

Analytic Hierarchy Process in Multiple-Criteria Decision-Making: A Model Example

Anna Siekelova^{1,*}, Ivana Podhorska¹, Jorma J. Imppola²

¹ University of Zilina, Faculty of Operation and Economics of Transport and Communications, Department of Economics, Univerzitna 8215/1, 010 26 Zilina, Slovakia

² Seinäjoki University of Applied Sciences, Business and Culture, P. O. Box 412, FI-65101, Seinäjoki, Finland

Abstract. Managers have to make decisions several times a day. The decision-making process can be defined as an essential activity realized by managers every day. Decisions can be implemented intuitively, or by using relevant decision-making methods. This depends on the nature of the decision, as well as the intensity of its possible future effects. The theory of decision-making can be defined as a relatively young discipline. It can be stated that decision-making is no longer an intuitive process. Most decision-making situations are of a multiple criteria character. In this contribution, the authors focus on multiple-criteria decision-making, to which several methods can be applied. In the practical part, the authors use Saaty's method, also known as the Analytic Hierarchy Process. Saaty is considered to be the most important researcher dealing with the issue of multiple-criteria decision-making. The set multiple-criteria decision-making problem was to choose one business partner out of eight under consideration. The decision-making criteria included selected financial indicators and non-financial criteria. The aim of the contribution is to use the Analytic Hierarchy Process to assess potential business partners and to select an optimal candidate.

Keywords: multiple criteria; decision-making; Analytic Hierarchy Process; AHP

1 Introduction

Decision-making is described as one of the most important activities performed by business managers, and is also referred to as the core of management. The decision-making process can be defined as a set of activities that leads to the resolution of a decision-making problem that involves at least two meaningful alternatives, of which the selected one offers the best result with respect to the set goal, as well as the possibility of its implementation [1]. There are two sides to the decision-making process, namely [2]:

1. *The content side of decision-making* - this reflects the differences of individual decision-making processes.
2. *The formal-logical (procedural) side of decision-making* - this expresses that individual decision-making processes have certain common features, regardless of their content.

The formal-logical side of decision-making is a precondition for claiming that there is a framework procedure for resolving a decision-making situation with different content. It is the basis for decision-making theory, which aims to create appropriate procedures (methods) for solving decision-making situations with the highest degree of universality. One of the biggest issues that decision-making theory deals with is the issue of multiple-criteria decision-making. The beginnings of multiple-criteria decision-making (hereinafter "MCDM") can be seen in Benjamin Franklin's decision-making process. He used a simple paper system for the decision-making process for important issues. On one side of the sheet of paper, he wrote the arguments in favour of a decision; on the other side, he wrote the arguments against the decision. Gradually, he deleted the arguments on each side which were of relatively equal importance. When all the arguments on one side were deleted, the side which had remaining arguments was the side of the argument that he would support. In 1785, Marie-Jean-Antoine-Nicolas de Caritat published "*Essay on the Application of Analysis to the Probability in Majority Decisions*". The most famous results of this study is the *Condorcet (voting) Paradox*. The development of studies dealing with multiple-criteria decision-making accelerated during the 1980s and early 1990s. Francis Edgeworth, a philosopher and political economist, significantly contributed to the development of statistical methods. He was the first who applied formal mathematical concepts to decision-making. The basis of modern MCDM theory is Pareto-optimality, also known as the concept of efficiency by Vilfredo Pareto. Frank Ramsey published an essay "*Truth and Probability*" dealing with the decision-making process with uncertain outcomes. It was the basis for later published study "*Foundation of Statistics*" by Leonard Savage. Studies by John von Neumann, Oskar Morgenstern, Gerard Debreu, as well as John Nash have become famous. They dealt with utility theory and influenced modern

