Modeling the evolution of socio-economic systems using the methods of stability theory

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Abstract. Justification of the state socio-economic policy on the basis of its modeling, as a dynamic economic system, should take into account, along with income, the stratification by the level of income of the population. Obtaining a characteristic of the evolution of such a system and the tendencies of its stability is possible through a mathematical model, in particular, in the form of a set of differential equations (DE). Such equations allow us to describe the dynamics of the evolution of a simulated socio-economic system (SES) taking into account state protectionist or fiscal policies. The method of SES study by the method of phase portraits based on the constructed mathematical model is proposed. Using statistical data, parametric estimates of the consequences of the SES development policy pursued by the state were obtained. Based on the results of the model study, recommendations for adjusting the socio-economic policy are proposed.

1 Introduction

The structure of the elements and internal connections SES, as a large system, can be considered as a modeled object, at various levels of aggregation.
The economic subsystem of the organization of the social structure can include interconnected production, financial and logistic objects that ensure the production and distribution of products, [4, 8] (Fig. 1). The study of the evolution of SES by analytical methods of mathematical modeling, in comparison with other approaches used in theoretical economics (cognitive, econometric, statistical, neural network modeling, etc.) allows you to obtain a qualitative picture of the phenomenon under study at different values of the main parameters of the modeled system [3, 9, 11].

In economic theory, mathematical models of models in the form of various types of DE are used, for example, the models of Solow Leontiev, Neumann, and others [3, 4, 6,10,]. However, most of the well-known economic and mathematical models do not allow to take into account explicitly the influence of state control mechanisms that determine the socio-economic policy of the development of society.

2 Research Methods

The study simulates closed SES [4], with limited economic and economic interactions with other arbitrary SES.

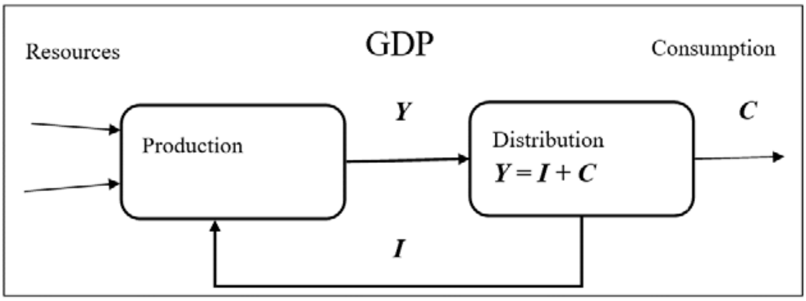

Fig. 1. Enlarged structural model of the state economy as a subsystem of society

2.1. Scientific hypotheses

As part of the study, the following scientific hypotheses were accepted:

The considered SES is closed and is exhaustively described by a pair of key indicators, namely:

- the total volume of production Q of production of material and material goods necessary for society, attributable to each individual SES, calculated per unit of time (analyzed for a year);
- the stratification of the income of community members in real consumption and the accumulation of material and spiritual wealth, assessed by a statistical indicator - the Gini coefficient J.

The validity of the formulated and accepted hypothesis is based on understanding the goal of a community that unites individuals, in such a provision for the most favorable life, which ensures the cumulative creation of material and spiritual benefits at the cost of minimal effort through a reasonable division of labor.

For each member of the community, a motivating factor in the development of production is a certain possibility of redistributing material and spiritual benefits in their favor, which can be produced additionally.

It is convenient to measure both indicators having the same dimension in units that do not depend on the socio-economic formation, the level of production, time, etc. Such an indicator exists and is used to characterize the development of a civilization - the amount of energy consumed by a civilization per unit of time [9]. This figure is adjusted to estimate the total energy consumption for each member of the community.

2.2 Basic approaches

The evolution of the key indicators adopted in clause 2.1 over time is described by the following system of two (by the number of indicators) ordinary differential equations

$$\begin{cases} \frac{dQ}{dt} = F_1(Q, \sigma) \\ \frac{d\sigma}{dt} = F_2(Q, \sigma), \end{cases} \quad (1)$$

where symbols F_1, F_2 - designate two continuous and time differentiable functions.

If the modeled and investigated SES is in a certain stationary or close to it state, characterized by practically unchanged values of the key indicators Q and σ for a short period of time perceived by society, then we can assume that both functions

$$\begin{cases} F_1(Q,\sigma) = 0 \\ F_2(Q,\sigma) = 0, \end{cases} \quad (2)$$

Expression in the general case (2.2) describes a system of algebraic equations, the solution of which will be pairs (Qi0, σi0) of numbers for i = 1, 2… N, where N is the number of real roots of equations (2.2). The area of change of indicators is non-negative values of Q and σ.

The accepted scientific hypothesis is based on the known character of changes in the simulated indicators Q, σ over time. In a fairly general case, the evolution process can be written using operator equations of the form (2.3)

$$\begin{cases} L_1(Q,\sigma,t) = 0 \\ L_2(Q,\sigma,t) = 0, \end{cases} \quad (3)$$

where L1, L2 are operators that implement the interaction between the SES key indicators over time. Expressions (2.1) are a special case of the considered operators. The adequacy of the model description of economic phenomena and processes occurring in the studied SES, using dependences (2.1), can be verified using statistical data.

