Modeling RMB Exchange Rate Volatility – Application of GARCH Family Models

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Abstract. The exchange rate risk caused by the two-way fluctuation of the RMB exchange rate will bring many effects. The volatility of the foreign exchange market is the most common feature of the financial market. Therefore, the research on the volatility of the RMB exchange rate is of great significance in economic and financial aspects. Through statistical analysis of the RMB exchange rate data, an ARMA model was established to eliminate the auto-correlation of the sequence, and the GARCH family model was combined to fit the data. Comparing different distribution hypotheses, the EGARCH model under the GED distribution determined by the information criterion can match well the financial time series peak and thick tail characteristics. That the RMB exchange rate has agglomeration volatility and leverage effect is also shown, and provides relevant suggestions for preventing RMB exchange rate risks.

1 Introduction

1.1 Research background and purpose

In the process of RMB internationalization, the fluctuation of exchange rate gradually intensified [1]. In 2020, the global financial market experienced significant fluctuations, but the overall flexibility of the RMB exchange rate continued to increase, and the market-oriented process of the exchange rate made steady progress. The fluctuation of RMB exchange rate depends on China's economic fundamentals in the long run, while it is more susceptible to market supply and demand and international foreign exchange market in the short run [2]. With the increasing influence of the global financial crisis and the impact of the uncertain event of COVID-19 on the exchange rate, exchange rate changes are more frequent, and the resulting exchange rate risk has a great impact on various industries. Large fluctuations of exchange rate will affect the profitability and even the survival of enterprises. Increased exchange rate risks will affect the investment decisions of enterprises and hinder the improvement of enterprise productivity [3]. Mastering exchange rate fluctuations and deepening the understanding and understanding of the exchange rate fluctuations can help to avoid exchange rate risks.

1.2 Reaearch content

Through literature review, many scholars have studied the characteristics of RMB exchange rate volatility in different periods. The RMB exchange rate reform in 2015 significantly improved the marketization degree of the RMB, and the COVID-19 epidemic caused increased fluctuations in the RMB exchange rate. Therefore, this paper selects the daily data of USD/RMB exchange rate from the exchange rate reform in 2015 to 2021 to conduct an empirical analysis on the time series of daily data of RMB exchange rate return by constructing GARCH higher-order model. The standard normal distribution, T distribution and GED distribution were selected to fit the residual sequence distribution [4]. Finally, the parameter significance and information criterion of each model are analyzed comprehensively to determine the optimal fitting model.

2 Theoretical basis of model

2.1 ARMA Model

ARMA model is a kind of common random time series model, which usually describes the evolution and change of things with the help of the random characteristics of time series. If the stationary random process has the characteristics of both autoregressive process and moving average process, it is not suitable to use AR(P) or MA(Q) models alone, but the two models should be mixed. Since this model contains two components, autoregression and moving average, denoted as ARMA(p,q), it is called autoregressive moving average model, and its specific form is as follows:

$$y_t = \phi_0 + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=1}^{q} b_j \varepsilon_{t-j} + \varepsilon_t \tag{1}$$

2.2 GARCH Model

In order to capture the conditional heteroscedasticity and more statistical features of financial time series, various

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ARCH models based on Engle's idea have been continuously developed since Engle [5] proposed the ARCH model. Bollerslev [6] extended the ARCH model to the general process, namely the GARCH model, whose conditional variance equation was set as:

$$\varepsilon_t = \sigma_t Z_t \tag{2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3}$$

In equations (2) and (3), σ_t^2 is the conditional variance, Z_t is the independent random variable with the same distribution, Z_t and σ_t^2 are independent of each other, p and q show the times of ARCH and GARCH terms respectively.

