

# An integrated assessment of the quality of teaching Mathematics in school

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**Abstract.** This article presents a diagnostic criteria-based methodology for a systematic and complex assessment of the quality of teaching Mathematics in school. This methodology may help assess the prospects of applied approaches to teaching Mathematics as well as monitor it and describe in terms of ensuring the high quality of general education in Mathematics. We used Kolmogorov's "convolution of qualities" as a basis for developing this methodology for the systematic and complex diagnostics of mathematical education in school. This methodology is used to assess the quality and optimization of complex objects in mechanics, chemical industry, economics, and higher education. The suggested integral assessment (systematic and complex diagnostics) was successfully applied to evaluate the quality of teaching Mathematics in schools of the Volgograd region. We defined three levels of mathematical education quality: discrete (minimal and restricted level), fragmentary (average functional level), and integrated and comprehensive (a rather high level).

## 1 Introduction

In Russia, the sociocultural, political, social and economic changes caused by globalization have triggered the need for changing the requirements to teaching Mathematics in school. The mathematical competence is believed to be helpful to comprehend modern world-scale information technology and ensure the social mobility of an individual. Nowadays, more and more people start to see the high quality education, particularly in Mathematics, as a means for social and professional growth. Teaching Mathematics in school depends on several factors: 1) the social-sector procurement, which is stipulated in legal documents, as well as social and personal needs; 2) the conditions that promote personal inclinations and needs in terms of studying Mathematics; 3) the possibilities to implement professional skills of teachers of Mathematics; 4) the possibility to implement educational, methodological, and scientific innovations in teaching as well as the possibility to use the school's resource potential. It is believed that every school has its own models and procedures for teaching Mathematics. In its modern sense, the quality of teaching Mathematics does not exclusively amount to obtaining the level of knowledge stipulated in the learning standards; it is also about the successful performance of school departments responsible for building competence.

These days, the school authorities understand the need to organize the study process in the most appropriate way. That is why the systematic and complex diagnostics of teaching Mathematics in school has become highly relevant. In addition, running diagnostics

regularly ensures the cyclical nature of the study process. In this way, new goals are set on a new, higher level.

There is a great body of research concerning the issue of the quality assessment of teaching Mathematics. Unfortunately, the suggested systems for monitoring and assessment fail to be unified or comprehensive. Usually, the quality of teaching is assessed under the framework of student rankings that include the results of school leavers' academic achievements (for example, in Russia, there is mass testing after the 9th grade that is compulsory for everyone) [1].

S. Yu. Sergeeva and Ye. D. Obrevko suggest using a process approach to the assessment of the quality of teaching Mathematics with regard to the three criteria groups: the quality of the study process, the quality of the results of the study process, the quality of the study conditions [2].

V. A. Yasvin, S. N. Rybinskaya, S. A. Belova, and S. Ye. Drobnov suggest using a complex ranking to the assessment of the quality of teaching Mathematics. This ranking will include the academic results according to the taught subjects and the indicators of schools' study conditions and potential for development [3].

Recently, the rank analysis has gained much attention in terms of the monitoring and assessment of the quality of education. This method is discussed in the works of R. V. Gurina and V. V. Bedash [4].

The question of how to assess the quality of teaching Mathematics in school is still highly relevant although there is a significant advancement in the research in this area.

## 2 Materials and Methods

Since we have to run diagnostics on the education system that has a complex structure, there can be some difficulty in developing a diagnostic criteria-based methodology. In this article, we suggest a methodology for an integrated assessment of the quality of teaching Mathematics in school. We used "a convolution of qualities" as a basis for developing this methodology [5]. The methodology includes the following provisions:

— The quality of teaching Mathematics is assessed according to a set of criteria. These criteria form a complex hierarchical system [6];

— Separate qualities form the general quality. Separate qualities are convolved into a single criterion according to Kolmogorov-Nagumo averages [5].

— Here, the average is regarded as a function with values always belonging to the interval that is taken by a certain set of argument values. Introducing a normal interval sets a common scope for all separate qualities. The use of averages helps present a single quality in terms of this scope.

— This methodology is used to assess the quality and optimization of complex objects in mechanics, chemical industry, economics, and higher education [7,8].

— According to the approach that we used to identify the criteria, there are four directions in the assessment of education quality:

— matching the objectives (correspondence of the level of knowledge and abilities as well as the physical and moral development of school leavers) and results (as a means to achieve objectives);

— the content of the Mathematics curriculum that ensures personal development;

— the nature of the study process that complies with the modern demands of such sciences as philosophy, psychology, pedagogy, and Mathematics teaching methodology;

— providing the conditions to ensure the achievement of the objectives in teaching Mathematics [9].