* Corresponding author: anna.siekelova@fpedas.uniza.sk

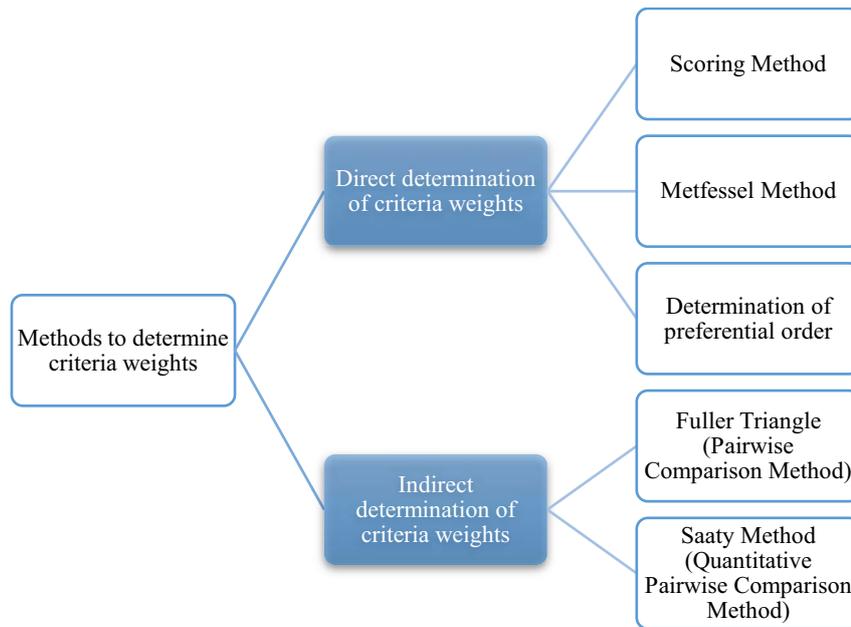


Figure 1. Methods used to determine criteria weights

Source: Authors.

