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The Practicality of Vasicek Model in China's Financial Market

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Abstract. In the changing financial market, the price of financial products fluctuates continuously over time. The study of the static term structure of the interest rate on the market can no longer satisfy the actual needs, and the dynamic model is imperative. Compared with the static term structure, the dynamic model introduces a stochastic differential term on the basis of the static term structure model of interest rate. This paper shows some relevant models including Vasicek Model, Single-Factor Dynamic Model, Multi-Factor Dynamic Model, and Kalman Filter method. To conclude, in the multi-factor dynamic interest rate term structure model, Kalman Filter has many benefits. However, it still has actual limitations. The multi-factor Vasicek model still needs some analysis to find the error and do the correction. In the research of bond pricing, risk management, and other aspects, the multi-factor model should be the main direction.

1. Introduction

With the development of the financial market, the continuous change of the market has a huge effect on the interest rate, exchange rate, and security price. At the same time, the financial market faces significant risks. The research of interest rate models and the term structure of interest rate is promoted by theoretical basis and practical application [1]. The theory of term structure of interest rate provides a crucial theoretical basis for the pricing of financial products. It is of great significance to the whole financial market. For practical significance, the study of the term structure of interest rate also contributes to the rationalization of asset prices, thus improving the effectiveness of the capital market. This paper firstly introduces the Vasicek Model, the Ornstein-Uhlenbeck (OU) process, and the application of Shanghai Interbank Offered Rate (SHIBOR). Then the single-factor model is analyzed. Some of its assumptions are not valid in China's financial market. For the multi-factor dynamic model, this paper points out the three-factor model and demonstrates the correction direction. Finally, the Kalman Filter is introduced. When applying the Kalman Filter, it is essential to transform the Vasicek model into the state space representation form. This paper shows that further study should surround the dynamic multi-factor term structure model.

2. Vasicek model

Interest rate models can be divided into two categories: the Gaussian model and the non-Gaussian model. Based on the Gaussian model, the instantaneous rates may be negative, but its derivative pricing has analytic properties. The Vasicek model, a form of one-factor short-term interest rate model that describes interest rate movements when there is only one source of market risk, is a conventional Gaussian model. The Vasicek model can be built by the OU process.

The OU process is a stochastic process with dynamics and defined by the stochastic differential equation: $dx_t = -\theta x_t dt + \sigma dW_t$. In this equation, θ and σ are parameters, and W_t denotes the Wiener process. When an additional drift is added, $dx_t = -\theta(x_t - \mu)dt + \sigma dW_t$. Since θ and μ are greater than 0, the drift term means that the process tends to the mean retractable. Up to μ, it means that x_t has moved away from μ and will also advance back to μ with time. The OU process has a good time-series correlation. The equation can be deduced by using the method of solving linear first-order differential equation [2].

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{1}$$

$$Y = \left(\int e^{\int P(x)dx} Q(x)dx + c\right)e^{\int -P(x)dx} \tag{2}$$

$$dx_t = -\theta(x_t - \mu)dt + \sigma dW_t \tag{3}$$

$$\therefore Q = \theta\mu + \sigma\frac{dW_t}{dt} \tag{4}$$

$$\begin{aligned}\therefore x_t &= \left(\int e^{\int \theta dt}\left(\theta\mu + \sigma\frac{dWt}{dt}\right)dt + c\right)e^{\int -\theta dt} \\ &= \left(\int e^{\theta t}\left(\theta\mu + \sigma\frac{dWt}{dt}\right)dt + c\right)e^{-\theta t} \\ &= \left(\theta\mu\int e^{\theta t}\, dt + \sigma e^{-\theta t}\int e^{\theta s}\, dWs\right) + ce^{-\theta t}\end{aligned} \tag{5}$$

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$$= \mu + \sigma \int_0^t e^{-\theta(t-s)}\, dWs + ce^{-\theta t}$$

When $t = 0, x_0 = \mu + c, c = x_0 - \mu$

$$\therefore x_t = \mu + (x_0 - \mu)e^{-\theta t} + \sigma \int_0^t e^{-\theta(t-s)}\, dWs \tag{6}$$

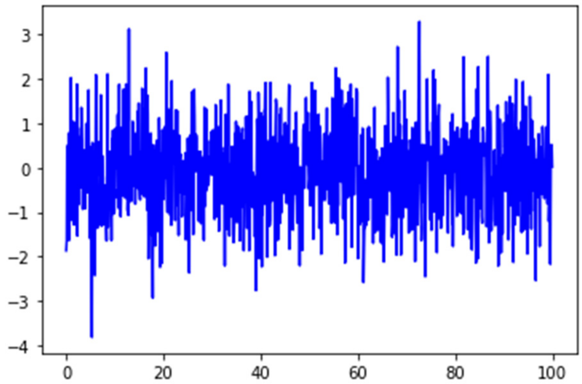

Fig. 1. The quantification of the OU process.