We have defined the criteria according to the aforementioned directions. Thus, we have determined the indicators according to these four criteria. The identified criteria are to some extent interrelated. They can have a low or high indicator, and this will be reflected in the integral assessment of the quality of teaching Mathematics. Each of the four criteria consists of indicators. This helps assess the quality of education accurately and comprehensively. The indicators are rated on a five-point scale with integers from 0 to 4.

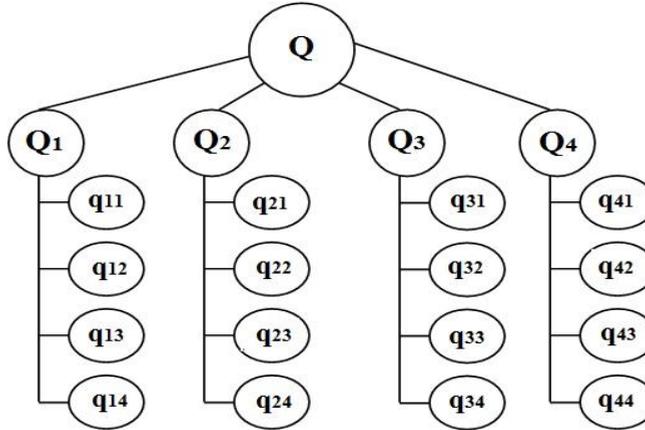
**Table 1.** Criteria and indicators.

	Criteria and indicators	Range
Q1	Scientific and objective criterion (the objectives of teaching Mathematics in school should meet the demands of the society and time)	0-16
q11	A target that is oriented at the demands of the society and time and that is common for every subsystem in the general educational institution	0-4
q12	Compliance between the targets and motives of every subsystem in the general educational institution	0-4
q13	Legal and conceptual framework (this implies developing a framework necessary to organize mathematical education as an innovational education system in the general educational institution)	0-4
q14	Involving teachers into developing and implementing the methodology for teaching Mathematics in general educational institutions	0-4
Q2	Action and organization criterion (integrating teaching approaches, methods, and techniques in terms of the focus on the individual) with regard to modern trends in teaching Mathematics	0-16
q21	Comprehensive approach to teaching Mathematics with regard to all its components	0-4
q22	Interdependence of integration and differentiation processes in teaching Mathematics	0-4
q23	Choosing the content, methods, and techniques with regard to the modern trends in teaching Mathematics; combining tradition and innovation in teaching	0-4
q24	Interactive approach to teaching Mathematics; adaptivity to societal changes (the ability to adapt easily to the upcoming changes)	0-4
Q3	Content and quality criterion (the comprehensive approach to personal development at Mathematics classes as well as a practice-oriented nature of teaching Mathematics; consequently, this will help a person play his or her social roles successfully)	0-16
q31	Diverse components of Mathematical curriculum that can be organized in various ways to meet students' demands, interests, and needs.	0-4
q32	Combining components of Mathematical curriculum with regard to changes in education stemming from the social and economic situation; aiming to develop meta-subject skills	0-4
q33	Students' pursuit for self-development and self-organization (reviewed through the results of learning Mathematics reflected in rankings)	0-4
q34	School management and management quality	0-4
Q4	Quality and workspace criterion (providing appropriate conditions for studying)	0-16
q41	Creating a favorable emotional microclimate and conditions for ensuring students' wellbeing <i>создание</i>	0-4
q42	Material and technical basis (the possibility to use the resources that the educational institution provides)	0-4
q43	The high level of teachers' expertise in teaching Mathematics; ensuring interaction between teachers during the education process	0-4
q44	Functional comfort and involvement in collaborative activities	0-4

With a hierarchical graph, we can analyze the qualities' indicators to create a unified criterion for the quality assessment (Figure 1). It is possible to express the relations between indicators and a unified quality provided the two following conditions are met:

- the common scale for all indicators and a unified quality is normalized ([0, 1] interval);
- average functions are used to create a unified criterion.

The integrated assessment of the quality of teaching Mathematics in school can be represented in the form of a four-level hierarchical system.



**Fig. 1.** Hierarchical structure of the integrated assessment of the quality of teaching Mathematics in school.

The integrated assessment of the quality of teaching Mathematics in school is defined by assessing the four integrated qualities of the system:  $Q = Q(Q_1, Q_2, Q_3, Q_4)$ .  $Q$  is combined of the following criteria:  $Q_1$  – scientific and objective criterion;  $Q_2$  – action and organization criterion;  $Q_3$  – content and quality criterion;  $Q_4$  – quality and workspace criterion.