2.3 Stability options for simulated systems

For each time moment t in the modeled evolution process, SES can be considered as quasi-stationary, and the set of indicators (Q_0, σ_0) characterizing its stationary state correspond to dependencies (2.2). According to the provisions of the theory of stability Lyapunov A.M. [1], the aforementioned quasi-stationary state can be characterized by either stable or unstable. Let us consider in more detail the calculated special cases of stability of the systems under study.

The possible variant of asymptotic stability corresponds to the state of stability of the studied SES, but does not determine its development. The case of non-asymptotic stability is characterized by a change in the simulated key indicators, but in a limited area. Here, at certain time intervals, both an increase and a decrease in the values of the mentioned indicators can be observed.

In contrast to the listed steady states, unstable cases correspond to monotonic variations in the simulated key SES indicators. Mathematically, this nature of the evolution of the modeled system can be interpreted from the standpoint of the accepted criterion of optimality. In the social sphere, the evolutionary dynamics of the SES characterizes the class value. From an economic standpoint, the SES behavior, assessed by phase portraits, can be interpreted both positively and negatively.

Below we formulate the problem of assessing the nature of the stationary state of the socio-economic system and the tendency of its development, as well as identifying the possibility of controlling the modeled tendency of the evolution of SES.

3 Results

3.1 Analytical solution by methods of stability theory

The solution of the problem is based on the application of the stability theory according to the first approximation [1]. for this purpose, equations in variations for an unperturbed system of differential equations (2.1) in the vicinity of the stationary state Q_0, σ_0 are compiled.

$$\begin{cases} \frac{d\delta Q}{dt} = a_{11}\delta Q + a_{12}\delta\sigma \\ \frac{d\delta Q}{dt} = a_{21}\delta Q + a_{22}\delta\sigma \end{cases} \tag{3.1}$$

where

$$a_{11} = \frac{\partial F_1}{\partial Q}\bigg|_{\substack{Q=Q_0 \\ \sigma=\sigma_0}} ;\ a_{12} = \frac{\partial F_1}{\partial Q}\bigg|_{\substack{Q=Q_0 \\ \sigma=\sigma_0}} ;\ a_{21} = \frac{\partial F_2}{\partial Q}\bigg|_{\substack{Q=Q_0 \\ \sigma=\sigma_0}} ;\ a_{22} = \frac{\partial F_2}{\partial Q}\bigg|_{\substack{Q=Q_0 \\ \sigma=\sigma_0}} . \tag{3.2}$$

In the general case, the parameters aij characterize the specifics of the modeled SES and may even have different signs. It is known from macroeconomic theory that most often, parameter a_{11} is negative, which is determined by the objectively existing need to minimize efforts spent on the creation and production of public goods.

Most often, parameter a_{21} is positive, which is determined by the objective desire of each individual to get the most out of the community, for example, from the state with its officials, from the generated and distributed stream of public goods. The value of a_{12} (in the case of a positive a_{12}) should be interpreted as the level of state protectionism for the manufacturing sectors of the real economy. Parameter a_{22} can be interpreted as the degree of income equalization of the population (rationing of wages and incomes, progressive taxation for wealthier strata, subsidies, etc.

The analytical solution of the obtained equations (3.1), characterizing SES, can be written [1]

$$\delta Q = U_0 e^{\lambda t}$$
$$\delta\sigma = V_0 e^{\lambda t} \tag{3.3}$$

where U_0, V_0 are constants determined from the initial conditions;
λ - system parameter.

The character of the stationary point Q_0, σ_0 depends on the value of the characteristic exponent λ, determined from the characteristic equation

$$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = 0 \tag{3.4}$$

For the specified signs of the coefficients a_{ij}, the roots λ_1, λ_2 of equation (3.4) are real numbers, if at least one of them is greater than zero, and this corresponds to the instability of the stationary state Q_0, σ_0.

3.2 Analysis of results and methods for adjusting socio-economic processes

Phase trajectories of socio-economic processes that establish the relationship between the indicators Q and σ are particularly clear.

$$\Phi(Q, \sigma) = 0. \tag{4.1}$$

To construct phase trajectories, the first equation (3.1) is divided into the second

$$\frac{d\delta Q}{d\delta\sigma} = \frac{a_{11}\delta Q + a_{12}\delta\sigma}{a_{21}\delta Q + a_{22}\delta\sigma}. \tag{4.2}$$

Equation (3.6) can be solved by numerical methods, but the qualitative form of the solution, which is convenient for analysis, can be obtained by the method of isogonal trajectories [1]. Two different cases are possible here, characterized by the corresponding phase portraits with the asymptotes KL, MN, when approaching which one of the indicators increases and the other decreases for the case $a_{12} / a_{11} < a_{22} / a_{21}$ (Fig. 2).

