A Multi-stage Scheme Decision Model for Complex Products Based on Prospect Theory and Earned Value Method

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Abstract. The development process of complex products is highly uncertain and risky, cause the balance among progress, cost and technology needs to be comprehensively considered. Therefore, how to make decisions on key development plans becomes a problem that needs to be studied and solved. The existing multi-stage decision model needs to determine the complete stage division, quantity and attribute value of all alternatives in each stage of the development process at the beginning of the decision. However, actually, given the characteristics of long cycle of complex products, high technical risks, and many uncertainties, it is difficult to give the above information in advance in practical work. Therefore, in view of the above shortcomings, this paper proposes a multi-stage scheme decision-making method based on prospect theory and earned value method. First, based on the current progress and cost performance of the project, the dynamic reference point is set by analyzing the deviation between the actual implementation and the plan; Then, the dynamic prospect values of the schedule, cost and technical indicators of each alternative are calculated, and the optimal attribute weights and the comprehensive prospect values of each alternative are obtained by constructing a linear programming model; Finally, the advantages and innovation of the proposed model are further demonstrated through specific application examples.

1 Introduction

Complex products refer to a class of products with complex composition, complex technology, complex development process, complex processing and manufacturing, and complex engineering management [1]. The development process is a systematic process with a long cycle and large consumption of people, money, and materials. Therefore, in order to maintain a relative balance between cost, progress, and technology, and ensure the successful completion of research and development tasks, how to choose the key R&D scheme has become the key problem to be studied and solved in complex product management [2].

As NASA pointed out [3], the life cycle development process of complex products is a multi-objective optimization problem, and the multi criteria decision theory has been widely used in the decision-making of key development programs. At present, scholars at home and abroad mainly adopt two research ideas: single-stage scheme decision-making and multi-stage scheme decision-making. The former includes the entropy based and improved TOPSIS scheme optimization model proposed by Su Xiang et al. [4] for engineering change of complex products; Jiang Yanping et al. [5] determined the prospect reference point based on the performance of the new product development scheme compared with the competitive product scheme, and proposed a decision-making method based on the prospect theory; Fan Zhiping [6] proposed a decision-making method based on prospect theory, which provides a basis for handling the risk type mixed multi-attribute decision-making problem in which the decision-maker gives expected information. The latter mainly includes Hao Jingjing et al. [7], who identified the development characteristics of each stage and proposed a decision-making method with multi-stage mixed attributes; Liu Yong et al. [8] conducted research on stage weights and dealt with multi-stage decision-making problems based on comprehensive preference relations; Meysam et al. [9] defined the concept of attribute dominance, which is used to deal with multi-stage decision-making problems in which attribute evaluation information is represented by interval triangular fuzzy numbers.

It can be seen that, on the one hand, the concept of "decision-maker's risk attitude affects decision results" has been widely recognized by the current academic community [10], and scholars have conducted a lot of research based on prospect theory and achieved fruitful results [11-16]; On the other hand, in view of the high uncertainty and risk in the development process of complex products and the complexity of the decision-making problem itself [17], it is difficult to achieve scientific and reasonable optimal decision-making through a round of analysis, so the single-stage scheme decision-making has very obvious limitations. However, through in-depth analysis of the current multi-stage scheme decision-making method, we can see that [18], Its common feature is that it is necessary to determine the complete stage division, quantity and attribute value of all alternatives in each stage of the development process at the beginning of the decision, so that the dynamic reference point, stage weight, attribute weight and

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other information can be calculated based on the above information. However, in fact, it is difficult to give the above information in advance during the actual development of complex products [19]. With the deepening of the project, the above information needs to be confirmed periodically according to the actual implementation of the project (such as a key experiment, a type of major accessories procurement). Therefore, the existing multi-stage scheme decision-making method cannot be well applied to the actual decision-making of the key development scheme of complex products.

In addition, in reality, people often encounter decision-making problems based on heterogeneous information [20], that is, evaluation values under different attributes are expressed as precise values, fuzzy numbers, interval numbers or linguistic values [21-22]. For example, the attribute values of development cost and progress are usually expressed in the form of triangular fuzzy numbers. For example, the cost data of an alternative is, indicating that the development cost of the scheme is 800000 yuan in the most pessimistic case, 400000 yuan in the most likely case, and 200000 yuan in the most optimistic case; For the attribute value of technical indicators, such as "corrosion resistance grade", the attribute value can be expressed with an accurate value, while the attribute value of "noise intensity" is often expressed with a language value. Therefore, how to unify the heterogeneous attribute values coexisting with the above multiple expressions is also one of the key problems to be solved in the decision-making method of complex product multi-stage development scheme.

Based on the above analysis, this paper proposes a multi-stage scheme decision-making model based on prospect theory and earned value method. The key difference between this model and existing methods is to solve the problem of unknown stage division, quantity and attribute value of alternatives in the multi-stage decision-making process. Firstly, this method unifies the attribute values of different forms of technical indicators into the form of interval numbers and normalizes them; Then, based on the current progress and cost performance of the project, the deviation between the actual implementation and the plan is calculated based on the earned value method to set a dynamic reference point; Then, according to the prospect theory, the dynamic prospect value of the progress, cost and technical indicators of each alternative is calculated. On this basis, the corresponding weight is calculated by building a planning model, and finally the comprehensive prospect value of each alternative is calculated to achieve optimal selection.