The quality of the model for teaching Mathematics in school (integrated assessment)  $Q$  is determined by the convolution of normalized criteria of the education system quality. Those criteria are determined by a convolution of the normalized indicators (four-level convolution):

$$Q_1 = Q_1(q_{11}, q_{12}, q_{13}, q_{14}), \tag{1}$$

$$Q_2 = Q_2(q_{21}, q_{22}, q_{23}, q_{24}), \tag{2}$$

$$Q_3 = Q_3(q_{31}, q_{32}, q_{33}, q_{34}), \tag{3}$$

$$Q_4 = Q_4(q_{41}, q_{42}, q_{43}, q_{44}). \tag{4}$$

The quality assessment of the scientific and objective criterion:

$$Q_1 = -\ln \left( \frac{\exp(-q_{11}) + \exp(-q_{12}) + \exp(-q_{13}) + \exp(-q_{14})}{4} \right) \tag{6}$$

The quality assessment of the action and organization criterion:

$$Q_2 = -\ln \left( \frac{\exp(-q_{21}) + \exp(-q_{22}) + \exp(-q_{23}) + \exp(-q_{24})}{4} \right) \tag{7}$$

The quality assessment of the content and quality criterion:

$$Q_3 = -\ln \left( \frac{\exp(-q_{31}) + \exp(-q_{32}) + \exp(-q_{33}) + \exp(-q_{34})}{4} \right) \tag{8}$$

The quality assessment of the quality and workspace criterion:

$$Q_4 = -\ln \left( \frac{\exp(-q_{41}) + \exp(-q_{42}) + \exp(-q_{43}) + \exp(-q_{44})}{4} \right) \tag{9}$$

$Q$  – the integrated quality indicator:

$$Q = -\ln \left( \frac{\exp(-Q_1) + \exp(-Q_2) + \exp(-Q_3) + \exp(-Q_4)}{4} \right) \tag{10}$$

According to each criterion, we assessed the system sensitivity as a normalized unified function in terms of different parameters. The selected parameters ensure the function’s appropriate sensitivity to changing the system’s input parameters. This means that the model describes the real situation in the correct way.

The suggested systematic and complex assessment of the quality of teaching Mathematics in school was developed with the software package VisSim; the system’s hierarchy was represented with interconnected MathCad blocks.

The suggested methodology and the automated program help quickly assess the mathematical education in school according to various indicators. In the future, the aforementioned systematic and complex diagnostics can be easily expanded by introducing new parameters to the system. Adding a new block (which optimizes the function) will help find optimal values of the parameters for decision-making. Moreover, this methodology will help assess the potential of mathematical education as well as to describe it and monitor in terms of ensuring the quality of education.

As an integrated approach, the mathematical education has the following levels of integrity: 1) non-integrated; 2) integrated; 3) wholly integrated [10]. Thus, we defined three levels of mathematical education quality: discrete (minimal and restricted level), fragmentary (average functional level), and integrated and comprehensive (a rather high level).

In the discrete level of mathematical education, its components are fragmentary and autonomous. There is no or little consistency in aims, values, approaches, and external connections within the education structure. The education patterns are monotonous. Moreover, the education process is cyclical.

In the fragmentary level of mathematical education, the system's elements are interconnected only locally. There is no systematical approach since adopting aims and other aspects of mathematical education is sporadic. Teachers and management take part in the organizational process only episodically.

In the integrated and comprehensive level of mathematical education, the approach to education is complex, comprehensive, and well-organized. There are di-verse education patterns that are relevant in terms of the current social and economic context.

The aforementioned levels are interconnected. Each of the subsequent levels includes the features of prior levels but improves them.

This criteria and diagnostic methodology for assessing the quality of mathematical education in school provides detailed characteristics of the educational initiatives in developing and implementing the mathematical curriculum. This is crucial for defining pedagogical goals and educational content as well as for selecting methods and technics to ensure high-quality education.

We have studied the quality of teaching Mathematics in schools in Volgograd and the Volgograd region. We have taken the academic year as a point of reference.

We have defined the objectives of the systematic and complex diagnostics:

- to define the quality levels of mathematical education in terms of quantitative and qualitative indicators;
- to represent the performance in terms of quantitative and qualitative indicators;

— to conduct a comparative analysis of the quality levels of mathematical education according to the type of the institution (lyceums, gymnasiums, secondary schools, urban schools, and rural schools);

— to define problems that teachers of Mathematics have to deal with, and to discuss possible solutions. New paragraph: use this style when you need to begin a new paragraph.

