A fuzzy model for assessing mathematical competences of university students

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Abstract. Within a competency-based approach to professional education, the paper outlines the concept of mathematical competence, grounds an urgency for effective mathematical competences and their assessment through a constructed fuzzy model. It promotes objective criteria for defining quantitative and qualitative indicators and assessing the level of all components. The topic addressed is novel in that it develops and implements a system for assessing the level of professional mathematical competences in tertiary education. The model constructed for evaluating the outcomes, based on the selected criteria, evaluates whether mathematical training opportunities are sufficient to satisfy the ultimate goal of training – the readiness of graduates for professional activities and on-fire specialties.

1 Introduction

Today, a series of national projects are being implementing in Russia, approved by the President of Russia V.V. Putin on May 7, 2018 No. 204 On National Goals and Strategic Objectives for the Development of the Russian Federation for the period up to 2024, including Education [1]. For this, several areas are being outlined, including training opportunities to produce highly qualified workforce attuned to modern standards and advanced technologies [2]. Assumedly, in the next decade, the implementation of educational projects will provide a leap in the field of education, and thereby provide a scientific, technological and socio-economic breakthrough in the development of Russia.

Pursuant to new educational conceptions, graduates should be able to apply their knowledge acquired for solving professional current tasks. Developing highly qualified professionals is crucial in university education. The goal is to train experts with a high level of general cultural and professional competency, which will guarantee each graduate to be competitive and successful in future professional activities.

A major problem today is the lack of interpretation of measurement results at all stages of training. At present, it is challenging to develop characteristics and criteria, indicators that assess the real status of both a single educational unit (for example, a core discipline – mathematics), and the entire system of professional education as such.

The problem concerned is relevant being in line with unified quality assurance (QA) requirements to professional university training in all domains, on which basis, a rationale for assessments, a unified calculation of mathematical indicators is provided.

Following the Federal State Educational Standard of Higher Education (FSES HE), work program and other regulatory documents, scientific research in the field of quality assurance, a number of contradictions can be articulated between:

- making sure that national standards ubiquitously comply with changing requirements of consumers;
- inconsistency between tuition fees and professional learning achievements;
- fragmentarily structured and classified quality indicators in the overall education management system;
- possibility of providing accommodations and modifications aligned with new requirements for university graduates by employers.

2 Materials and Methods

The term ‘competences’ was first used by R.W. White considering student abilities shaped by high motivation at a university [3].

E.F. Zeer and E.E. Symanyuk define competences as “integrative continuity of knowledge, skills and abilities that ensure professional activity” and ability to put competences acquired in practice [4, 26]. The key component of competences is expertise (mastering certain techniques, methods, actions in solving problems).

However, there is a conceptual difference in the categories of ‘competency’ and ‘competences’.

S.A. Voronov claims that “competency as a category reflects some essential characteristics of personal ability to effectively solve problems in a certain field of activity”, while “competence as a category reflects some

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essential characteristics of personal ability to choose proper means and methods applicable for problem solving" [5, 170].

R. Boyatsis defines competency through the ability to apply knowledge and skills acquired in new professional settings [6].

N.O. Vasetskaya claims that professional competency is determined by "the desire to fulfill one's potential, a creative attitude to work" [7, 104].

In the system of higher professional education, competences are defined as abilities to apply knowledge, skills and personal qualities for successful activity in a certain sphere. The FSES HE establishes a group of necessary competences (general cultural, general professional and professional) that must be shaped during university studies to develop competitive graduates.

Clearly, the quality of education cannot be evaluated by state attestation commissions alone or final qualification results. These final stages of education are no doubt essential and important, but to develop professional competences and determine demands for students in the educational industry, the quality of education should be assured by not only separate educational activities, but also address the processes to take place in structural divisions.

The FSES HE, based on the profile of training [8] and the developed curriculum for the discipline Mathematics, defines the goals and content of the discipline, the information volume, the level of knowledge, skills and abilities to be mastered by students, as well as criteria for learning achievements and assessment tools in the discipline.

