish the production volume. The decline of the number n may even lead to higher profits of firms with pricing strategy, while firms with a production volume strategy will receive less income. This raises the question – do consumers benefit from the introduction of electronic trade? At least this model gives a positive answer on this question.

General decline in prices caused by changes in technology of trading, loosening the restraints of a typical consumer's budget. In this regard, real consumer welfare increases. While the firms that implemented e-trade technology, get a strategically disadvantageous situation, consumers will benefit from the introduction of the new electronic production and marketing. In such a situation, market efficiency increases whereas the price of allowances at zero marginal costs decrease.

Further research should endeavour to a complete and comprehensive economic and mathematical research of the whole complex of problems, including, on the one hand, the activities of enterprises in the field of e-trade, and, on the other, the use of information technology in these enterprises and organizations. In general, we are talking about a toolkit developing that can be used to analyze the complexities of e-trade, and which will be the basis for developing mechanisms for effective governance and decision-making at the e-trade enterprises level.

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