2 Principle and method

2.1 Trigonometric Fuzzy Numbers and Related Operations

Definition 1 Let $a=[a^l,a^m,a^u]$,where $0<a^l\le a^m\le a^u$,then call a is a triangular fuzzy number. The operation rules of triangular fuzzy numbers are as follows:

If $a=[a^l,a^m,a^u]$, $b=[b^l,b^m,b^u]$ are triangular fuzzy numbers, then:

$$a+b=[a^l,a^m,a^u]+[b^l,b^m,b^u]=[a^l+b^l,a^m+b^m,a^u+b^u]$$

$$a-b=[a^l,a^m,a^u]-[b^l,b^m,b^u]=[a^l-b^l,a^m-b^m,a^u-b^u]$$

$$a\times b=[a^l,a^m,a^u]\times[b^l,b^m,b^u]=[a^lb^l,a^mb^m,a^ub^u] \quad \frac{1}{a}=[\frac{1}{a^l},\frac{1}{a^m},\frac{1}{a^u}]$$

$$\lambda a=[\lambda a^l,\lambda a^m,\lambda a^u],\quad \lambda\ge 0$$

Assume $d(a,b)$ is a triangular fuzzy number, the distance between a and b [7]is:

$$d(a,b)=\sqrt{\frac{1}{3}[(a^l-b^l)^2+(a^m-b^m)^2+(a^u-b^u)^2]}$$

According to literature [7], the size of triangular fuzzy numbers can be compared by calculating the possibility degree. The relevant calculation formula is as follows:

$$P(a\ge b)=\begin{cases}1, & a^m\ge b^m\\ \dfrac{(b^l-a^u)}{(a^m-a^u)-(b^m-b^l)}, & a^m<b^m,\ b^u\ge a^l\\ 0, & others\end{cases}$$

For two triangular fuzzy numbers a and b, if $P(a\ge b)\ge P(b\ge a)$, it is deemed $a\ge b$; Otherwise, if $P(a\ge b)\le P(b\ge a)$, it is considered $a\le b$.

2.2 Transformation between exact number, linguistic variable and interval number

Quantitative indicators and qualitative indicators are two major categories of decision attribute indicators. As the name implies, the former is an indicator that can be quantified and accurately measured, and its measurement results are reflected in accurate quantities (accurate numbers, interval numbers, etc.); The latter is an indicator that cannot be directly quantified, needs to be indirectly expressed by means of fuzzy expressions or other quantitative indicators, and cannot be directly measured. For the treatment of qualitative indicators, the specific problems of decision experts are generally analyzed. However, considering the fuzziness and descriptive characteristics of such indicators, their measurement results are usually given in the form of linguistic values. Based on the need of generalization and operability, this paper refers to the research work in [23-26], and uniformly converts all decision attribute values into interval number form, whose interval number matrix can be expressed as $A=[a_{ij}^L,a_{ij}^U]_{m\times n}$,the value of $[a_{ij}^L,a_{ij}^U]$ is

$$[a_{ij}^L,a_{ij}^U]=\begin{cases}[x_{ij},x_{ij}], \textit{The attribute value is an exact value}\\ [x_{ij}^L,x_{ij}^R], \textit{The attribute value is interval number}\end{cases}$$

For the form that the decision attribute value is a language variable value, suppose that the set of uncertain language terms of τ is $L=\{l_0,l_1,...,l_\tau\}$, then the language variable $l_p(p=0,1,2,...,\tau)$ can be converted to interval number by the following formula $a_l=[a_l^L,a_l^U]$:

$$\begin{cases} a_0^L = 0 \\ a_l^U = a_l^L + 1/\tau, 0 \le l \le \tau \\ a_l^L = a_{l-1}^U, 1 \le l \le \tau \\ a_\tau^U = 1 \end{cases} \quad (1)$$

2.3 Earned value management method

The earned value management method was born in the 1960s [27], and was organized and developed by the US Department of Defense. Since its birth, it has been widely concerned by domestic and foreign academia and industry. The earned value management method can objectively measure and reflect the progress and cost of the project by analyzing the data of existing parameters, and provide the basis for managers to evaluate the project stage. In addition, the earned value management method can also predict the future development trend of the project from both the progress and cost aspects by combining the existing data and earned value data, and further provide a reference for managers to control the project and achieve goals. The three basic parameters of the earned value management method are: the budget cost (PV) of the planned workload, the budget cost (EV) of the completed workload, and the actual cost (AC) of the completed workload.

PV=Planned Progress × Planned cost

EV=Actual Progress × Planned cost

AC=Actual Progress × actual cost

On this basis, the progress performance index SPI and the cost performance index CPI are defined. When SPI (the project earned value is higher than the planned value) or CPI (the budget cost is higher than the actual cost), the actual progress of the project is ahead of schedule or the actual cost is lower than the budget cost; When SPI (project earned value compared to planned value) or CPI (budget cost compared to actual cost), the actual progress of the project is consistent with the planned progress or the actual cost is exactly consistent with the budget cost; When SPI (project earned value compared with planned value) or CPI (budget cost compared with actual cost), it indicates that the actual progress of the project lags behind or the actual cost exceeds the budget cost.

$$SPI = EV/PV$$
$$CPI = EV/AC$$

At the same time, SPI and CPI can be used to predict the cost and duration status when the whole project is completed, including the estimated completion cost FCAC and the estimated completion duration ECD. Among them, TBC represents the total planned cost of the project and OPD represents the total planned duration of the project.

$$FCAC = TBC/CPI \quad (2)$$

$$ECD = OPD/SPI \quad (3)$$

3 A Multi stage Scheme Decision Model for Complex Products Based on Prospect Theory and Earned Value Method

3.1 Problem description

Suppose $S = \{s_1, s_2, \ldots s_n\}$ represents *n* stages in the development process of complex products, and S_t represents the *t* stage.