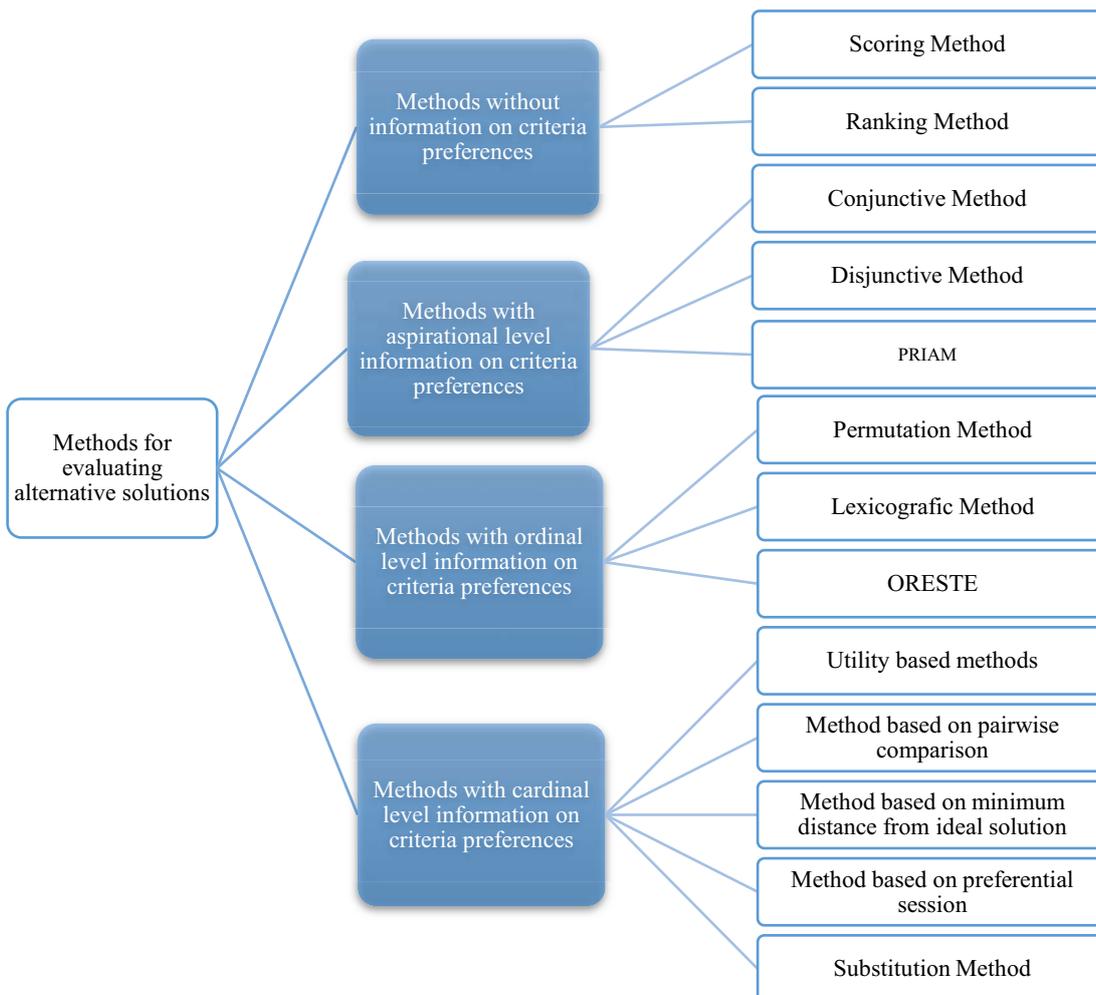


Figure 2. Methods used to evaluate alternative solutions

Source: Authors.

In this contribution, the authors focus on the application of the Analytic Hierarchy Process (hereinafter “AHP”) on a model example of a MCDM problem. The AHP method is a pairwise comparison based methodology.

2 Methodology

AHP belongs to those method with a cardinal level of information on criteria preferences based on a pairwise comparison. In the 1970s, the principles of analytical thinking led to the development of a useful model for the quantitative solution of MCDM problems, also known as Analytical Hierarchy Process, by American professor Thomas Saaty. Saaty is considered to be the most important researcher dealing with the issue of MCDM. AHP consists of the decomposition of complex MCDM problems into partial components, which create the hierarchical structure of the problem. Subsequently, Saaty's quantitative pairwise comparison procedure is applied to each level of the hierarchical structure. Saaty and Kearns [13] distinguish three stages of the AHP method, namely:

Hierarchy

Priority

Consistency

The *hierarchy* of the AHP method usually consists of the goal, the alternatives for achieving the goal and the criteria that apply to the individual alternatives of the goal. It is a so-called three-level hierarchy, whereby the peak is the goal (the 1st level), followed by the criteria (the 2nd level), and lastly, the alternative solutions to the problem (the 3rd level). In specific cases, it is possible to divide the criteria into sub-criteria, i.e. a so-called four-level hierarchy.

In the AHP, priorities are used for the pairwise comparison of criteria, as well as for the pairwise comparison of the alternatives. Professor Saaty set the following scale for priority description.

“1” Both compared elements (criteria/alternatives) contribute equally to achieving the goal.

“3” Opinions and experiences gently prefer one element over another.

“5” Opinions and experiences strongly prefer one element over another.

“7” Opinions and experiences very strongly prefer one element over another.

“9” Opinions and experiences completely prefer one element over another.

It is also possible to use “2”, “4”, “6”, or “8” to express an intermediate level of preference.

The consistency can be defined as follow. If the element k_i is s_{iq} - times more important than the element k_q , and the element k_q is s_{qj} - times more important than the element k_j , it should be able to state that k_i is $s_{ij} = s_{iq} * s_{qj}$ -times more important than k_j . In practice, it is very difficult to meet this condition [14].

The AHP method can be divided into the following steps:

Define the problem to be solved

Choosing a business partner was defined as the MCDM problem. A selected company operating in the construction industry is looking for a new business partner. Eight business partners are under consideration. For the purposes of this contribution, the company, as well as potential business partners, are identified as follows: the company choosing the business partner is marked as *Company 1*, potential business partners are marked as PBP 1 – 8. In making its decision, Company 1 takes into account several financial indicators and non-financial criteria.

The selected and calculated financial indicators are presented in Table 1. The selected non-financial criteria were previous experience with a business partner, the length of time the business partner has been operating on the market, the costs of delivering materials, entries in the credit register or register of debtors.

