Production Structure and Multiple Equilibria - A DSGE Perspective

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Abstract. This paper integrates the Austrian school's production structure theory and financial market theory, considers the impact of heterogeneity of firms' production cycles on economic fluctuations, constructs a DSGE model that incorporates production structure, and uses the model to analyse the existence of multiple equilibria and endogenous financial economic cycles. Compared with previous DSGE models, this paper highlights the asymmetry of firms' responses due to the existence of different production cycles and considers emotional shocks as the main source of economic volatility, as Keynes' "animal spirits" suggest.

1 Introduction

The mechanism of economic cycles has always been a major and highly controversial issue in macroeconomics. Three hypotheses about the mechanism of economic cycles were put forward at the very beginning of macroeconomics, which still have a profound influence on the frontier research in recent years: Keynes' "animal spirits" theory that irrational speculative behavior causes economic cycles. Fisher's "debt-deflation cycle" points to the principle of rapid economic decline in the face of negative shocks. Hayek's "Hayek's triangle" proposed the hypothesis that money injection breaks the tradition that the economy is in general equilibrium, considers the actual situation such as incomplete information and speculative behavior, and proposes new mechanisms of endogenous economic cycles such as financial instability, external financing uplift, and balance sheet channel, which makes the theory and model of economic cycles change accordingly. However, the economic cycle theories of Hayek and the Austrian School are yet to be mathematically realized and are less valued by the current economic cycle models.

Coevolving with economic cycle theory are economic cycle models. The mathematical theory of savings[31] pioneered a dynamic economic model of savings growth and rising welfare, and Samuelson and Solow built a complete model of savings accumulation and economic growth based on this model, taking into account the effects of commodity heterogeneity, which was carried forward by later economic cycle models.[³³] The RBC model and its successor, the DSGE model[¹⁸][²⁰], have been the most influential in the field of economic cycles. The RBC model maintains the view that the economy in general equilibrium and naturally assumes that economic cycles are not inefficient[⁶] and do not depart from Pareto optimality. However, general equilibrium presupposes that all people have the same beliefs[¹], so most of the later researchers no longer follow this assumption. To explain the high correlation between nominal and real variables, the NK-DSGE model introduces the assumptions of price and wage stickiness, and the CEE model takes into account changes in capital adjustment costs and capital utilization, as well as the effects of expectations and information[⁹][¹⁸][¹⁹] after the Great Recession of '08, the DSGE model, one, incorporates the ideas of financial economic cycle theory by introducing into the model expectations bias and expectations shocks, credit constraints, and After the Great Recession of 2008, the DSGE model, on the one hand, incorporates the financial economic cycle theory perspective by introducing expectation bias and expectation shocks, credit constraints, and liquidity constraints into the model to explain the endogenous financial instability, and on the other hand, it starts to consider the heterogeneity of household utility functions and corporate managers to make the model more realistic.

It should be noted that DSGE models have been developed to provide plausible explanations for many economic phenomena such as self-fulfilling expectations, economic fluctuations in the economy, perverse price declines after monetary expansion, and the correlation between various sectors of the economy in fluctuations, but they still lack explanations for phenomena that are highly correlated with economic cycles such as managerial externalities, correlation between fixed asset prices and economic fluctuations. At the same time, researchers of other economic cycle models have come up with many new theories and models on these issues that are highly innovative and have explanatory power. One of
the most prominent contributors was Roger E. A. Farmer[30], who developed a systematic theory of belief functions to explain financial instability, multiple equilibria, and the fulfillment of self-fulfilling prophecies, and was an early adopter of the non-deterministic economic system that had long been neglected by DSGE. After the Great Recession, SCHMITT-GROHE used a full information approach to filter expected volatility and demonstrated that expected volatility contributes 50% of economic volatility.[21] ANGELETOS innovatively developed a two-stage model to explain that imperfect information and sentiment shocks can endogenize economic cycles.[22][23] ZHENG proposed that fixed asset prices affect economic cycles through a credit constraint channel and BERAJA uses regional differences to better explain the U.S. economic cycle than the nominal rigidity used in the NK-DSGE model.[17] Simsek finds that belief heterogeneity leads to speculative behavior, and then builds a macroeconomic model of belief dynamics adjustment, which points out the conditions for the occurrence and collapse of financial bubbles.[32]

