Mathematics Case Teaching Method and Its Characteristics and Significance

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Abstract. Based on the teaching needs of applying the Nonlinear Functional Analysis, a new type of discussion-type teaching method was refined to strengthen the mathematical foundation and train the ability of scientific research, which is called the Mathematical Case Teaching Method (MCTM). It is an open teaching mode which is centered on an important principle in mathematics, with the results of applied research as the material, drawing on the concept of Case Teaching Method (CTM) for case design, and on the basis of students' full research. It has similar but not identical characteristics and teaching advantages with traditional teaching methods.

1 Introduction

Based on the Linear Functional Analysis, the Nonlinear Functional Analysis focuses on the analysis of the ideas, theories, methods and new research results of modern nonlinear functional analysis, which is a developing modern mathematics discipline. The Nonlinear Functional Analysis not only has the characteristics of basic theory like a general mathematics course, but also has a very powerful function of application. The application of the Nonlinear Functional Analysis in other disciplines can be seen from literature [1-4] and its citations. Every research work in [1-4] was the application research result of a principle in the Nonlinear Functional Analysis. The purpose of this course is twofold. First, it provides a basic course of modern nonlinear functional analysis for graduate students, especially doctoral students. Second, it provides an applied technology of modem nonlinear functional analysis for graduate students to solve nonlinear problems in scientific research. To put it bluntly, it is to enable the students to learn to use the principles of the Nonlinear Functional Analysis to engage in the scientific research of nonlinear problems.

The demand of scientific research puts forward new requirements and challenges to the teaching of how to apply mathematical theory to solve practical problems, which urge current teaching to change from single theory teaching to simultaneous teaching of theory and application technology.

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2 Proposal of the concept

When it comes to the teaching of application techniques, people can easily think of the examples and their designs in Mathematics. No one would dispute that examples are the most successful teaching of applied technique in Mathematics. However, the examples can only deal with the simple application problem of the important principles in Mathematics, and usually do not involve too much knowledge and application problems in other disciplines, so they are very "small" and far from the teaching needs of scientific research. From this point of view, the design of examples belongs to the category of theoretical teaching. So, what method is suitable for teaching the applied technique of the Nonlinear Functional Analysis and helps to carry out scientific research training? We might think of CTM. The success of CTM in law, medicine and business teaching is universally praised. As we all know, case teaching method takes typical historical events as the material of case study, and in order to achieve the open effect of case teaching, the selected cases usually do not design a unique goal. In fact, it is difficult to achieve a unique goal because of uncertainty. In other words, the case does not hope to obtain the best result, but only hope to obtain better ones. From a mathematical point of view, the materials in CTM are similar to the results of project study in mathematics. However, there is a significant feature of project study in Mathematics, that is, the certainty (uniqueness) of the goal, which seems not completely consistent with the case in CTM. In addition, as a project study of applied mathematics, it inevitably involves multiple disciplines, often adopting specific research methods and technical routes, usually in the form of academic monographs, which are obviously too "large" to facilitate the studying of basic theory. Therefore, it is not appropriate to apply the research results of the Nonlinear Functional Analysis as a monograph to CTM.

Can we find a balanced teaching method between examples and monographs that can not only strengthen the basic ability but also train the scientific research ability? After years of teaching experiments, combining the advantages of examples and the design concept of CTM, the author summarizes and puts forward a balanced teaching method which applies the important principles of the Nonlinear Functional Analysis to scientific research, and calls it "Mathematical Case Teaching Method". Generally speaking, MCTM is a kind of discussion teaching method which is centered on a principle in Mathematics, and forms teaching cases with the application research results in other disciplines as materials. It uses the concept of CTM for reference to design cases, and carries out teaching activities on the basis of students full own research.

From the above description, we can see that MCTM conforms to the knowledge view of constructivism. The knowledge view of constructivism subverts the traditional teaching paradigm. It changes the teaching goal from knowledge mastery to scientific accomplishment, the teaching content from simplicity and certainty to problem and generative, and the teaching method from imprecation to inquiry[5,6]. MCTM is to train and develop students ability in scientific research, expand students cognition in application of mathematics, and let students to get practical experience of scientific research in open teaching. In short, MCTM tries to achieve the goal of "double-base training" (basic knowledge and basic scientific research skills).

3 The function and significance of MCTM

3.1 To reform classroom teaching method and train students thinking and innovation ability

Thinking and innovation are the two most important objectives of talent training. However, today graduate education mostly adopts the traditional teaching mode of "teacher teaching + students listening + examination". In addition, the teaching of less class hours and more content causes students to be in a state of half-understanding. Such a teaching is not conducive to the cultivation of thinking and innovation ability. MCTM based on application belongs to the category of open process teaching in essence, which is a powerful supplement to process teaching. Open teaching is based on the process of knowledge formation, which investigates the source of the problem, the composition of the problem and the whole process of its solution. It emphasizes the interaction between teachers and students. Therefore, MCTM can well improve students thinking ability, and enable students to experience the creative process, and to improve the hands-on ability and creative ability.