Table 1. Selected financial indicators used to describe potential business partners

Financial indicator	Calculation
Quick ratio	$\frac{\text{Financial accounts} + \text{Short – term receivables}}{\text{Short – term foreign resources} + \text{Short – term accrued liabilities}}$
Short–term debt ratio	$\frac{\text{Short – term foreign resources} + \text{Short – term accrued liabilities}}{\text{Total assets}}$
Credit indebtedness	$\frac{\text{Credit} + \text{Loans}}{\text{Total assets}}$
Days sales outstanding	$\frac{\text{Trade receivables}}{\text{Revenues from sales of goods, products and services}}$
Days payable outstanding	$\frac{\text{Trade payable}}{\text{Total cost}}$
EBT in three years	Earnings before taxes
Return on assets	$\frac{\text{Earnings after taxes}}{\text{Total assets}}$
Return on sales	$\frac{\text{Earnings after taxes}}{\text{Sales}}$
Indicator characterizing the imminent decline of the company	$\frac{\text{Equity}}{\text{Liabilities}}$

Source: Authors.

1. Develop hierarchical framework

The three-level hierarchical framework was defined (see Figure 3).

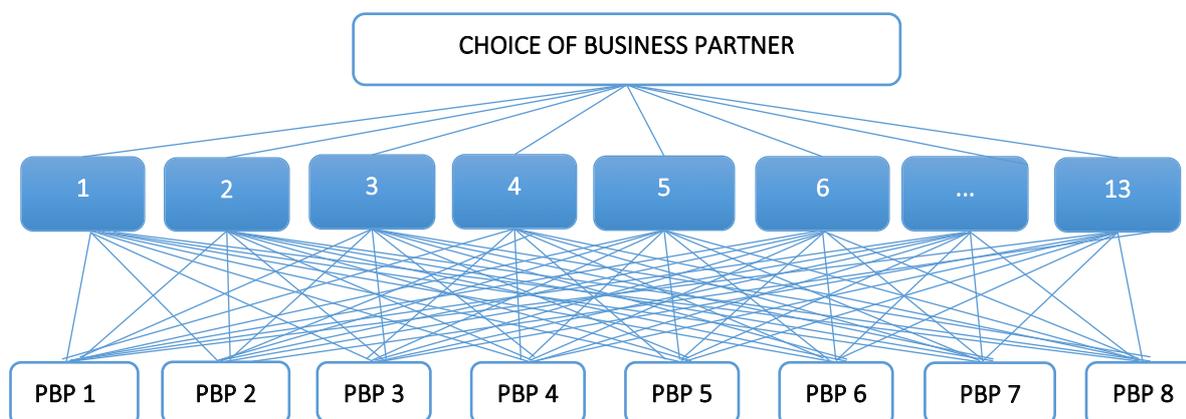


Figure 3. Hierarchical framework

Source: Authors.

Note: 1 – 4 describe the non-financial criteria in the following order: previous experience with a business partner (1); length of time the business partner has been operating on the market (2); costs of delivering materials (3); entries in the credit register or register of debtors (4); 5 – 13 describe the financial criteria in the same order as in Table 1.

2. Construct pairwise comparison matrix

Preferences between individual criteria are shown in the so-called pairwise comparison matrix (Saaty's matrix). Saaty's matrix $S = \{s_{ij}\}$ is a matrix that contains elements s_{ij} . s_{ij} expresses the intensity of the preference between the objects k_i and k_j . It holds that if the object k_i (the object in the row) is more important than k_j (the object in the column), then $s_{ij} \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, on the other hand $s_{ij} = \frac{1}{s_{ji}}$. The following formula is used to construct Saaty's matrix [15].

$$s_i \begin{pmatrix} 1 & s_{ij} \\ 1/s_{ji} & 1 \end{pmatrix} \quad (2)$$

Where:

$k_i; k_j$ are criteria or alternatives;

$s_{ij}; 1/s_{ji}$ expresses the intensity of the preference between the objects k_i and k_j .

Saaty's scale is used to express the intensity of the preference between the objects.