The equation of the Vasicek model is $dr_t = a(b - r_t)dt + \sigma dW_t$, of which b is the long-term average, a is the regression speed, and σ is the standard deviation parameter, which affects the fluctuation of the interest rate. The fluctuation amplitude has the characteristics of instantaneous random flow. a and b can be seen as the θ and μ in the OU process. The parameters and the initial condition r0 are completely dynamic and change instantaneously [3].

According to Ito lemma, a method of differentiating the functions for random processes,

$$d[e^{\theta t} r_t] = \theta e^{\theta t} r_t dt + e^{\theta t} dt = \cdots\cdots = e^{\theta t}[\theta \mu dt + \sigma dWt] \tag{7}$$

$$s \le t, e^{\theta t} r_t - e^{\theta s} r_s = \int_s^t e^{\theta u} \theta \mu du + \int_s^t e^{\theta u} \sigma dWu \tag{8}$$

$$e^{\theta t} r_t e^{-\theta t} = e^{\theta t} r_s e^{-\theta t} + e^{-\theta t}(\int_s^t e^{\theta u} \theta\, \mu du + \int_s^t e^{\theta u}\, \sigma dWu) \tag{9}$$

$$r_t = r_s e^{-\theta(t-s)} + \mu(1 - e^{-\theta(t-s)}) + \sigma \int_s^t e^{-\theta\ (t-u)}\, dWu \tag{10}$$

r_t is conditional on r_s, following a normal distribution,

$$E[r_t | r_s] = r_s e^{-\theta(t-s)} + \mu(1 - e^{-\theta(t-s)}) \tag{11}$$

According to Ito Isometry,

$$Var[r_t | r_s] = var[r_s e^{-\theta(t-s)} + \mu(1 - e^{-\theta(t-s)}) + \sigma \int_s^t e^{-\theta\ (t-u)}\, dWu] \tag{12}$$

$$= E[-(\sigma \int_s^t e^{-\theta\ (t-u)}\, dWu)^2]$$

$$= \frac{\sigma^2}{2\theta}[1 - e^{-2\theta(t-s)}]$$

Assume that r=0.05, μ=0.15, σ=0.05, using T=10, N=200, θ=0.004, 0.04, 0.4, the quantification is as follows,

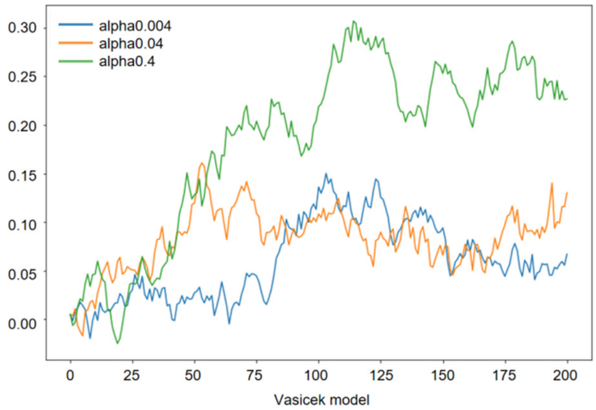

Fig. 2. The quantification of Vasicek.

The Vasicek model can be used in the valuation of financial derivatives, though it may get negative interests.

Empirical studies point out that the Shanghai Interbank Offered Rate (SHIBOR) of march maturity has the characteristics of mean reversion and thick tail. The Vasicek model has a good ability to depict and describe the dynamic characteristics of the SHIBOR market interest rate. It also has good applicability in studying the characteristics of the interest rate in China [4].

SHIBOR is a daily reference rate based on the interest rates at which banks offer to lend unsecured funds to other banks in the Shanghai wholesale (or "interbank") money market. There are eight SHIBOR rates, with maturities ranging from overnight to a year. They are calculated from rates quoted by 18 banks, excluding the four highest and the four lowest rates, and then averaging the remaining data. During the simulation of 1Y SHIBOR using the single-factor Vasicek model, the result cannot completely match the actual situation [3].

3. Single-Factor dynamic model and vasicek model

In the changing financial market, the price of financial products fluctuates continuously over time. The study of the static term structure of the interest rate on the market can no longer satisfy the actual needs, and the dynamic model is imperative. Compared with the static term structure, the dynamic model introduces a stochastic differential term on the basis of the static term structure model of interest rate [5]. It can well describe the characteristics of the yield curve over time. In the term structure model of the interest rate, there is usually no arbitrage model or general equilibrium model. The former is based on the expectation theory, and the latter is based on the liquidity preference principle. Because of the number of different factors in the model, the model can be divided into the single-factor and multi-factor dynamic interest rate models to explore the dynamic term structure.

In the single–factor dynamic term structure model of interest rates, the instantaneous interest rate is expressed as $dr_t = \mu r_t dt + \sigma r_t dWt$. $\sigma r_t dWt$ is the diffusion equation of the single-factor dynamic interest rate model, and $\mu r_t dt$ is the drift equation. The changes of these two function equations will continuously affect the instantaneous interest rate. Wt is the stochastic interest rate disturbance term in the model. It can be seen that the current data mainly influences the formula, and there is no relationship with the historical data. As a consequence, the instantaneous interest rate in the single-factor dynamic interest rate model is consistent with the Markov property. When a random process is given the present state and all the past states, the conditional probability distribution of its future state depends only on the present state, and the random process has Markov property [6].