3.2 To strengthen basic training and enhance scientific research ability

Foundation is the source of acquiring new knowledge, but also the cornerstone of scientific research. Because of the teaching design and practice centered on mathematical principles, MCTM can further promote students understanding and cognition of mathematical knowledge and help students lay a solid mathematical foundation. At the same time, MCTM is designed on the base of the existing research results which include some successful methods, study skills and experience. Therefore, MCTM can greatly help the students learn to use mathematical theory to analyze, study and solve practical problems, train the students ability to engage in scientific research.

3.3 To promote the ability of independent thinking and cultivate the spirit of cooperation

The ability of independent thinking is the prerequisite for the growth of talents, and the spirit of cooperation is the booster for the growth of talents. Using MCTM, after gaining certain cognition of the case by self-study, the students will join the group for collective discussion. In the process of communicating and discussing with others, the students will gain more information and ideas to solve problems, and finally achieve the vision of common progress.

3.4 To change from focusing on knowledge into focusing on ability

In the traditional concept, people cognition of mathematical ability seems to always stay at the level of being able to do mathematical exercises. As a result, traditional mathematical teaching only pays attention to the teaching of mathematical knowledge and mathematical exercise, but neglects the training of mathematical comprehensive ability. This almost stubborn concept not only misunderstands mathematics as a useless subject, but also misleads people cognition of Mathematics, and so greatly hurts the enthusiasm of those students who love Mathematics. MCTM is to train the students ability to use mathematical knowledge comprehensively, and to change from teaching mathematical knowledge into teaching both basic and application, so as to achieve the purpose of training students comprehensive ability of Mathematics. This teaching method tries to change the situation

that students can only do mathematical exercises, let students learn to "read living books", and establish the confidence that mathematics is useful, and cultivate the ability of students to solve practical problems.

3.5 To change from passive leaning into active learning

The original intention of MCTM is to break the teaching mode of teachers speaking and students listening, and make the classroom become truly student-centered place of teaching. In such a teaching mode, from the moment students get the case, the focus of teaching will no longer be on the teacher side, but on the students side, and the classroom has become a student-centered teaching space. MCTM is to let students, through independent thinking or cooperation, learn to find problems, and use the clues given by the case to explore the way to solve the problem, put forward policies to solve the problem, and form the final conclusion. The teaching process of a mathematical case is also the active learning process of the students.

4 Differences between MCTM and traditional teaching methods

Although MCTM emphasizes basic teaching, it is different from the teaching of basic knowledge in the traditional sense, and it is not like the traditional method to directly teach point-to-point knowledge. For the basic knowledge, teachers do not participate in direct teaching, but let students learn by themselves or cooperatively.

Although MCTM is a kind of discussion-based teaching method, it is impossible to conduct traditional classroom teaching because of the large "case". In practice, teaching becomes a model in which students study by themselves in advance and then focus on discussion under the teaching guidance.

MCTM always focuses on the application of an important principle and has a clear goal, therefore, CTM that may expand the problem infinitely cannot be completely applicable to MCTM. In other words, if CTM is to pursue a "better" result, then MCTM is to pursue a "best" result.

The differences between MCTM and traditional teaching methods are summarized in the following table 1.

Table 1. Comparative table of traditional teaching methods and MCTM

process	traditional teaching method	MCTM
Teaching of knowledge	Point-to-point,focus on	Self-study, auxiliary teaching
Involved disciplines	Single-discipline	Multi-discipline
Restricts in application	Simple applications within the course,such as discussion teaching	Unrestricted, emphasis on scientific research in other disciplines
Preset results	Better, like CTM	Best
Time for preparation	Shorter, it can even unfold in class	Too long to follow

5 Conclusion

This paper puts forward an extended Case Teaching Method, namely Mathematics Case Teaching Method, expounds the purpose and significance of this method, and compares it with traditional teaching method. The biggest advantage of this method is that it can help students to carry out scientific research training.

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Reference

1. Z.Li, X.Yuan, Q.Zhang, Existence of critical points for noncoercive functionals with critical Sobolev exponent. Applicable Analysis,2021, DOI: 10.1080/00036811.2021.1892078
2. Y.Zhou, X.Tang, On existence of periodic solutions of a kind of Rayleigh equation with a deviating argument, Nonlinear Analysis, 2008, pp. 2355–2361.
3. K.Zhou, Y.Zhou, Special Manasevich -Mawhin continuation theorems with applications (in Chinese), Mathematical Theory and Applications, 2020, 40(1), pp. 19-33.
4. X.Tang, Y.Zhou, Periodic solutions and global attractivity of a class of nonlinear functional differential equations (in Chinese), Acta Mathematica, 2006, 49(4), pp. 899-908.
5. R. Liu, constructivism: View of knowledge, view of learning and view of teaching (in Chinese), People's Education, 2005 (17), pp.9-11.
6. W. Yuan, From Receiving to Construction the transformation of knowledge view and the reconstruction of science teaching paradigm (in Chinese), Global Education, 2005(2), pp.18-23.