- $A = \{\overset{t}{a}_1, \overset{t}{a}_2, \ldots \overset{t}{a}_m\}$: represents the collection of alternatives at S_t stage, where $\overset{t}{a}_i$ represents the *ith* alternative for S_t stage, $i = 1, 2, \ldots, m$.
- $Z = \{Z_1, Z_2, \ldots, Z_h\}$: represents a collection of natural states, where Z_g represents *g*th stage, P_g indicates the probability of occurrence of natural state Z_g, meeting the conditions of $P_g \ge 0$ and $\sum_{g=1}^{h} P_g = 1$.
- $C = [\overset{t}{c}_{ig}]_{m\times h}$: represents the planned cost of alternative $\overset{t}{a}_i$ under natural state Z_g in S_t stage. For the convenience of expression, the specific cost composition of $\overset{t}{c}_{ig}$ is not subdivided in this paper. In view of the fuzziness and uncertainty of alternatives in the development process, we consider that $\overset{t}{c}_{ig}$ is a triangular fuzzy number, namely $\overset{t}{c}_{ig} = [\overset{t}{c}{}_{ig}^{l}, \overset{t}{c}{}_{ig}^{m}, \overset{t}{c}{}_{ig}^{u}]$, indicates the most optimistic input cost, the most likely input cost and the most pessimistic input cost, $\overset{t}{c}{}_{ig}^{l} \ge \overset{t}{c}{}_{ig}^{m} \ge \overset{t}{c}{}_{ig}^{u} \ge 0,\ i = 1, 2, \ldots, m$, $g = 1, 2, \ldots, h$.
- $D = [\overset{t}{d}_{ig}]_{m\times h}$: represents the planned progress of alternative $\overset{t}{a}_i$ under natural state Z_g in S_t stage. Similar to planned input cost, we consider that $\overset{t}{d}_{ig}$ is also a triangular fuzzy number, namely $\overset{t}{d}_{ig} = [\overset{t}{d}{}_{ig}^{l}, \overset{t}{d}{}_{ig}^{m}, \overset{t}{d}{}_{ig}^{u}]$,meeting the conditions of $\overset{t}{d}{}_{ig}^{l} \ge \overset{t}{d}{}_{ig}^{m} \ge \overset{t}{d}{}_{ig}^{u} \ge 0$ and $i = 1, 2, \ldots, m, g = 1, 2, \ldots, h$.
- $Q = [\overset{t}{q}_{ijg}]_{m\times k\times h}$: represents decision matrix for technical indicators of alternative $\overset{t}{a}_i$ at S_t stage, $\overset{t}{q}_{ijg}$ represents the attribute value of the *jth* technical index of alternative $\overset{t}{a}_i$ under natural state Z_g. In this paper, according to the reality, the type of $\overset{t}{q}_{ijg}$ has the form of exact value, interval number, linguistic value or triangular fuzzy number.
- $w^t = (w_1^t, w_2^t, w_3^t)$: indicates the weight of cost, progress and technical indicators at S_t stage, meeting $0 \le w_1^t, w_2^t, w_3^t \le 1$ and $w_1^t + w_2^t + w_3^t = 1$.

- $\overset{t}{OC}, \overset{t}{BC}, \overset{t}{OD}, \overset{t}{BD}$: respectively represents the actual cost expenditure, planned cost expenditure, actual progress and planned progress of the project at S_t stage.

3.2 Normalization of technical index attribute value

First, heterogeneous technical index attribute values need to be converted, that is, different types of attribute values are expressed (converted) into interval numbers. This paper adopts the method proposed in subsection 1.2. For the case that the attribute value is an accurate number, it is converted into an interval number with the same upper limit and lower limit according to formula (1); If the attribute value is a language variable, it will be converted into the corresponding interval number according to formula (2).

Then the attribute values in the form of interval numbers are normalized. At the same time, this paper uses formulas (5) and (6) to transform the above attribute value primitive matrix into interval number form based on the range transformation method $Q=[\overset{t}{q}_{ijg}]_{m\times k\times h}$, normalize to matrix $B=[\overset{t}{b}_{ijg}]_{m\times k\times h}$, where interval number $\overset{t}{q}_{ijg}=[\overset{t}{q}{}^{L}_{ijg},\overset{t}{q}{}^{U}_{ijg}]$, normalize to interval number $\overset{t}{b}_{ijg}=[\overset{t}{b}{}^{L}_{ijg},\overset{t}{b}{}^{U}_{ijg}]$, and $0\le \overset{t}{b}{}^{L}_{ijg}\le \overset{t}{b}{}^{U}_{ijg}\le 1$. The influence of dimension on attribute data is eliminated.

When attribute j is a utility type technical indicator:

$$\begin{cases} \overset{t}{b}{}^{L}_{ijg}=\dfrac{\overset{t}{q}{}^{L}_{ijg}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}}{\max\limits_{i}\{\overset{t}{q}{}^{U}_{ijg}\}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}} \\ \overset{t}{b}{}^{U}_{ijg}=\dfrac{\overset{t}{q}{}^{U}_{ijg}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}}{\max\limits_{i}\{\overset{t}{q}{}^{U}_{ijg}\}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}} \end{cases} \tag{4}$$

When attribute j is a cost based technical indicator:

$$\begin{cases} \overset{t}{b}{}^{L}_{ijg}=\dfrac{\max\{\overset{t}{q}{}^{U}_{ijg}\}-\overset{t}{q}{}^{U}_{ijg}}{\max\limits_{i}\{\overset{t}{q}{}^{U}_{ijg}\}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}} \\ \overset{t}{b}{}^{U}_{ijg}=\dfrac{\max\{\overset{t}{q}{}^{U}_{ijg}\}-\overset{t}{q}{}^{L}_{ijg}}{\max\limits_{i}\{\overset{t}{q}{}^{U}_{ijg}\}-\min\limits_{i}\{\overset{t}{q}{}^{L}_{ijg}\}} \end{cases} \tag{5}$$