Vasicek added a linear function to the drift function term in the single-factor dynamic interest model, then there was the Vasicek model. The value of the instantaneous interest rate and drift rate in the Vasicek model will reach the historical average level. In the Vasicek model, the price of risk in the market is usually set to a constant [6]. Setting the drift term as a function of the interest rate makes the model have the mean-reversion property. It makes the Vasicek model more consistent with the actual situation of the economic market. But this model can not exclude the possibility of a negative short-term interest rate and the drift function which is a linear equation does not coincide with the actual situation.

4. Multi-factor dynamic model and vasicek model

In a rapidly changing market economy, the factors affecting the dynamic process of the interest rate are more than a short-term interest rate. Considering the short-term interest rate only is obviously unable to meet the requirements of the market. Scholars began to introduce multi-factor dynamic interest rate models, which are the most widely used. The affine interest rate model is the most famous [7]. The equation is as follows:

$$r_t = \xi_0 + \sum_{k=1}^{n} \xi_k x_{tk} \quad (13)$$

$$dx_t = K^*(\theta^* - x_t)dt + H \times \sqrt{s_t} dw_t^* \quad (14)$$

The basic assumptions of the Vasicek model are: 1) due to the Markovian nature of instantaneous interest rate changes, the determination of the interest rate level is only affected by the current moment, and has nothing to do with historical interest rate data and future data; 2) the price function P(t, T) of a zero-coupon bond with maturity T represents the estimated function of $\{R(t^*), t \le t^* \le T\}$. Since R(t) is assumed to be Markovian, $R(t^*)$ is related to R(t); 3) the market is efficient. Full information is in the market, and there are no transaction costs; speculators expect the same thing about the same financial asset; speculators must be rational [8].

The Vasicek model has several characteristics: firstly, the instantaneous interest rate is an unobservable value, so after the simulation of the instantaneous interest rate, the interest rate curve needs to be restructured according to the path of the rate. Secondly, the price of bonds at different time points can also be calculated by P(t, T) formula of the Vasicek model [9]. However, the shortcoming of the model is that the fitting of the curve cannot be consistent with the market curve. Therefore, the Vasicek model is not a risk-free arbitrage model, but an equilibrium model [1]. From the perspective of risk management, Vasicek can effectively explain the change of the curve. As long as it is not user product pricing, this model is still useful. Finally, this model follows the normal distribution of the Wiener process and may have negative values. For the economic system of China, the Vasicek model is suitable for the national debt market of China [10]. Because the simulating result of the single-factor model is not ideal, the number of factors was increased and the three-factor Vasicek model was introduced.

5. Kalman filter method and vasicek model

The Kalman filter method was first proposed by Rudoff Emil Kalman. The Kalman filter method uses the state space function to describe the dynamic system, and the estimation method of model parameters mainly uses the prediction error decomposition to calculate the likelihood function value [1]. Compared with other classical time series models, the filtering model has the following advantages: firstly, the Kalman filter can comprehensively describe the state of a system with small present and past information sets, so there is no need to find a large amount of historical data. Using the Kalman filter to calculate the parameter matrix can find the most preferred estimate process. Secondly, the Kalman filter can not only describe the internal situation of the whole system but also express very comprehensive information, and it can well describe the relationship between the input and output variables [11]. Finally, in classical time series models, the modeling process is often based on some basic assumptions, however, it is hard to meet all the assumptions at the same time. There are not so many restrictions in the Kalman filter. Using the Kalman filter to calculate the parameter matrix can find the most preferred estimate process.

Because the Kalman filter method requires the model to be transformed into the state space form, it is necessary to transform the Vasicek model into the state space representation form when solving the parameters' value of the three-factor Vasicek model. It can be discussed from the equation of state and the equation of observation.

6. Conclusion

The value of capital is reflected in many aspects, the value of its time efficiency has always been one of the core problems in financial research, and the important tool to describe it is the dynamic term structure of interest rates. Hence, the term structure of interest rates must be involved in the pricing of underlying bonds or financial derivatives. The Vasicek model built by the OU process can be used in the valuation of financial derivatives, but there may be negative interest. The single-factor Vasicek model only includes the short-term interest rate, and some of its assumptions are not valid in the Chinese financial market [6]. The multi-factor dynamic interest rate term structure model plays a guiding role. Although the Kalman filter has many advantages, it also has certain limitations. For instance, the Kalman filter is generally based on the Markov property, which indicates that the future state and the past state described by the system are independent of each other. If a system does not satisfy the Markov property, it is not suitable to demonstrate by the Kalman filter. The three-factor Vasicek model still needs some empirical analysis to find the pricing error and do some corrections. Then the pricing accuracy can be significantly increased [1]. In conclusion, when interest rates need to be used in bond pricing, risk management, determination of benchmark interest rates, and other aspects, the dynamic multi-factor term structure model of interest rates should be taken as a main research direction.

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