GARCH model can well explain the characteristics of volatility aggregation of financial asset return series, but it can not explain the leverage effect of financial time series, that is, negative shock or bad news produces greater volatility than positive shock or good news. Zakoian [7] proposed TARCH model to capture investors' asymmetric response to information, and its conditional variance was set as:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_i I_{t-i}) \ \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{4}$$

In equation (4), I_{t-i} is a dummy variable, when $\varepsilon_{t-i} < 0$, $I_{t-i} = 1$; Otherwise, $I_{t-i} = 0$. As long as $\gamma \neq 0$, asymmetric effect exists. Good news ($\varepsilon_{t\,i} > 0$) and bad news ($\varepsilon_{t\,i} < 0$) have different effects on conditional variance: good news has only a α-fold impact, while bad news has a ($\alpha+\gamma$) fold impact. If $\gamma>0$, it indicates that the main effect of asymmetric effect is to increase the fluctuation. If $\gamma<0$, the effect of asymmetry is to reduce the fluctuation. In order to overcome the non-negative constraint of GARCH model parameters, Nelson [8] proposed the EGARCH model, which reflects the asymmetric effect of positive and negative return on assets, and its conditional variance is set as follows:

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2) \tag{5}$$

Since this is modeling $\ln(\sigma_t^2)$, the conditional variance is positive even if the parameter estimate is negative. Therefore, the EGARCH model does not require artificial assumption of non-negative constraint constraints on model parameters. At the same time, if $\gamma=0$, it indicates that there is leverage effect. If $\gamma=0$, there is no asymmetric effect[9].

3 Empirical Analysis

3.1 Sample selection

In order to enhance the marketization of the dollar-RMB exchange rate, the central bank announced the reform of the RMB exchange rate system. After the exchange rate reform, RMB exchange rate fluctuated greatly. In consideration of this situation, this paper selected the central parity of USD/RMB exchange rate from January 1, 2017 to December 31, 2021 as the sample data, with a total of 1217 sample observation values. In the form of natural logarithmic rate of return:

$$r_t = \ln p_t - \ln p_{t-1} \tag{6}$$

p_t is the midpoint of exchange rate between US dollar and RMB on the t_{st} day, and P_{T-1} is the midpoint of exchange rate between US dollar and RMB on the $t\text{-}1_{st}$ day. As shown in Fig.1, the sequence chart of logarithmic return rate obviously presents the phenomenon of volatility cluster. The large and small fluctuations show the phenomenon of aggregation, which indicates that the logarithmic return rate series of USD/RMB exchange rate has heteroscedasticity.

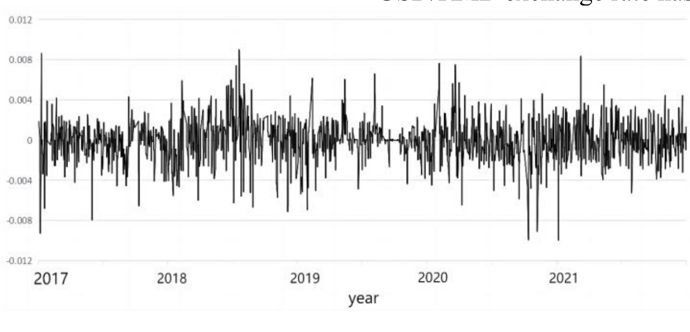

Fig. 1. Log return (Eviews)

3.2 Statistical characteristics

The statistical results in Fig.2 show that compared with the standard normal distribution curve, the logarithmic rate of return has an obvious characteristic of sharp peak and thick tail. The standard normal distribution and T distribution were selected in this paper to fit residual sequence distribution.

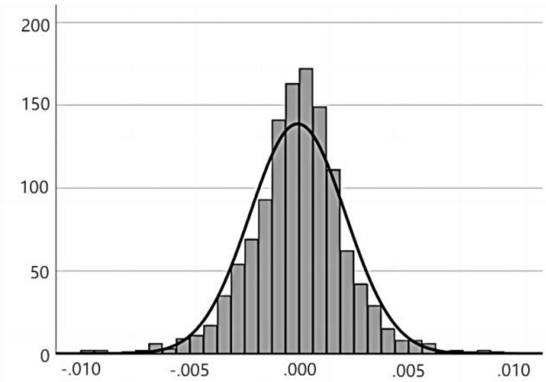

Fig. 2. Frequency (Eviews)

3.3 ARMA Model

3.3.1 Correlation test and stability test

The ARIMA model is constructed to test whether the logarithmic return rate series of RMB exchange rate is stable. The first step is to test the data for autocorrelation.