3. Perform judgement for pairwise comparison

The result of this step determines the weights of individual criteria (if criteria are compared) or the utility of the individual alternatives with respect to a given criterion (if alternatives are compared). Saaty defined a procedure which is based on the calculation of the eigenvector of a matrix according to the following formula.

$$S \cdot v = \lambda_{max} \cdot v \quad (3)$$

Where:

S Saaty's matrix;

v eigenvector of Saaty's matrix;

λ_{max} the largest eigenvalue of Saaty's matrix.

The values of the eigenvector must be standardized. This method of calculation is the most complex of all. Precisely because of its complexity, it is used only in specialized decision support programs. The geometric mean method is considered to be a simpler method that produces almost identical results, and for which specialized programs are not needed.

$$G_i = \sqrt[n]{\prod_{j=1}^n s_{ij}} \quad (4)$$

Where:

G_i geometric mean expresses the weights of individual criteria or determines the utility of individual alternatives;

s_{ij} elements of Saaty's matrix;

n number of criteria or alternatives.

4. Check for consistency

Consistency was described within the definition of the three stages of the AHP method. The consistency ratio expresses the level of consistency [16].

$$CR = \frac{CI}{RI} \quad (5)$$

$$CI = \frac{\lambda_{max} - k}{k - 1} \quad (6)$$

Where:

CR the consistency ratio;

CI the consistency index;

RI the random index;

λ_{max} the largest eigenvalue of Saaty's matrix;

k the number of objects - criteria/alternatives.

The random index was obtained by generating a large number of reciprocal random matrices (matrix elements came from a cardinal scale) and then averaging the eigenvalues of the matrices. A value of 1.56 was set for the number of 13 objects (the number of criteria). A value of 1.40 was set for the number of 8 objects (the number of alternatives – potential business partners) [16].

Steps 3 – 6 are performed for all levels of the hierarchy. By multiplying the utility of the individual alternatives with respect to the given criteria by the weights of these criteria, the total utility of the given alternative is determined. The final step is the selection of the optimal alternative based on the value of total utility.

3 Results and Discussion

A multiple-criteria matrix containing a description of the MCDM problem was created.

Table 2. Multiple-criteria matrix

Criteria/Alternatives	Non-financial criteria				Financial indicators								
	Previous experience	Length of operation	Costs of delivery	Entries in the credit/debtors registers	Quick ratio	Short-term debt ratio	Credit indebtedness	Days sales outstanding	Days payable outstanding	EBT	Return on assets	Return on sales	Decline of the company
PBP1	Yes	5	1.5	No	0.23	0.80	0.09	23	50	1	0,12	0,15	1.20
PBP2	Yes	8	1.3	No	0.40	0.55	0.10	15	48	1	0,09	0,10	0.95
PBP3	No	7	1.9	No	0.15	0.25	0.32	45	16	0	0,01	0,05	0.88
PBP4	No	3	1.1	No	0.25	0.40	0.12	33	89	0	0,02	0,06	2.1
PBP5	No	2	1.0	No	0.33	0.10	0.18	88	123	1	0,15	0,19	0.99
PBP6	Yes	4	1.4	No	0.59	0.85	0.26	62	23	1	0,18	0,25	1.85
PBP7	Yes	5	1.2	No	1.20	0.50	0.58	19	59	0	0,05	0,03	0.99
PBP8	No	9	1.9	No	0.90	0.75	0.42	37	28	0	0,09	0,02	1.01

Note: Previous experience is expressed verbally as yes or no; Length of operation is expressed in years; Costs of delivery are expressed in thousands of euros; Entries in the credit register or register of debtors are expressed verbally as yes or no; Financial indicators are expressed according to the calculation. If EBT was positive during the last 3 years, the value 1 is entered in the table, otherwise the value 0.

Source: Authors.

For the purposes of this contribution, the company, as well as potential business partners, are identified as follows: the company choosing the business partner is marked as *Company 1*, potential business partners are marked as PBP 1 – 8. In making its decision, Company 1 takes into account several financial indicators and non-financial criteria.