This paper argues that combining the aforementioned research in the fields of financial economics and information economics with the DSGE model will serve to further optimize it. At the same time, the business cycle theory of the Austrian school had excellent explanatory power at the time but has not been widely studied in the field of financial and economic cycles, but is instead valued in the field of management. This is also a promising new direction for DSGE model improvement. Therefore, based on the current research, the contributions of this paper are: first, considering the effects of different production structures, this paper considers the effects of the production cycle as proposed by the Austrian school that goods with longer production cycles differ in economic volatility. [22][23] ZHENG proposed that fixed asset prices affect economic cycles through a credit constraint channel and BERAJA uses regional differences to better explain the U.S. economic cycle than the nominal rigidity used in the NK-DSGE model.[17] Simsek finds that belief heterogeneity leads to speculative behavior, and then builds a macroeconomic model of belief dynamics adjustment, which points out the conditions for the occurrence and collapse of financial bubbles.[32]

2 Models

2.1. Exogenous demand and prices

The production cycles of different products differ significantly, from a few hours for bread to several years for ships. Suppose that there are J goods in the market with different production cycles. First assume that capital is homogeneous (i.e., the capital used to produce different goods is the same; to liberalize the above assumption, one can just assume that it costs to convert capital on different production lines to each other).

production function:

\[ y_{j,t+1} = A_{j,t} f(k_{j,t}, l_{j,t}) \]  

\[ E(\pi_{j,t}) = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]  

\[ y_{j,t+1} \] is the output of the product of production cycle \( j \), in period \( t+1 \). \( A_{j,t} \) is the technology of the product in period \( t \). \( K_{j,t} \) and \( L_{j,t} \) are the amount of capital and labor, respectively, invested into the product in period \( t \).

The capital stock follows the following equation.

\[ K_{j,t} = (1 - \delta)K_{j,t-1} + I_{j,t} \]  

There is a capital price in the market. The enterprise must acquire new capital for investment at the capital price. However, the non-cash capital held by the firm has low liquidity, so the price at which the firm sells the old asset should be less than the price at which it acquires the new asset. We measure this by Tobin's q, i.e., price of old assets sold = q × price of new assets acquired. (This is also a manifestation of barriers)

We assume here first that the prices of goods, capital and labor are exogenous, then the profits accumulated by a firm going through a production process are:

\[ \pi_{j,t} = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]  

If the firm wants to ensure that its capital quantity at the beginning of each production round is constant, then \( y_{j,t} = [1 - (1 - \delta)]k_{j,t-1} \). For simplicity, let \( \Delta = 1 - (1 - \delta) \). Then the profit function can be rewritten as

\[ f(K_{j,t-1}, l_{j,t-1}) = k_{j,t-1}^{\alpha} l_{j,t-1}^{1-\alpha} \]  

Assume that the production function obeys the Cobb-Douglas functional form:

\[ E(p_{j,t}) = p_{j,t} y_{j,t} \]  

The firm's expectations are assumed to be static, i.e., the price at the beginning of production is assumed to be equal to the price at the end of production. Thus there are:

\[ E(\pi_{j,t}) = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]  

So the expected level of profit is:

\[ \pi_{j,t} = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]  

The profit-optimal capital and labor use is as follows:

\[ k_{j,t-1} = \frac{1+\alpha}{1-\alpha} p_{k,t} A_{j} \]  

Assuming that the demand function is an exogenously given, downward sloping demand curve:

\[ p_{j,t} = a - by \]  

Since the payoffs to scale are constant, it is clear that the optimal output of the firm is:

\[ y = \frac{a}{2b} \]