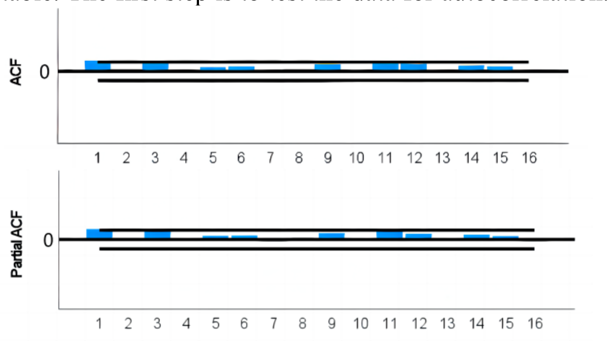

Fig. 3. Correlation test (SPSS)

It can be seen from the Fig.3 that almost all numbers of the autocorrelation graph (ACF) and partial autocorrelation graph (partial ACF) of the sequence are located within the confidence interval, indicating that the sequence is basically stable. The next step is to build the ARIMA model.

Model Statistics

model	predictive variables	degree of fitting		Ljung-Box Q(18)			number of outliers
		Stationary R squared	R squared	statistics	DF	significance	
log return	1	.000	.000	25.585	18	.110	0

Fig. 4. ARIMA (SPSS)

SPSS was used for modeling and the data in the Fig.4 was obtained. R squared is less than 0.001, therefore, the logarithmic yield sequence is a stationary series supporting the building of the ARMA model. According to Table 1, with 95% confidence, P values of Q statistics are less than 0.05 for both low-order autocorrelation coefficient and partial autocorrelation coefficient, indicating that this sequence has obvious autocorrelation characteristics. To deter- mine the optimal lag order, it is necessary to judge the fitting results of each model by information criterion.

Table 1. Correlation test of series (the first ten lines)

lagged value	AC	PAC	Q	P
1	0.064	0.064	4.977	0.026
2	0.013	0.009	5.183	0.075
3	0.051	0.050	8.336	0.040
4	0.006	0.000	8.382	0.079
5	0.024	0.023	9.100	0.105
6	0.029	0.024	10.159	0.118
7	0.007	0.011	10.218	0.177
8	0.014	0.012	10.442	0.235
9	0.043	0.039	12.747	0.174
10	0.005	0.000	12.777	0.236

Table 2. ARMA model information criteria

AR	MA	AIC	SC
1	1	-9.419593	-9.407011
1	2	-9.412251	-9.399669
1	3	-9.414660	-9.402078
2	1	-9.412251	-9.399669
2	2	-9.414651	-9.402069

According to AIC or SC information criteria, the optimal order of ARMA model is determined, as shown in Table 2. As can be seen from the comparison results in Table 2, the AIC value ARMA(1,1) model is the lowest. Therefore, when constructing logarithmic return series model, this model is of first priority to fit the mean equation.

3.3.2 Residual test

From Table 3, the sample correlation functions of residual sequences are all within the 95% confidence interval, and the corresponding P value is greater than 0.05. Therefore, there is no correlation among the residual sequences of the model estimation results, and the model is tested reliable.

Table 3. Residual test of ARMA model

lagging value	AC	PAC	Q-Stat	Prob
1	0.030	0.030	1.1014	—
2	-0.022	-0.023	1.6752	—
3	0.018	0.020	2.0920	0.148
4	-0.027	-0.029	2.9782	0.226
5	-0.007	-0.005	3.0417	0.385
6	-0.001	-0.002	3.0428	0.551
7	-0.038	-0.037	4.7987	0.441
8	-0.016	-0.014	5.1076	0.530
9	0.016	0.015	5.4127	0.610
10	-0.023	-0.024	6.0814	0.638

3.4 GRACH Model

3.4.1 Construction of ARMA-GRACH Model

Using the LM method to determine whether the residual sequence has an ARCH effect, the resulting probability value is 0, rejecting the null hypothesis that there is no ARCH effect, indicating that the heteroscedasticity of the logarithmic yield of the RMB exchange rate can be constructed by the ARMA model to fit the heteroscedasticity of the logarithmic yield of the RMB exchange rate by using the ARMA model to determine whether the residual series has an ARCH effect.