3.3 Dynamic reference point setting

According to the idea of prospect theory, the evaluation results of decision-makers mainly focus on the "gain" or "loss" of the scheme performance relative to a certain reference level, but the design of reference points cannot only rely on subjective factors such as the subjective preference and psychological state of decision-makers, especially for more complex decision-making problems, it is difficult for decision-makers to determine a scientific and reliable reference level only based on subjective opinions [7]. Therefore, many scholars determine the decision reference point from the perspective of mining decision information. From the existing literature, the current reference point selection method still focuses more on the single stage decision-making problem, and the setting ideas are traditional zero point, mean value, positive and negative ideal schemes and so on; However, when facing multi-stage decision-making problems, dynamic reference points are mainly set based on the attribute values of alternatives at each stage. As mentioned above, the above methods are difficult to be applied to the actual decision-making of key development schemes for complex products.

This paper believes that due to the characteristics of complex products such as long cycle, high technical risk and many uncertain factors, the relative balance between schedule, cost and technology is the most important issue for decision-makers. Therefore, when setting dynamic reference points, it is necessary to consider not only the advantages and disadvantages of technical indicators between different schemes, but also whether the cost and schedule can be controlled within the scope allowed by the project plan according to the current development trend, which is the advantage of earned value management. Therefore, the schedule deviation degree $\Delta\overset{t}{Dur}$ and cost deviation degree $\Delta\overset{t}{Cos}$ in the S_t stage of complex product development process are defined based on the earned value method.

Definition 2: Progress deviation degree of complex products $\Delta\overset{t}{Dur}$,represents the difference between the estimated completion period of S_t stage and the total planned project duration (OPD).

$$\Delta\overset{t}{Dur}=ECD_t-OPD$$

Definition 3: The deviation degree of complex product cost $\Delta\overset{t}{Cos}$, represents the difference between the estimated completion cost of S_t stage and the total planned cost of the project (TBC).

$$\Delta\overset{t}{Cos}=FCAC_t-TBC$$

Using Eqs. (3) and (4), $FCAC_t$ and ECD_t can be calculated. When $\Delta\overset{t}{Dur}$ or $\Delta\overset{t}{Cos}$ >0, represents current cost overrun of the project $\Delta\overset{t}{Cos}$ or overdue construction period $\Delta\overset{t}{Dur}$;When $\Delta\overset{t}{Dur}$ or $\Delta\overset{t}{Cos}$ =0, represents that the actual implementation of the current progress and cost of the project is consistent with the plan; When $\Delta\overset{t}{Dur}$ or $\Delta\overset{t}{Cos}$ <0,represents that the current cost of the project has a balance $\Delta\overset{t}{Cos}$ or Construction period ahead of plan $\Delta\overset{t}{Dur}$.

As shown in Figure 1 below, for the decision-maker, the ideal state of the project under the schedule and cost criteria in the multi-stage decision-making process should be that the actual implementation is consistent with the plan. If the actual cost or progress of a stage is greater than the planned value, it means that the project is in the overdue or overspent state, and the remaining duration and cost need to be reduced; If the actual cost or progress of a stage is less than the planned value, it indicates that the project is in balance, and the remaining duration and cost can be increased accordingly. In view of this, this paper defines the dynamic reference points and for the cost and schedule $\overset{t}{D}_{dur}$, $\overset{t}{C}_{cost}$ of S_t stage, as shown in definitions 4 and 5.

Definition 4 The dynamic reference point of progress criteria $\overset{t}{D}_{dur}$ for S_t stage is:

$$\overset{t}{D}_{dur} = OPD - \overset{t}{O}D - \Delta\overset{t}{D}ur$$

Definition 5 The dynamic reference point of cost criteria $\overset{t}{C}_{cost}$ for S_t stage is:

$$\overset{t}{C}_{cost} = TBC - \overset{t}{O}C - \Delta\overset{t}{C}os$$

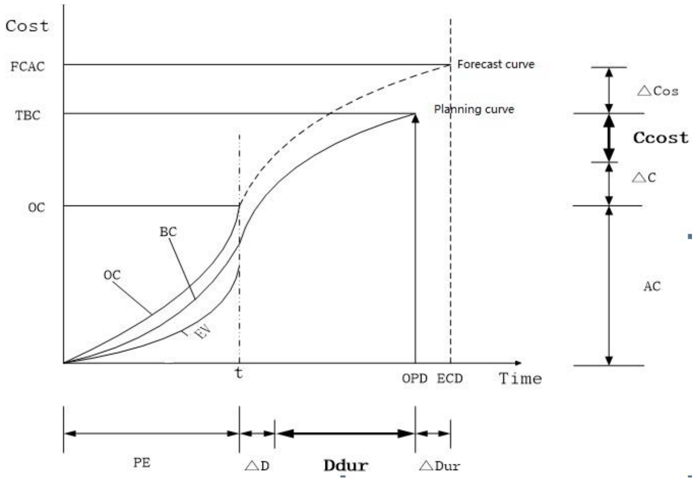

Fig. 1. Cost and Progress Dynamic Reference Point Setting Based on Earned Value Method (Self-drawn)