2.2 Endogenous model

As Say's law says, “A product is no sooner created, than it, from that instant, affords a market for other products to the full extent of its own value”. Any commodity needs to be exchanged for other commodities. Therefore, the demand for all goods comes from the production of other goods. Suppose that the utility functions of all people in the market are homogeneous and obey the following form:

\[ U_{t} = \sum_{j=1}^{J} c_{j,t} + \omega \sum_{j=1}^{J} l_{j,t} \]

where \( c_{j,t} \) and \( l_{j,t} \) are the consumption of good \( j \) and the amount of labor in the production of good \( j \), respectively, and \( \omega \) is the coefficient. In this paper, we accept the Austrian school that goods with longer production cycles

\[ y_{j,t+1} = A_{j,t} f(k_{j,t}, l_{j,t}) \]  

\[ E(\pi_{j,t}) = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]  

\[ \pi_{j,t} = p_{j,t} y_{j,t} - p_{k,t} A_{j} k_{j,t} - \omega c_{j,t} + \lambda \]

\[ k_{j,t-1} = \frac{1+\alpha}{1-\alpha} p_{k,t} A_{j} \]  

Assuming that the demand function is an exogenously given, downward sloping demand curve:

\[ p_{j,t} = a - by \]  

Since the payoffs to scale are constant, it is clear that the optimal output of the firm is:

\[ y = \frac{a}{2b} \]
are more pedestrian and bring higher levels of welfare. Therefore, the utility function needs to satisfy the feature that a unit of a commodity with a longer production cycle will bring a greater marginal utility. However, it cannot take the common form \( u = x^a y^b \), because this form would lead to zero utility without holding any of the goods. The function also needs to satisfy diminishing marginal utility. The problem of "capital reversing" (Samuelson, 1965), which was raised in the Cambridge capital controversy, is discussed in the appendix of this paper.

The purpose of such an equation is to make the prices of products and labor not depend on exogenous assumptions, but on endogenous parameters that are spontaneously formed in the market. Clearly, at the time of profit maximization is between the price of the good and the cost of production margin spontaneously formed in the market. Therefore, the utility function needs to satisfy the feature consistent with the general DSGE model with the addition of consideration for goods with different production cycles.

### 2.3 Incomplete information, belief functions, and expected volatility

Both the RBC and NK-DSGE models assume that firms plan their production based on fundamental quantities (utility function, production function, and technological constraints), which actually follows the rational expectations theory proposed by Lucas. This theory essentially assumes that the equilibrium state is unique and therefore human estimates naturally fluctuate around this equilibrium value (or correct value). However, empirical studies have demonstrated the existence of multiple equilibria in the economy, i.e., that human beliefs have a significant impact on the equilibrium performance of the economy.

To address the above issues, this paper follows the model idea of ANGELETOS and combines Roger Farmer’s belief function theory, assuming that the production process of the firm is divided into two stages, the first stage receives external signals and determines the current production plan based on the belief function; the second stage places the product on the market, conducts information exchange and product trading. Taking into account the differences in the production cycles of different firms, this structure is shown in the figure.

The obvious fact is that the longer the time, the greater the degree of incompleteness of information. It is also for this reason that companies with long production cycles tend to be more cautious in determining their production schedules. Before starting production of a good, the entrepreneur should observe the supply of other goods on the market and make an estimate of the demand for this product based on the basic volume, and then start production.

The more the supply of other goods, the more the demand for this good will be. Therefore, the demand function for this good is assumed to be of the following form:

\[
D_t = f(Q_t, p_t)
\]
where: \( Q = (Q_1 \ldots Q_n) \), representing the output of n products.

The function satisfies the following mathematical characteristics:

\[ \frac{\partial \eta_j}{\partial q_j} > 0 \quad (j \neq i), \quad \frac{\partial \eta_j}{\partial p_i} < 0. \]

Following the belief function given by Farmer, it is assumed that the firm first plans production and then determines the price based on the demand curve and production. The production planning function is as follows.

\[ y_{t+1}^e = a y_{t+1}^e + (1 - a) M_t(r, w, e, E_t(Q)) \]

where: \( 0 < a < 1 \).