According to the residual series correlation chart, it is determined that the ARMA(1,1)-GARCH(1,1) model depicts the volatility characteristics of the RMB exchange rate, and the estimated results are shown in Table 4.

Table 4. Model estimation

parameter	normal	T	GED
AR(1)	0.974		
MA(1)	-0.953		
ω	$3.02*10^{-7}$	$1.04*10^{-7}$	$1.41*10^{-7}$
α	0.089	0.069	0.073
β	0.848	0.914	0.899

As can be seen from Table 4, all model parameters are significant in the 95% confidence interval compared with different distribution assumptions. Moreover, the model coefficient $\alpha+\beta<1$ satisfies the stationarity condition of GARCH model.

However, LM-ARCH test was conducted on the residual after fitting, and the P value was less than 0.05, so the null hypothesis was rejected, indicating that the variance equation of GARCH model fitting rate of return still needs to be improved. Considering the asymmetry and leverage effect of financial time series, TARCH model and EGARCH model are added for comparative analysis.

3.4.2 Construction of ARMA-EGARCH Model

In Table 5, the parameters of the estimated model are significant within 95% of the confidence intervals. First, the LM ARCH test is performed on the fitted residuals, and the low-order EGARCH models fail the test, so they are excluded. Secondly, through the principle of information criteria, the high-order EGARCH models are compared, and the EGARCH (2,1) model under the condition of GED distribution can better fit the data. Finally, an estimate of ARMA (1,1)-EGARCH (2,1) can be obtained:

$$\ln\hat{\sigma}_t^2 = -0.279 + 0.274\frac{|\hat{\varepsilon}_{t-1}|}{\hat{\sigma}_{t-1}} - 0.132\frac{|\hat{\varepsilon}_{t-2}|}{\hat{\sigma}_{t-2}} - 0.066\frac{\hat{\varepsilon}_{t-1}}{\hat{\sigma}_{t-1}} + 0.977\ln\hat{\sigma}_{t-1}^2 \quad (7)$$

Table 5. Model estimation (EGARCH)

	EGARCH(1,1)		EGARCH(1,2)			EGARCH(2,1)		
Para-meter	T	GED	N	T	GED	N	T	GED
ω	-0.232	-0.338	-0.835	-0.308	-0.456	-0.522	-0.202	-0.279
α1	0.147	0.158	0.252	0.205	0.227	0.294	0.251	0.274
α2	—	—	—	—	—	-0.136	-0.113	-0.132
β1	0.98	0.972	0.482	0.575	0.5	0.957	0.983	0.977
β2	—	—	0.449	0.399	0.462	—	—	—
γ	-0.032	-0.025	-0.027	-0.048	-0.044	-0.067	-0.069	-0.066

In the estimation of the EGARCH model's fitted variance equation, the coefficient estimate for the asymmetric term is -0.066, less than zero and statistically significant. The good news produced a 0.208 (0.274-0.066) shock, and the bad news generated a 0.340 (0.274+0.066) shock, thus verifying that the bad news had a greater impact on the fluctuations of the RMB exchange rate, that is, there was a so-called "leverage effect".

4 Conclusion

In this paper, by comparing the model results under different distribution assumptions, the yield sequence after logarithmic difference is a stationary series, and its peak is described and statistically analyzed, and its peak is much greater than the peak of the normal distribution, and the sequence shows a peak thick tail distribution, indicating that the sequence does not follow the normal distribution. The sequence is autocorrelation tested, which shows autocorrelation, so choose an ARMA model to eliminate the autocorrelation of the mean equation. By constructing the ARMA-EGARCH composite model to fit the variance equation, the empirical results show that there is a leverage effect on the fluctuation of the RMB exchange rate, and the impact of negative news has a greater impact on the fluctuation of the RMB exchange rate.

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