It can be seen that, for the planned cost $\overset{t}{c}_{ig}$ of alternatives $\overset{t}{a}_i (i=1,2,...m)$ in the S_t stage in the natural state Z_g, when $\overset{t}{c}_{ig} > \overset{t}{c}_{cost}$, it represents that the planned input cost of the alternative $\overset{t}{a}_i$ is greater than the cost reference point $\overset{t}{c}_{cost}$ .At that time, the psychological perception of the decision-maker was "loss"; When the planned input cost of the alternative $\overset{t}{a}_i$ is less than the cost reference point $\overset{t}{c}_{cost}$, and the psychological perception of the decision-maker is "benefit". According to the prospect theory, the dynamic value function and probability weight function of the cost criterion of alternatives $\overset{t}{a}_i$ in the natural state Z_g under the multi-stage scenario are given, as shown in Definition 6:

Definition 6 The dynamic value function of cost $\overset{t}{V}_{i1}(c_{ig})$ and probability weight function of cost $\pi_{i1}(P_g)$ for alternatives $\overset{t}{a}_i$ in the natural state Z_g for S_t stage is:

$$\overset{t}{V}_{i1}(c_{ig}) = \begin{cases} (d(\overset{t}{c}_{ig}, \overset{t}{c}_{cost}))^{\alpha}, & \overset{t}{c}_{ig} \le \overset{t}{c}_{cost} \\ -\theta(d(\overset{t}{c}_{ig}, \overset{t}{c}_{cost}))^{\beta}, & \overset{t}{c}_{ig} > \overset{t}{c}_{cost} \end{cases}$$

$$\pi_{i1}(P_g) = \begin{cases} (P_g)^{\gamma} / [(P_g)^{\gamma} + (1-P_g)^{\gamma}]^{1/\gamma} & \overset{t}{c}_{ig} \le \overset{t}{c}_{cost} \\ (P_g)^{\delta} / [(P_g)^{\delta} + (1-P_g)^{\delta}]^{1/\delta} & \overset{t}{c}_{ig} > \overset{t}{c}_{cost} \end{cases}$$

Correspondingly, $0 < \gamma < 1$, $0 < \delta < 1$. As the value for parameter α、β、θ、γ、δ , the existing research has produced innovative research results[28] - [29], which can be obtained through empirical research $\alpha = \beta = 0.88$, $\theta = 2.25$, $\gamma = 0.61$, $\delta = 0.69$.

Definition 7 The dynamic prospect value $\overset{t}{V}_{i1}(a_i)$ of S_t stage alternatives $\overset{t}{a}_i (i=1,2,...m)$ under cost criteria is:

$$\overset{t}{V}_{i1}(a_i) = \sum_{g=1}^{h} \overset{t}{V}_{i1}(c_{ig}) \pi_{i1}(P_g)$$

Similarly, for the planned progress $\overset{t}{d}_{ig}$ of S_t stage alternatives $\overset{t}{a}_i$ in the natural state Z_g , this paper gives the dynamic prospect value under the progress criteria in the multi-stage situation, as shown in Definition 8:

Definition 8 The dynamic value function of progress $\overset{t}{V}_{i2}(d_{ig})$ and probability weight function of progress $\pi_{i2}(P_g)$ for alternatives $\overset{t}{a}_i (i=1,2,...m)$ in the natural state Z_g for S_t stage is:

$$\overset{t}{V}_{i2}(d_{ig}) = \begin{cases} (d(\overset{t}{d}_{ig}, \overset{t}{d}_{dur}))^{\alpha} & \overset{t}{d}_{ig} \le \overset{t}{d}_{dur} \\ -\theta(d(\overset{t}{d}_{ig}, \overset{t}{d}_{dur}))^{\beta} & \overset{t}{d}_{ig} > \overset{t}{d}_{dur} \end{cases}$$

$$\pi_{i2}(P_g) = \begin{cases} (P_g)^{\gamma} / [(P_g)^{\gamma} + (1-P_g)^{\gamma}]^{1/\gamma} & \overset{t}{d}_{ig} \le \overset{t}{d}_{dur} \\ (P_g)^{\delta} / [(P_g)^{\delta} + (1-P_g)^{\delta}]^{1/\delta} & \overset{t}{d}_{ig} > \overset{t}{d}_{dur} \end{cases}$$

Definition 9 The dynamic prospect value $\overset{t}{V}_{i2}(a_i)$ of S_t

stage alternatives $\overset{t}{a}_i (i=1,2,...m)$ under progress criteria is:

$$\overset{t}{V}_{i2}(a_i)=\sum_{g=1}^{h}\overset{t}{V}_{i2}(d_{ig})\pi_{i2}(P_g)$$

On this basis, this paper further discusses the setting of the dynamic reference point of technical indicators at St moment. Following the traditional idea of positive and negative ideal scheme setting, the positive and negative ideal points are selected as the dynamic reference points of the technical indicators at the stage s_t time by comparing the attribute values of the technical indicators of the standardized alternative schemes. The calculation formulas of the positive and negative ideal points are as follows:

$$\overset{t}{r}_j^{+} = [\overset{t}{r}_j^{+L}, \overset{t}{r}_j^{+U}] = [max(\overset{t}{b}_{ijg}^{L}), max(\overset{t}{b}_{ijg}^{U})], i = 1,2,\dots m \tag{6}$$

$$\overset{t}{r}_j^{-} = [\overset{t}{r}_j^{-L}, \overset{t}{r}_j^{-U}] = [min(\overset{t}{b}_{ijg}^{L}), min(\overset{t}{b}_{ijg}^{U})],\ i = 1,2,\dots m \tag{7}$$

In order to obtain the value function, this paper uses the distance between the attribute value of technical indicators in each alternative and the positive ideal point to represent the income region in the value function; Similarly, the distance between it and the negative ideal point is used to represent the loss area in the value function, and the distance between $j(j=1...k)$ technical index attribute value $\overset{t}{b}_{ijg}$ of the alternatives $\overset{t}{a}_i(i=1,...m)$ and positive, negative ideal points $d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{+})$, $d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{-})$ are calculated separately.