When curricula are developed for the discipline Mathematics in accordance with the FSES HE requirements, the learning outcomes should be clearly defined: competences, expertise, knowledge, and skills acquired.

The paper aims to define the concept of mathematical competence, to determine the key indicators and assessment criteria for attainment level of a selected number of competences to measure student performance.

The paper is concerned with professional training opportunities for university students using systematic quality assurance tools.

The paper explores the effective development of competences and assessment tools through a constructed fuzzy model.

The objectives are:

- analyze the status of the problem based on scientific
- theoretical and applied research;
- explore standards and key documents for general and vocational competences;
- describe the concept of mathematical competence and identify a number of competences within the discipline of Mathematics;
- articulate quantitative and qualitative criteria for selected competences;
- build a mathematical fuzzy model based on the selected criteria.

T.A Tabishev introduces the concept of professional mathematical competence as a set of domain knowledge, abilities and skills, as well as expertise and structured abilities [9]

The concept of mathematical competence here implies the ability of students to create mathematical models of applied problems, to analyze and transform through graphic and computer capabilities and to apply the acquired mathematical knowledge and skills in professional activities and everyday life.

Mathematical competences (Table 1) developed in professional training were selected relying on the charts of indicators for assessing general, general professional and professional competences aligned with the Federal State Educational Standards of Higher Education in the leading specialties at the Admiral Ushakov Maritime State University.

Table 1. Mathematical competences developed in professional training at university.

Mathematical competences	
MC - 1	thorough knowledge of the core sections of higher mathematics to the extent required for mastering mathematical methods;
MC - 2	mastery and use of information capabilities for professional tasks;
MC - 3	ability to study and analyze technical and scientific literature;
MC - 4	perspectives on the problem, the ability to state and correlate with theoretical material;
MC - 5	ability to select and explore the object of study, analyze its parameters, select the ones required, determine the optimal ratios of parameters representing various systems;
MC - 6	ability to draw up algorithms or sequences of actions;
MC - 7	ability to collect, analyze and arrange data, to state the problem properly;
MC - 8	ability to use computer capabilities in solving mathematical professional problems, constructing diagrams, graphs and statistical calculations;
MC - 9	ability to build mathematical models to describe and study applied processes;
MC - 10	ability for abstract considerations, logical conclusions;
MC - 11	ability for graphical representation of objects and use of geometric methods in solving applied problems, graphical interpretation of real-life processes;
MC - 12	good command of basic mathematical methods for solving problems;
MC - 13	ability to transfer mathematical processes to non-mathematical environments;
MC - 14	ability to interpret professional problems into mathematical language;

MC - 15	mastery of mathematical symbols, culture and literacy;
MC - 16	ability to prove a formulated theory with mathematical statements;
MC - 17	to interpret the outcomes meaningfully;
MC - 18	to master the basic experimental techniques of data processing;
MC - 19	to predict the results of research using mathematical algorithms and methods.

Obviously, the system for assessing the attainment level of mathematical competences is somewhat subjective. The need to develop a system for assessing the attainment level of mathematical competences in professional tertiary training is attributed to the following reasons:

– to improve the quality of professional tertiary education;
– to develop competences in accordance with the FSES HE, which university graduates should have in engineering and technical fields of study;
– the main objectives of the mathematical discipline, aimed at developing the abilities and skills of students to use the acquired mathematical knowledge and skills in solving applied and professional problems;
– willingness of students to independently find and solve standard and non-standard problems;
– to use new information and technical capabilities to find the better solution to the tasks.

3 Results and Discussion

Mathematical training at the university is geared not simply to teach the discipline, but also to develop student intellectual and creative capacity. Students should master not only the system of actions, but also learn how to compare it with an applied or professional setting in which information and skills acquired should be used.