— In this diagnostics, we complied with the following requirements:

— the "pedagogical" nature of the diagnostics: not only did we aim to obtain certain data but also to analyze them and use further;

— feasibility: methodology does not require time-consuming processing;

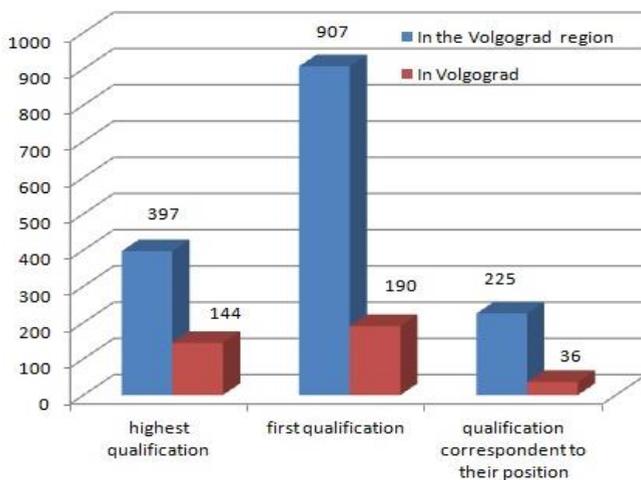
— objectivity ensured by expert assessment;

— relevance of the applied methodology reflecting various conditions and factors that affect mathematical education in school.

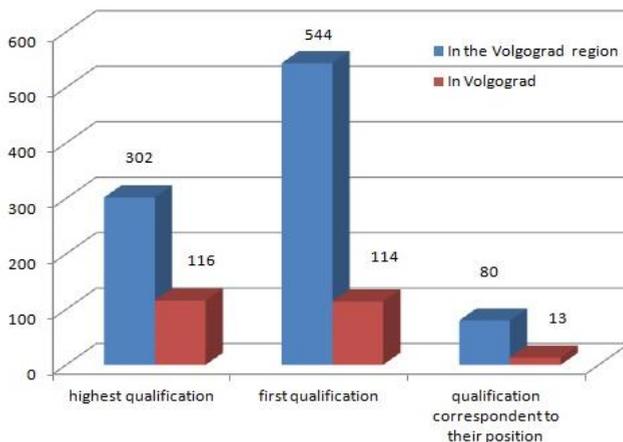
### 3 Results

It took us five years (2014-2019) to complete the diagnostics. The schools-participants are located in 46 administrative units (AU), namely 8 city districts, 12 (33) municipal regions, 5 urban and rural settlements (Volzhsky, Kamyshin, Mikhaylovka, Uryupinsk, Frolovo).

In the Volgograd region, the mathematical curriculum for 5th-9th grades is to be taught in a five-year time frame. For 10th-11th grades, it is two years. Mathematics is taught at basic, specialized (10th-11th grades), and advanced (10th-11th grades) levels. According to the specific features of the implemented programs, the teacher can decide on lesson plans and the number of hours for every topic. The schools can also introduce additional hours. In the Volgograd region, there are 1932 teachers of Mathematics in secondary school, 397 teachers have the highest qualification, 907 teachers have the first qualification, 225 teachers have a qualification correspondent to their position; 1785 teachers have a degree in higher education (92%). There are 1080 teachers of Mathematics in higher school, 302 teachers have the highest qualification, 544 teachers have the first qualification, 80 teachers have a qualification correspondent to their position; 1052 have a degree in higher education (97%). In Volgograd, every second teacher has the highest qualification.



**Fig. 2.** Qualitative characteristics of the workforce in secondary school teaching according to the qualification (Mathematics).



**Fig. 3.** Qualitative characteristics of the workforce in higher school teaching according to the qualification (Mathematics).

**Table 2.** Territorial division of schools.

Region	Number of schools	
	Number	%
Volgograd	75/145	0,75/21,29
Urban settlements	13/75	0,13/11,01
Municipal regions	12/461	0,12/67,70
Total	100/681	0,15/100,00

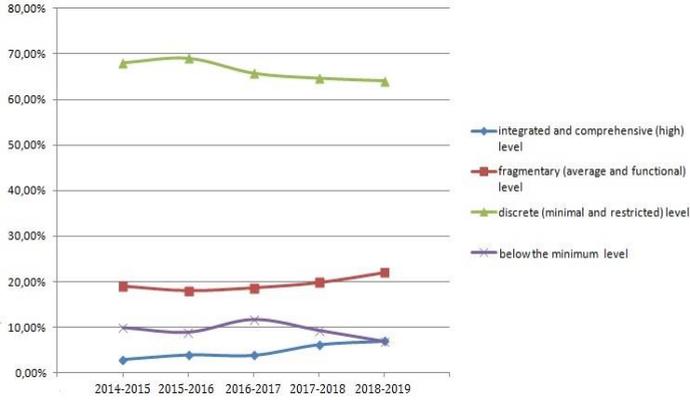
**Table 3.** Division of schools according to their type.