$$d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{+}) = \frac{\sqrt{2}}{2}\sqrt{(\overset{t}{b}_{ijg}^{L} - \overset{t}{r}_j^{+L})^2 + (\overset{t}{b}_{ijg}^{U} - \overset{t}{r}_j^{+U})^2} \tag{8}$$

$$d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{-}) = \frac{\sqrt{2}}{2}\sqrt{(\overset{t}{b}_{ijg}^{L} - \overset{t}{r}_j^{-L})^2 + (\overset{t}{b}_{ijg}^{U} - \overset{t}{r}_j^{-U})^2} \tag{9}$$

Then, according to the prospect theory, we can get the dynamic prospect value of the attribute value $\overset{t}{b}_{ijg}$ of the j th technical indicator of alternative $\overset{t}{a}_i$, as shown in Definition 10:

Definition 10 The dynamic value function of the j th technical indicator $\overset{t}{V}_{i3j}(b_{ijg})$ for alternative $\overset{t}{a}_i(i=1,2,...m)$ in the natural state Z_g for S_t stage is:

$$\overset{t}{V}_{i3j}^{-}(b_{ijg}) = -\theta d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{+})^{\beta}$$
$$\overset{t}{V}_{i3j}^{+}(b_{ijg}) = d(\overset{t}{b}_{ijg},\overset{t}{r}_j^{-})^{\alpha}$$

The corresponding probability weight vectors of attribute values of different technical indicators of equipment selection scheme $\overset{t}{a}_i$ are:

$$\overset{t}{\pi}_{i3j}^{+}(P_g)=[\overset{t}{\pi}_{i31}^{+}(P_g),\overset{t}{\pi}_{i32}^{+}(P_g),...,\overset{t}{\pi}_{i3k}^{+}(P_g)]$$
$$\overset{t}{\pi}_{i3j}^{-}(P_g)=[\overset{t}{\pi}_{i31}^{-}(P_g),\overset{t}{\pi}_{i32}^{-}(P_g),...,\overset{t}{\pi}_{i3k}^{-}(P_g)]$$

where $\overset{t}{\pi}_{i3j}^{+}(P_g)$ and $\overset{t}{\pi}_{i3j}^{-}(P_g)$ represent respectively probability weight of the profit and loss faced by the alternative $\overset{t}{a}_i(i=1,2,...m)$ when it is in the natural state Z_g of attribute j, the corresponding calculation formula is:

$$\overset{t}{\pi}_{i3j}^{+}(P_g) = \begin{cases}(P_g)^{\gamma}/[(P_g)^{\gamma}+(1-P_g)^{\gamma}]^{1/\gamma}, & \overset{t}{V}_{i3j}(b_{ijg}) \geq 0 \\ 0 & \overset{t}{V}_{i3j}(b_{ijg}) < 0\end{cases} \tag{10}$$

$$\overset{t}{\pi}_{i3j}^{-}(P_g) = \begin{cases}0, & \overset{t}{V}_{i3j}(b_{ijg}) \geq 0 \\ (P_g)^{\delta}/[(P_g)^{\delta}+(1-P_g)^{\delta}]^{1/\delta} & \overset{t}{V}_{i3j}(b_{ijg}) < 0\end{cases} \tag{11}$$

Definition 11 The dynamic prospect value $\overset{t}{V}_{i3}(a_i)$ of S_t stage alternatives $\overset{t}{a}_i(i=1,2,...m)$ under technical index criteria is:

$$\overset{t}{V}_{i3}(a_i)=\sum_{g=1}^{h}\sum_{j=1}^{k}\overset{t}{V}_{i3j}^{+}(b_{ijg})\overset{t}{\pi}_{i3j}^{+}(P_g)+\sum_{g=1}^{h}\sum_{j=1}^{k}\overset{t}{V}_{i3j}^{-}(b_{ijg})\overset{t}{\pi}_{i3j}^{-}(P_g)$$

In order to eliminate the impact of dimension differences on the results, the dynamic foreground values of each indicator (cost, schedule, etc.) are normalized into $\overline{\overset{t}{V}}_{i1}(a_i)$, $\overline{\overset{t}{V}}_{i2}(a_i)$ and $\overline{\overset{t}{V}}_{i3}(a_i)$. The corresponding normalized formula is shown in Formula (13) where $\overline{\overset{t}{V}}_{iv}(a_i)=\frac{V_{iv}(a_i)}{\left|\overset{t}{V}_v\right|^{max}},\quad i=1,2,...,m,\quad v=1,2,3$.

$$\left|\overset{t}{V}_v\right|^{max} = max\left|\overset{t}{V}_{1v}(a_1)\right|, \left|\overset{t}{V}_{2v}(a_2)\right|, \dots \left|\overset{t}{V}_{mv}(a_m)\right|, \quad v=1,2,3 \tag{12}$$

3.4 Calculation of comprehensive prospect value of alternatives

According to the idea of prospect theory, combined with the dynamic prospect value defined under the criteria of cost, progress and technical indicators, the comprehensive prospect value of different alternatives in the multi-stage scenario can be obtained, as shown in Definition 12.