\( Q = (Q_1 \ldots Q_n) \), representing the production of n products, respectively. \( E_t(Q) \) on the other hand, represents the expectation for other products in the current period. \( r, w \) are the prices of capital and labor, and \( e \) represents the stochastic shock observed by this firm when making the production plan. Because the production of other goods is not necessarily the same in each period, and this variation is a stochastic shock to demand in the perspective of this firm. \( m \) is the production plan that determines the price based on the demand curve and production.

Then analyze the function \( E_t(Q) \). The firm’s expectation of the output of other firms depends not only on the basic quantity, but also on the information received by the firm, which can be divided into two categories, the first of which is a positive shock and the second a negative one. This shock is independent of the changes in the basic quantities (utility, capital, technology, etc.) of the firm. It is precisely because the firm is under incomplete information that it is affected by emotional shocks. Thus there are:

\[ E_t(Q) = Q_{t-1} + \xi_t \]

where \( Q_{t-1} \) is the output of other firms in the previous period and \( \xi_t \) represents the sentiment shock to the firm. This is the production plan of the firm considering the belief function. In the next section of the paper, the formula, financial markets and speculative behavior are introduced to explain the endogenous mechanisms of financial economic cycles.

### 2.4 exacerbated economic volatility - financial speculation, bubbles and animal spirits

A recent study by Alp Simsek [32] asserts that the source of financial speculation is heterogeneous beliefs (heterogeneous beliefs). This paper follows the above view. There is a large body of previous research on the formation of belief heterogeneity and its principles leading to financial speculation [37][10][13][15], which will not be elaborated much in this paper.

This paper follows Alp Simsek's assumptions: (1) There are two types of speculators in the market, optimists and pessimists. (2) The market prohibits shorting, optimists can add leverage, but pessimists can not short (here shorting only refers to borrowing assets for sale, the sale of their own holdings is not shorting in this paper). These two assumptions are important because the pessimists are not allowed to short, which means that the pessimists' attitude is often not reflected in the market.

Assume that the ratio of optimists at the beginning is \( \alpha \) and the ratio of pessimists is \( 1 - \alpha \). The marginal pessimists will change to optimists when capital prices continue to rise to the point where speculation can benefit, and the marginal optimists will change to pessimists when the potential risks in the market are generally perceived. The above hypothesis is consistent with the basic characteristics of a financial bubble, i.e., a financial bubble accumulates when marginal investors are more optimistic than the average optimism, and vice versa, asset prices fall.

In this paper, the speculative bubble model is based on the five-stage theory of financial bubbles proposed by Minsky[15][27]. Since most capital as a factor of production is very illiquid and does not need to be considered in the speculation problem, it can be assumed that there is only one type of capital in the market that can be used for speculation. The change in the price of this capital depends on the buyers and sellers, and the buying and selling behavior depends on the expectations of the price of the capital. The expectation of the price of the capital in a given period depends on the previous period's price, the theoretical price calculated on the basis of the underlying quantity and the sentiment shock. Therefore, the speculative market is modeled as follows.

\[ \Delta p_t = \Delta p_t(Q_{b_t} - Q_{b_s}) \]

\[ Q_{b_t} = Q_{b_t}(E_p, \sigma_t) \]

\[ Q_{s_t} = Q_{s_t}(E_p, \sigma_t) \]

\[ E_{p_t} = \Delta p_{t-1} + (1 - a) f + \xi_t \]

\[ \sigma_t = \sigma(p_{t-1}, f, \xi_t, Q_{s_{t-1}}) \]

\[ \Delta p \] is the net price change function of the capital, \( Q_b \) is the total buy volume function of the capital, \( Q_s \) is the total sell volume function of the capital, \( \sigma_t \) is the risk aversion, \( f \) is the theoretical price of the capital calculated from the fundamental quantity, and \( \xi_t \) is the sentiment shock. Typical examples of sentiment shocks are those resulting from the birth of a new technology or a new product, such as the meta-universe and chatgpt. Due to the presence of shorting restrictions, the case of negative sentiment shocks is not considered in this paper.