Type	Number of schools	
	Number	%
State schools	75/605	0,11/88,84
Lyceums	10/17	0,015/2,50
Gymnasiums	15/21	0,22/2,94
Total	681	0,15/100,00

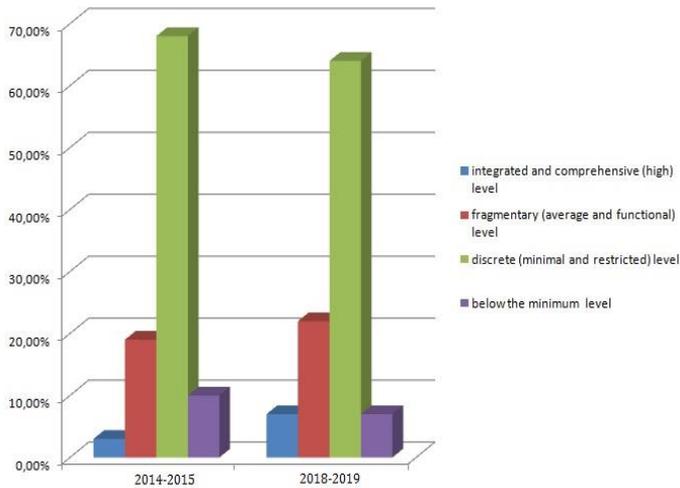
In this diagnostics, we take into account that teachers of Mathematics have taken courses in developing an innovative mathematical curriculum at the Volgograd State Academy of Postgraduate Education (VGAPO) [11,12].

**Table 4.** Quality dynamics of mathematical education in schools.

Organizational levels of mathematical education in school	2014-2015	2015-2016	2016-2017	2017-2018	2018-2019
integrated and comprehensive (high) level	3	4	4	6	7
fragmentary (average and functional) level	19	18	19	19	22
discrete (minimal and restricted) level	68	69	67	62	64
below the minimum level	10	9	12	9	7



**Fig. 4.** Generalized graph of quality dynamics of mathematical education in schools.



**Fig. 5.** Diagram of quality dynamics of mathematical education in schools.

As a result, we have obtained a unified indicator of the quality of the mathematical education  $Q = 0,52$  (52%) in schools in the Volgograd region. Thus, we can state that the level of mathematical education is not sufficient enough.

The results of the Russian State Exam on Mathematics seem to support this claim. The quality of knowledge is 42-42%. In recent years, the level of school leavers' mathematical education amounts to basic knowledge.

**Table 5.** Division of schools according to their type.

levels of mathematical education in school	2014-2015	2015-2016	2016-2017	2017-2018	2018-2019
integrated and comprehensive (high) level	3	4	4	6	7

**Table 5.** Continued.

fragmentary (average and functional) level	19	18	19	19	22
discrete (minimal and restricted) level	68	69	67	62	64
below the minimum level	10	9	12	9	7

## 4 Conclusions

The diagnostics of the quality of mathematical education in schools in the Volgograd region has helped define the reasons inhibiting the goals stated in the Federal Standards [13] and Concept for the mathematical education [14].

According to the results, we have made the following conclusions. School teachers still have a tendency to avoid changes coming from the outside. The majority of teachers seem not to be ready to implement innovative methods and techniques; they tend to use unified approaches and reproductive activities that do not let the students unleash their potential. Moreover, teachers do not seem to act cooperatively and work as a team to develop creative projects.

There are several reasons that inhibit the quality of mathematical education. For example, the authoritarian management style, mismanagement of resources, and disbalance in interests of actors in the educational system. In order to eliminate these problems, the following actions can be taken: to shift from the dominant management style to improve the quality of education; to change the teachers' attitude to the aim set in the curriculum; to shift from the traditional pedagogical environment to the innovative one; to develop professional competence in developing integral educational process according to the modern realia.

The suggested procedures for the quality assessment of teaching Mathematics in school can be used as a methodology for further studies.

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