The selected and calculated financial indicators are presented in Table 1. The selected non-financial criteria were previous experience with a business partner, the length of time the business partner has been operating on the market, the costs of delivering materials, entries in the credit register or register of debtors.

The pairwise comparison matrix for the criteria was created. In the matrix, the criteria are numbered as in Figure 3.

Table 3. Pairwise comparison matrix for the criteria

Criteria	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	3	1/2	1/7	1/3	3	1/3	1	1	1/5	1	1	1/7
2	1/3	1	1/5	1/7	1/3	1/2	1/5	1/3	1/3	1/5	1/3	1/3	1/7
3	2	5	1	1/7	1/3	1	1/3	1	1	1/5	1	1	1/7
4	7	7	7	1	3	5	7	7	5	7	5	5	1
5	3	3	3	1/3	1	3	1	1/3	1/3	1/5	1	1	1/7
6	1/3	2	1	1/5	1/3	1	1/3	1/3	1/3	1/5	2	2	1/7
7	3	5	3	1/7	1	3	1	2	2	1/5	2	2	1/7
8	1	3	1	1/7	3	3	1/2	1	1	1/5	1/3	1/3	1/7
9	1	3	1	1/5	3	3	1/2	1	1	1/5	1	1	1/7
10	5	5	5	1/7	5	5	5	5	5	1	5	5	1/3
11	1	3	1	1/5	1	1/2	1/2	3	1	1/5	1	1	1/7
12	1	3	1	1/5	1	1/2	1/2	3	1	1/5	1	1	1/7
13	7	7	7	1	7	7	7	7	7	3	7	7	1

Source: Authors.

Saaty's matrix $S = \{s_{ij}\}$ is a matrix that contains elements s_{ij} . S_{ij} expresses the intensity of the preference between the objects – criteria. Formula 2 was used to construct Saaty's matrix. Saaty's scale was used to express the intensity of the preference between the objects.

According to Formula 4, the weights of the individual criteria were calculated.

Table 4. Standardized weights of the criteria

Criterion	Weights	Standardized weights	Criterion	Weights	Standardized weights
1	0.6210	0.0327	8	0.6757	0.0356
2	0.2920	0.0154	9	0.8211	0.0432
3	0.6603	0.0347	10	2.7287	0.1436
4	4.3834	0.2307	11	0.7154	0.0376
5	0.8278	0.0436	12	0.7154	0.0376
6	0.5169	0.0272	13	4.8616	0.2559
7	1.1823	0.0622			

Source: Authors.

According to Formula 3, the largest eigenvalue of Saaty's matrix λ_{max} was calculated, the consistency of which was subsequently checked according to Formulas 5 and 6.

$$S \cdot v = \lambda_{max} \cdot v \implies \lambda_{max} = \frac{S \cdot v}{v}$$

$$\lambda_{max} = 14.67$$

$$CI = \frac{14.67 - 13}{13 - 1} = 0.1392$$

$$CR = \frac{0.1392}{1.54} = 0.0903 < 0.1$$

The same procedure was chosen for the evaluation of the preferences between the individual alternatives in terms of criteria.

Table 5. Evaluation of the preferences between the individual alternatives in terms of previous experience

Alternative	PBP1	PBP2	PBP3	PBP4	PBP5	PBP6	PBP7	PBP8	Utility	Standardized utility	λ
PBP1	1	1	5	5	5	1	1	5	2.2361	0.2083	8.00
PBP2	1	1	5	5	5	1	1	5	2.2361	0.2083	8.00
PBP3	1/5	1/5	1	1	1	1/5	1/5	1	0.4472	0.0417	8.00
PBP4	1/5	1/5	1	1	1	1/5	1/5	1	0.4472	0.0417	8.00
PBP5	1/5	1/5	1	1	1	1/5	1/5	1	0.4472	0.0417	8.00
PBP6	1	1	5	5	5	1	1	5	2.2361	0.2083	8.00
PBP7	1	1	5	5	5	1	1	5	2.2361	0.2083	8.00
PBP8	1/5	1/5	1	1	1	1/5	1/5	1	0.4472	0.0417	8.00
Sum	X	X	X	X	X	X	X	X	10.7331	1	λ_{max}