Definition 12 The comprehensive prospect value of alternatives $\overset{t}{a}_i(i=1,2,...m)$ $\overset{t}{V}(a_i)$ at S_t stage is:

$$\overset{t}{V}(a_i)=\sum_{v=1}^{3}\overset{-t}{V}_{iv}(a_i)w_v^t,\quad i=1,2....,m$$

Among it, the weights of cost, schedule and performance can be obtained by subjective assignment through expert experience or by building a mathematical model. The former is subjective and arbitrary, so this paper uses the latter to determine the weight value. According to the idea of prospect theory, the prospect value of the scheme can be positive or negative, and it will be adjusted with the development of the R&D process. Considering the fairness of the scheme, this paper takes the maximization of the overall prospect value of the scheme as the objective function to build a linear programming model:

$$max\overset{t}{V}=max\overset{t}{V}(a_i)=max\sum_{i=1}^{m}\sum_{v=1}^{3}\overset{-t}{V}_{iv}(a_i)w_v^t$$

$$s.t.\begin{cases}\sum_{v=1}^{3}w_v^t=1\\0\le w_v^t\le 1\end{cases}\tag{13}$$

The conditional extreme value Lagrangian multiplier method is used to construct the Lagrangian function[30]:

$$L(w,\lambda)=\sum_{i=1}^{m}\sum_{v=1}^{3}\overset{-t}{V}_{iv}(a_i)w_v$$

$$L(w,\lambda)=\sum_{i=1}^{m}\sum_{v=1}^{3}\overset{-t}{V}_{iv}(a_i)w_v+\lambda(\sum_{v=1}^{3}w_v^2-1)$$

Calculate the partial derivative of the above equation and make:

$$\frac{\partial L}{\partial w_v}=\sum_{i=1}^{m}\sum_{v=1}^{3}v_{iv}+\lambda=0$$

$$\frac{\partial L}{\partial \lambda}=\sum_{v=1}^{3}w_v^2-1=0\tag{14}$$

The optimal solution of the criterion weight is obtained and normalized to obtain the optimal criterion weight:

$$w_v=\frac{\sum_{1=1}^{m}v_{iv}}{\sum_{v=1}^{3}\sum_{i=1}^{m}v_{ij}},v=1,2,3\tag{15}$$

To sum up, the decision-making steps of the multi-stage scheme decision-making model for complex products based on prospect theory and earned value method are as follows:

Step 1: Unify the attribute values of different types of technical indicators in the alternatives into the form of interval numbers according to formulas (1) and (2), and normalize the above attribute values using the range transformation method using formulas (5) and (6);

Step 2: according to definitions (4) and (5), calculate the dynamic reference point under the cost and schedule criteria at time s_t, and calculate the dynamic reference point under the technical index criteria at time s_t by formulas (7) and (8);

Step 3: according to definitions (6) - (11), get the dynamic prospect value of each alternative at time s_t under the criteria of cost, schedule and technical indicators, and use formula (13) for standardization;

Step 4: get the criterion weight of cost, schedule and technical indicators according to Formula (14) - (16), and calculate the prospect value of each alternative using Definition (12);

Step 5: Rank and optimize each scheme based on the prospect value of the scheme.

4 Case study

In this part, the potential application of the above methods is illustrated against the background of the selection of the development scheme of a large container ship contracted by a shipbuilding company. According to the contract signed with the shipowner, the total cost of the project is planned to be 300 million yuan and the total construction period is planned to be 1095 days. The overall manufacturing plan is prepared according to the main performance and technical requirements of the large container ship. After the completion of an important experimental test, the initial scheme could not meet the performance requirements. To ensure the smooth completion of the project, the project management team urgently convened the overall team and worked out three alternatives (a_1,a_2,a_3), including:

Table 1. Project Current Status Information

	Percent Complete	Planned cost (10000 yuan)	Actual cost (10000 yuan)	Actual progress (days)	Total planned cost of the project (10000 yuan)	Total planned duration of the project (days)
Current information	95%	17000	17836	392	30000	1095

Table 2. Cost and Progress Data of Alternatives

Alternatives	Cost (10000 yuan)			Progress(days)		
Z_g	Best	Better	Medium	Best	Better	Medium
P_g	0.1	0.6	0.3	0.1	0.6	0.3
a_1	[8620,8750,9600]	[8700,8950,9680]	[8800,9000,9700]	[640,656,685]	[650,667,689]	[652,670,692]
a_2	[8800,9400,11000]	[8890,9530,11200]	[8900,9680,11360]	[636;651,670]	[643,657,672]	[649,660,680]
a_3	[8920,9780,11400]	[9000,10000,12000]	[9200,10280,12360]	[620,638,650]	[628,640,654]	[630,645,660]

Scheme a1: reduce the spacing of bone materials, adopt automatic submerged arc welding +a small amount of manual welding for assembly and welding of components, adopt sectional forward outfitting in sectional manufacturing, adopt gravity launching, and adopt dual island layout with separate residential area and engine room.

Scheme a2: appropriately increase the plate thickness, adopt semi-automatic submerged arc welding +a small amount of manual welding for assembly and welding of components, adopt sectional forward outfitting in sectional manufacturing, and adopt dual island layout with separate residential area and engine room for traction drainage.

Scheme a3: increase the size of components far away from the neutral shaft and appropriately increase the plate thickness. The semi-automatic submerged arc welding+CO_2 double wire welding is used for assembly and welding of components. The sectional reverse outfitting is used for the outfitting in the sectional manufacturing, and the traction type drainage is used. The residential area and the engine room are separated in the dual island layout.

The grades of relevant technical indicators are: weldability, corrosion resistance: poor, average, good, excellent; Structural strength grade: weak, medium, strong, stronger; Stability: Grade I, II, III, IV and V.

Firstly, the attribute values of different types of technical indicators in the alternatives are unified into interval number form according to formulas (1) and (2), and the above attribute values are normalized based on the range transformation method using formulas (5) and (6). The corresponding results are shown in Table 4.