A competence-based model of a modern professional can be represented as a system of the following components:

- engagement, which is a set of incentives, needs, etc. fostering a positive attitude towards mathematical knowledge and aimed at developing mathematical competences;
- cognitive, exhibited in learning activities and including new knowledge gained individually as a result of self-improvement;
- activity-oriented, consisting of practical applications of mathematical knowledge in applied and professional tasks;
- informational, representing the ability to use information resources and technologies to solve applied problems;
- creative, involving the ability to solve non-standard problems, apply non-standard methods, find unknown relationships, new approaches to solving problems;
- reflexive, including the ability to evaluate results obtained, align them with changing conditions, study the use and application of findings, predict outcomes.

The practical implementation of quality assessment models within one discipline, and the university at large, is possible as long as comprehensive data on all intermediate and final indicators are collected and put into the overall management system.

Learners' four-semester achievement levels in the Mathematics discipline are measured during the following types of intermediate, current and final assessments at the Admiral Ushakov Maritime State University:

- ongoing assessment: multi-level tasks and assignments; tests; business game; case studies; quizzes; model calculations;
- interim assessment: colloquium; credit tests;
- final assessment: exam.

The FSES HE and curriculum provide a scale for assessing learning outcomes at each stage of activity and describe indicators corresponding to this scale.

Based on the assessment criteria, for assessing the level of mathematical competences the authors propose to include [10]:

- rating results;
- results of combined tests for applied tasks;
- development and solution of applied problems using information technologies;
- participation in conferences;
- number of essays and papers;
- participation in olimpiads;
- overall portfolio of achievements;
- etc.

For such activities, a lead Mathematics teacher is required to adopt a slightly different approach to planning the teaching process, including active and interactive classes (gaming, case studies, projects, research, interactive modeling, computer modeling, PP presentations, etc.). [11] Accordingly, such activities need to be assessed.

To assess the level of mathematical competences, it is necessary to determine objective criteria and indicators to characterize all components as fully as possible.

The selected criteria should specify the relationship between all components of the system. The criteria here will imply properties and attributes to evaluate its status, changes and levels of functioning.

Quantitative and qualitative indicators will give an insight into the level of each component, thereby measuring it against each criterion. Indicators evaluating mathematical competence should rely on the following characteristics [12]:

- the correctness of mathematical model constructed and investigated;
- several methods considered for solving the problem

and proving a rational for the most efficient;

- the use of a set of actions and operations, and the selection of the most effective solution;
- subject to current knowledge, creation and development of new algorithms, techniques and methods of solution.

The authors used the ideas proposed by A.V. Belikov, P.Yu. Romanov, A.V. Usova, T.E. Klimova to develop the criteria and indicators for assessing mathematical competences acquired in tertiary professional training [13, 14, 15].

Based on the described structural components (Table 1), the authors offered the following criteria that describe the level of students' mathematical competences (Table 2):

1) effective use of mathematical knowledge and skills in solving applied tasks through information technology;
2) application of acquired mathematical knowledge and skills in special disciplines and for preparing a final qualification thesis.

The following criteria are selected:

- completeness of mathematical operations performed in solving applied tasks through information technology;
- mastery of mathematical theories, laws, methods, operations, using computer capabilities;
- solidity of the acquired mathematical knowledge and skills, and their application in other areas;
- awareness of acquired mathematical skills to be applied in professional activities.

Table 2. Quantitative and qualitative descriptions of the criteria.

Effective use of mathematical knowledge and skills in solving applied problems through information technology		
	Completeness	Mastery
Low level (0)	Not all actions are performed or performed with shortcomings. Creative, research or scientific activity is initiated and supervised by the teacher	Templated activities. Preference is given to solving the same type of problems and learned ways of doing things. Knowledge is haphazard. A student has difficulty doing self-study assignments
Intermediate level (1)	All actions are performed, not so accurate, though. Knowledge acquired is consistent and scientifically rigour. Research and scientific activities are initiated the student, but under the guidance and control of the teacher	Preference is given to algorithmic patterned tasks. Attempts are made to solve unknown, non-standard problems, but has difficulties in choosing a rational method and finding an effective solution
High level (2)	All actions are performed correctly and in full. Knowledge acquired is consistent, deep and scientifically rigour. Preference is given to perform research activities independently	Preference is given to non-standard problems and non-standard methods of solution. The ability to see and develop one's own way of finding the effective solution
Use of acquired mathematical knowledge and skills in special disciplines and for preparing a final qualification thesis		
	Solidity	Awareness
Low level (0)	No transfer of knowledge and skills to new conditions, and no use of the studied methods in new domains	Has difficulty in providing a rationale for actions. Low involvement in professional and personal development
Intermediate level (1)	Desire to transfer knowledge and skills to new conditions, but only within a given topic, unit	Actions are reasonable, but rationale is somewhat inaccurate. Need for self-improvement is realized
High level (2)	Knowledge and skills are transferred to other disciplines that have a professional focus, to solving professional problems	Preference is given to non-standard tasks. Actions performed have a complete and error-free rationale, and proposed methods provide effective solutions. Awareness of the goals and objectives of research activities