Source: Authors.

$$S \cdot v = \lambda_{max} \cdot v \implies \lambda_{max} = \frac{S \cdot v}{v}$$

$$\lambda_{max} = 8.00$$

$$CI = \frac{8 - 8}{8 - 1} = 0.00$$

$$CR = \frac{0.00}{1.40} = 0.00 < 0.1$$

By multiplying the utility of the individual alternatives with respect to the given criteria by the weights of these criteria, the total utility of the given alternative is determined. The final step is the selection of the optimal alternative based on the value of total utility. Based on the results of the AHP method, PBP1 was recommended to the company as the optimal business partner.

What follows is a summary of advantages and disadvantages of the AHP method:

- The strength of the AHP method is that no special knowledge is required to apply the method, unlike some other methods of multiple-criteria decision-making. Quantitative pairwise comparisons made by the decision-maker are relatively simple and acceptable, as only two elements are always used, where the strength of the preference is relatively easy to determine. The intensity of importance is determined verbally by the decision-maker, who then assigns the appropriate numerical degree to the verbal preference according to Saaty's scale. A great benefit of the method, in the first phase of the decision-making process, is the creation of the hierarchical structure. This makes the problem clearer, making the individual parts of the problem more understandable, which contributes to a more comprehensive understanding of the problem as a whole. By creating and analysing the hierarchy, it is possible to see if all the important aspects of the problem have been included in the model. The AHP method can work with different types of criteria. Therefore, the method can combine criteria defined both verbally or numerically without any additional modifications. The method offers verification of the consistency of the decision-maker's judgments. The positive thing about consistency is that a certain degree of inconsistency is also accepted because it is not always possible to achieve full consistency. In the case of a large inconsistency of the matrix, it is possible to rethink the individual comparisons. This leads to better concentration, as well as the decision-maker being able to find new information in the reassessment, which may lead to them determining a different intensity of importance. The advantage of the AHP method is also the fact that it has a broad spectrum of applications. For example, the method can also be used for problems where there is a risk.

- The disadvantage of the analytical hierarchical process may be the number of pairwise comparisons. This is because it is necessary to compare the individual criteria and then compare the alternatives with respect to a specific criterion. If the decision-making problem contains a large number of criteria as well as alternatives, the decision-making process can be time consuming, and over time, the inconsistency of the matrices may begin to increase due to the loss of attention and lack of concentration of the subject. A minor drawback is also the basic scale (nine-point scale). When comparing the criteria, respectively alternatives, the decision-maker must determine the strength of the preference between the two criteria, respectively alternatives. With a large number, whether criteria or alternatives, a given scale may not be enough if it is necessary to sufficiently differentiate the individual elements. This shortcoming is followed by another weakness in the form of consistency, which is a relatively common problem when using the AHP method. If the nine-point scale is not enough, the full consistency of the matrix cannot be achieved either. On the other hand, the method also respects a certain degree of inconsistency, which can be considered an advantage. The analytical hierarchical process has a major drawback in that if the decision-maker uses the method to produce the results, but decides to add a new alternative to the model or remove the old alternative, the preferential order of other alternatives can change without changing the values of pairwise comparisons with regards to individual criteria or their preferences. Long analysis has shown that the order of the alternatives may change due to the lack of consistency of the intensities. Although it may sound nonsense to introduce or remove alternatives from the evaluated model, in practice such a need may arise.

4 Conclusion

The aim of the contribution was to use the Analytic Hierarchy Process to assess potential business partners and select the optimal alternative. MCDM is the most common type of decision-making process in business practice. AHP is an MCDM method originally developed by Prof. Thomas Saaty. The method derives ratio scales from paired comparisons. The ratio scales are derived from the principal eigenvectors and the consistency index is derived from the principal eigenvalue. The AHP method can be defined as one of the most commonly used MCDM methods in practice. The problem to be solved, as well as the steps of AHP method, are described in the methodology part of contribution. The problem to be solved was defined as follows: a selected company operating in the construction industry is looking for a new business partner. Eight business partners are under consideration. Using the AHP method, PBP1 was recommended to the company as the optimal business partner.