Speculators are themselves participants in the market, and since the previous sections did not deal with the issue of profits earned by firms, it is assumed here that a portion of the money used to purchase investments comes from the profits of firms. The capital available for speculation is finite, and according to the previous function, either a sentiment shock or a fundamental-driven increase in the price of capital stimulates speculative behavior. \( \alpha \) is the degree of risk aversion of the speculator, and in competitive markets is closely related to the amount of capital sold, the degree of deviation from the fundamental quantity price, and random sentiment shocks.

### 3 Economic Fluctuation Analysis

Exogenous fluctuations originate from stochastic perturbations in technology, sentiment, and utility, and are generally consistent with a first-order autoregressive process. The NK-DSGE model has been well described.
for the sources of exogenous shocks. The present model follows the previous description of exogenous shocks.

The present model can explain the endogeneity of economic cycles and the existence of multiple equilibria, in particular, this existence is related to the production cycle. (1) If a short-cycle good is subjected to a positive sentiment shock, which in turn raises production, the long-cycle good will observe a rise in demand prior to production, which in turn raises production accordingly, reaching a new equilibrium. A long-cycle good that receives a positive sentiment shock to raise production will also stimulate demand for the short-cycle good, reaching a new equilibrium, but the effects of the two on each other are not perfectly symmetric.

(2) When subjected to a sentiment shock, speculators raise their expectations of the price of capital, which in turn increases purchases, which in turn further increases the price of capital, which in turn further pushes up expectations, and leads to the formation of a financial bubble. But as the financial bubble inflates, the risk increases, and eventually the financial bubble bursts when a certain percentage of speculators who sell off assets reach.

(3) Financial speculation increases the total wealth during the boom, which in turn will stimulate the demand and production of goods during the boom (income effect). However, different commodities have different production cycles, and their speed of response in the face of demand stimulus is different.

In summary, both multiple equilibria and endogenous economic cycles are present in this model. For the mathematical description of the above analysis, this paper is arranged in the appendix.

4 Conclusion

This paper constructs a DSGE model that considers different production cycles and uses the above model to analyze the response of products with different production cycles when they face shocks, explaining the mechanisms by which two endogenous economic cycles, namely multiple equilibria and financial economic cycles, arise. The main contribution of this paper is to consider the production structure, which has been neglected by DSGE models for a long time. The production structure is derived from the theory of Pombavik, but the main point of the theory is that the production structure becomes more circuitous over time and thus provides more efficient and useful goods. However, the model in this paper does not consider the production structure itself as a variable, which is a major shortcoming of the model.

Appendix

For simplicity, the changes in profits and capital volumes of individual firms are analyzed assuming first that the level of technology is constant and that there is a continuously growing demand shock for all goods and that all firms can start the next round of production only when the current round is over.

Figure1. changes in firm profits under demand stimulus at j = 1.

For enterprises with long production cycles, the problem that must be noted is that profits are not available in the current period. Assuming that the firm plans production from the moment j=0, the firm has only expenses and no income from j=1 until the moment before the end of the firm's production. For a firm with a long production cycle, it is assumed that it acquires capital and then hires labor at the beginning of the production cycle. Assuming that the firm's production according to the expected plan is y, the first period is as follows:

\[ \pi_{j,1} = -p_k I \]  
\[ \pi_{j,2} = w l \]  
...  
\[ \pi_{j,j-1} = w l \]  
\[ \pi_{j,j} = p_j y - p_k \Delta_l I \]  

where \( p_k \) represents the price of capital, \( w \) is the wage level, \( L \) is the amount of labor employed, \( p \) is the price of the product, \( y \) is the output, and \( \Delta_l \) represents the depreciation of capital at the end of a production cycle. The difference between subsequent periods and the first period is that there is no more initial capital acquisition step (only the depreciation of the previous capital needs to be added).

The expected output and the planned amount of capital and labor, obey the relationship: \( I = k^{-1}(y) \) \( I = L^{-1}(y) \).

where \( k^{-1}(y) \) represents the inverse function of the relationship between output and capital and \( L^{-1}(y) \) represents the inverse function of the relationship between output and labor. Assuming the same structure of different production functions, analyze the change in profit of such firms under demand stimulation.
References