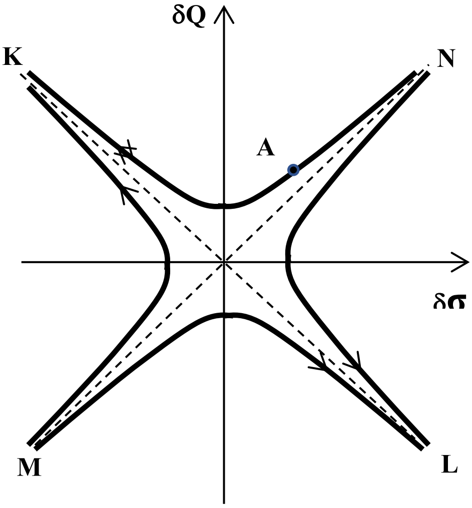

Fig. 2. Phase portrait of socio-economic systems

It should be noted that the phase trajectories constructed using equation (3.1) are valid in the case of linearization of equation (2.1), and the formulated conclusions take place only for small values of δQ and δσ. The direction of movement of the representing point A along the constructed phase trajectories depends on the sign of the right-hand sides (3.1), and its speed depends on the moduli of solutions λ_1 and λ_2 of the characteristic equation (3.4).

In the constructed mathematical model of the SES evolution, the parameters a_{12}, a_{22} depend on the degree of state participation in managing the development of the system. In the complete absence of such participation, $a_{12} = a_{22} = 0$. Then the phase portrait degenerates and has the form shown in Figure 3a.

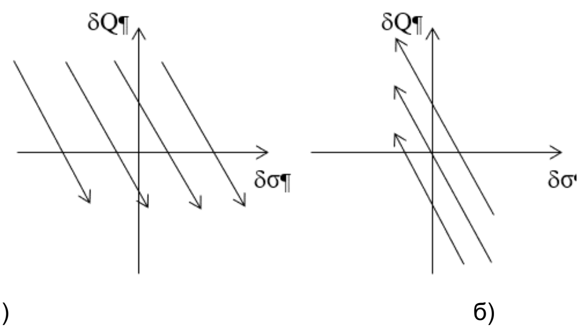

Fig. 3. Phase portraits of degenerate economic systems

a) $a_{12} = a_{22} = 0$; b) $a_{22} >> a_{21}$; $a_{12} >> a_{11}$

The analysis shows that production is falling and income stratification is increasing. If we assume that equation (3.1) is valid for a sufficiently large range of changes δQ, $\delta\sigma$, then bearing in mind that Q > 0, σ > 0, we can conclude that this method of functioning of the socio-economic system leads to an increase in the stratification to a certain limit and a reduction in production to zero.

If the state completely subordinates the management of the socio-economic system, then $a_{12} >> a_{11}$, $a_{22} >> a21$, and then the phase portrait differs from the previous one in the direction of movement of the image point along the phase trajectory (fig. 3-b).

In this case, the income stratification is reduced to zero, and the production of material and spiritual goods tends to a certain value.

According to the authors, these phenomena are qualitatively confirmed by such well-known real economic projects as" shock therapy " and total planned regulation of the economy.

The discussed mathematical model of socio-economic development of society allows purposefully managing such development.

Indeed, if, for example, as a result of measurements of Q, σ in a certain period of time, it is established that the socio-economic system is located at point A (Fig. 1-a), for which

$$K_A = \frac{\delta Q_A}{\delta \sigma_A} < K_1 \tag{4.3}$$

where

$$K_1 = \frac{a_{11} - a_{22}}{2a_{21}} + \frac{\sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}}{2a_{21}} \tag{4.4}$$

this corresponds to the trend of decreasing production and increasing stratification in the income of community members (with a slight decrease at the initial stage).

In the case when the community is faced with the goal of changing the emerging trend to the opposite of production growth and a decrease in income stratification, then it is necessary to change the coefficients a_{12}, a_{22} so that it corresponds to the nature of the phase portrait shown in Fig. 1-b. Geometrically, such a change in the slopes corresponds to a reversal of the asymptotes. In order to practically achieve this goal, it is necessary to increase support for the poor and reduce preferences for producers.

Thus, the set of parameters of the constructed model, determined according to statistical data, characterize the influence of state economic policies on the level of social orientation of society. A larger value of the parameter a_{12} corresponds to an increase in the degree of state support for their producers. On the other hand, a smaller value of the parameter a_{22} reduces the level of income stratification of the population.

4 Conclusions

1. The mathematical model proposed in the study, which characterizes the evolution of SES through a system of two ordinary DE, allows us to study the influence of the state, as a regulator of financial and economic policy, on the results of its implementation for society. With the help of the proposed model, the results are obtained that characterize at a qualitative level the evolution of its indicators in a wide range of variations in the macroeconomic parameters of the modeled SES.
2. The values of the set of parameters of the SES model make it possible to assess the emerging state of the modeled system and the degree of stability of its evolution. Acting on the exogenous parameters a_{12} and a_{22}, one can influence the nature of the phase portraits and, accordingly, the required evolutionary trends of the modeled SES.

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