There are a variety of software applications available for the AHP method, of which Expert Choice was the first to be developed. Expert Choice was created in the 1980s by Thomas Saaty, the creator of the AHP method, in order to help organizations around the world make better, more transparent, and faster decisions. Expert Choice combines team collaboration tools and proven math techniques to enable the best decision to be made to achieve a goal.

Despite the shortcomings of the AHP method, it is one of the most popular and one of the most objective methods of multiple-criteria decision-making.

References

1. F. C. Zola, J. C. Colmenero, F. V. Aragao, T. Rodrigues, A. Braghini, Multicriterial model for selecting a charcoal kiln. *Energy*. **190**(1), 1–14 (2020)
2. E. Mastrocinque, F. J. Ramirez, A. Honrubia-Escribano, D. T. Pham, An AHP-based multi-criteria model for sustainable supply chain development in the renewable energy sector. *Expert Systems with Applications*. **150**, 59–76 (2020)
3. M. Koksalan, J. Wallenius, S. Zionts, *Multiple Criteria Decision Making: From Early History to the 21st Century*. Singapur: World Scientific. 1–198 (2011)
4. H. C. Liao, X. L. Wu, DNMA: A double normalization-based multiple aggregation method for multi-expert multi-criteria decision making. *Omega-International Journal of Management Science*. **94**, 23–38. (2020)
5. H. C. Liao, X. L. Wu, X. M. Mi, F. Herrera, An integrated method for cognitive complex multiple experts multiple criteria decision making based on ELECTRE III with weighted Borda rule. *Omega-International Journal of Management Science*. **93**, 44–52 (2020)
6. C. Fu, W. J. Chang, S. L. Yang, Multiple criteria group decision making based on group satisfaction. *Information Sciences*. **518**, 309–329 (2020)
7. R. Costache, C. T. Bui, Identification of areas prone to flash-flood phenomena using multiple-criteria decision-making, bivariate statistics, machine learning and their ensembles. *Science of the Total Environment*. **712**, 269–281 (2020)
8. M. Haddad, D. Sanders, G. Tewkesbury, Selecting a discrete multiple criteria decision making method for Boeing to rank four global market regions. *Transportation Research Part A: Policy and Practice*. **134**, 1–15 (2020)
9. D. Lamba, D. K. Yadav, A. Barve, G. Panda, Prioritizing barriers in reverse logistics of E-commerce supply chain using fuzzy-analytic hierarchy process. *Electronic Commerce Research*. **20**(2), 381–403 (2020)
10. B. U. Elvarsson, S. Agnarsson, S. Gudmundsdottir, J. Vidarsson, Using multi-criteria analysis the assess impacts of change in ecosystem-based fisheries management: The case of the Icelandic cod. *Marine Policy*. **116**, 120–132 (2020)
11. J. Benitez, S. Carpitella, A. Certa, J. Izquierdo, Constrained consistency enforcement in AHP. *Applied Mathematics and Computation*. **380**, 85–101 (2020)
12. T. Si, C. B. Wang, R. Q. Liu, Y. S. Guo, S. Yue, Y. J. Ren, Multi-criteria comprehensive energy efficiency assessment based on fuzzy-AHP method: A case study of post-treatment technologies for coal-fired units. *Energy*. **200**, 159–172 (2020)
13. T. L. Saaty, K. P. Kearns, *Analytical Planning. The Organization of Systems*. Oxford: Pergamon Press. (1985)
14. T. L. Saaty, *Multicriteria Decision Making: The Analytic Hierarchy Process. Planning, Priority Setting, Resource Allocation*. New York: McGraw-Hill. (1980)
15. T. L. Saaty, Decision making with the analytic hierarchy process. *International Journal of Services Sciences*, **1**(1), 83–98 (2008)
16. T. L. Saaty, *Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process*. Pittsburgh: RWS Publications. (2000)