Then, based on the earned value method, the current cost and schedule data of the project are processed to obtain the corresponding status information, and the dynamic reference points D_{dur}^{t} and C_{cost}^{t} under the cost and schedule criteria at time s_t are calculated according to the definitions (4) and (5).

Calculate the dynamic reference point under the technical criteria of the time s_t according to Formula (7) and (8)

According to the definitions (6) - (11), the dynamic prospect value of each alternative at time s_t under the criteria of cost, schedule and technical indicators is obtained, and the formula (13) is used for normalization. The specific results are shown in Table 6.

Finally, take the maximization of the overall prospect value of the scheme as the objective function to calculate the weight of the cost, schedule and technical indicators, use the definition (12) to calculate the prospect value of each scheme, and then sort out the best scheme.

According to formula (16), w_1^t=0.346 , w_2^t=0.312 , w_3^t=0.342 . According to definition (12), the foreground values of each scheme are: (-0.615, -0.670, -0.294), which is easy to know that scheme 3 is the best scheme.

Table 3. Technical Index Data of Alternatives

Alternatives	Weldability			Noise			Square coefficient			Corrosion resistance			Structure strength			Stability		
Z_g	Best	Better	Medium	Best	Better	Medium	Best	Better	Medium	Best	Better	Medium	Best	Better	Medium	Best	Better	Medium
P_g	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3
a_1	Good	Average	Average	[42,65]	[45,68]	[53,76]	[0.585,0.640]	[0.594,0.639]	[0.602,0.640]	Good	Average	Average	Stronger	Average	Average	IV	III	II
a_2	Excellent	Good	Average	[50,70]	[52,75]	[52,80]	[0.574,0.620]	[0.581,0.642]	[0.590,0.645]	Good	Good	Average	Stronger	Stronger	Average	IV	III	III
a_3	Excellent	Good	Average	[52,73]	[55,80]	[55,82]	[0.540,0.598]	[0.543,0.60]	[0.572,0.670]	Excellent	Good	Good	Strongest	Strongest	Average	V	IV	III

Table 4. Standardization of Technical Indexes of Alternatives

Alter-natives	Weldability			Noise			Square coefficient			Corrosion resistance			Structure strength			Stability		
Z_g	Good	Me-dium	Poor	Good	Me-dium	Poor	Good	Me-dium	Poor	Good	Me-dium	Poor	Good	Me-dium	Poor	Good	Me-dium	Poor
P_g	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3
a_1	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.672]	[0,0.657]	[0.033,0.8]	[0.45,1]	[0.,0.639]	[0.602,0.657]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]
a_2	[0.5,1]	[0.5,1]	[0,0.5]	[0.258,0.903]	[0.2,0.857]	[0,0.933]	[0.34,0.8]	[0.581,0.642]	[0.2,0.857]	[0,0.5]	[0.5,1]	[0,0.5]	[0,0.5]	[0.5,1]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]
a_3	[0.5,1]	[0.5,1]	[0.5,1]	[0.29,1]	[0.286,1]	[0.1,1]	[0.31,0.7]	[0.543,0.60]	[0.286,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0,0.5]	[0.5,1]	[0.5,1]	[0.5,1]

Table 5. Project Cost, Progress Status Information and Dynamic Reference Points Based on Earned Value Method

CPI	SPI	FCFA	ECD	$\triangle Dur_t$	$\triangle Cos_t$	C_{cost}^{t}	D_{dur}^{t}
0.905	0.950	33131.889	1152.632	3131.886	57.632	9032.111	645.368

Table 6. Dynamic Reference Points of Technical Criteria Based on Positive and Negative Ideal Points

	Weldability			Noise			Square coefficient			Corrosion resistance			Structure strength			Stability		
Z_g	Good	me-dium	poor	Good	me-dium	poor	Good	medium	poor	Good	me-dium	poor	Good	me-dium	poor	Good	me-dium	poor
P_g	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3	0.1	0.6	0.3

Positive	[0.5,1]	[0.5,1]	[0.5,1]	[0,0.743]	[0,0.657]	[0,0.8]	[0,0.58]	[0,0.576]	[0,0.576]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]	[0.5,1]
Negative	[0,0.5]	[0,0.5]	[0,0.5]	[0.290,1]	[0.286,1]	[0.1,1]	[0.451,1]	[0.515,0.97]	[0.306,1]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]	[0,0.5]

Table 7. Alternative Cost, Schedule and Technology Dynamic Value Function

	Cost			Progress			Technology		
Alternatives	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
Dynamic foreground value	199.462	-1303.650	-1745.270	-44.789	-30.395	9.066	-5.827	-3.398	-0.204
Normalized dynamic foreground value	0.114	-0.747	-1	-1	-0.679	0.202	-1	-0.583	-0.035

5 Conclusion

This paper focuses on the key development scheme decision-making problem in the complex product development process. Aiming at the shortcomings of the current multi-stage decision-making method, that is, the stage division of the whole decision-making process and the evaluation matrix of each stage need to be given in advance, but in fact, it is difficult to give the above information in the actual development process. A multi-stage scheme decision-making method based on prospect theory and earned value method is proposed. First, based on the current progress and cost performance of the project, the dynamic reference point is set by analyzing the deviation between the actual implementation and the plan; Then, the dynamic prospect values of the schedule, cost and technical indicators of each alternative are calculated, and the optimal attribute weights and the comprehensive prospect values of each alternative are obtained by constructing a linear programming model; Finally, the advantages and innovation of the proposed model are further demonstrated through specific application examples.

In this paper, Earned Value Method is introduced into the multi-stage scheme decision making problem of complex products, which provides a new research idea and analysis method for it. Later, we will continue to study the problems in the fields of improving completion forecast, optimizing the setting of dynamic reference points, multi-stage group decision making and so on.

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