The fuzzy theory is used to effectively display inaccuracies of intermediate results, measurements and maintain the adequacy of the constructed model with minor changes in the experimental conditions [16].

The use of linguistic variables is of particular interest enabling to determine the level of certain mathematical competencies in conditions of inaccurate quantitative assessments. Therefore, based on the quantitative and qualitative criteria, linguistic input and output variables will be evaluated by qualitative terms.

Let us assume that the input variables are assessed on the scale given in Table 3: L-low, I-intermediate, H-high

Thus, it is possible to define some expressions, allowing fuzzy linguistic statements into certain mathematical expressions.

Based on the quantitative and qualitative characteristics of the criteria, we can define expressions that interpret linguistic statements in the form of the following mathematical expressions:

$$F = f(X, Y) \quad (1)$$

where $X = f_X(x_1, x_2)$ and $Y = f_Y(y_1, y_2)$.

For each criterion *X*, *Y*, we will construct the corresponding matrices (Tables 3 and 4):

Table 3. The ratio of indicators by quantitative levels

Completeness (x_1) / Mastery (x_2)	**L**	**I**	**H**
L	L	L	I
I	L	I	H
H	I	H	H

Solidity (y_1) / Awareness (y_2)	**H**	**C**	**B**
L	L	L	I
I	L	I	H
H	I	H	H

Let us make a matrix against the results *F*, presenting its correspondence on a three-point scale by levels: *L* – low, *I* – intermediate, *H* – high.

Table 4. Correlation of criteria by quantitative levels.

Effective use of mathematical knowledge and skills in solving applied problems through information technology (X) / Application of acquired mathematical knowledge and skills in special disciplines and for preparing a final qualification thesis (*Y*)	***L***	***I***	***H***
L	L	L	I
I	L	I	H
H	I	H	H

Table 5. The system for assessing the level of mathematical competences.

Levels	*Indicators of mathematical competences against criterion 1*	
	Completeness	*Mastery*
Low (0)	0.4 ≤ k < 0.6	0.4 ≤ k < 0.6
Intermediate (1)	0.6 ≤ k < 0.8	0.6 ≤ k < 0.8
High (2)	0.8 ≤ k ≤ 1.0	0.8 ≤ k ≤ 1.0

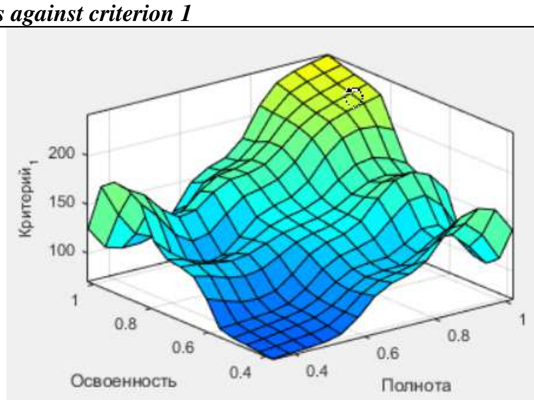

Fig.1. Three-dimensional graph of membership functions for indicator X.

Levels	*Indicators of mathematical competences against criterion 2*	
	Awareness	*Solidity*
Low (0)	Activity is not conscious enough	0.4 ≤ k < 0.6
Intermediate (1)	Activity is basically conscious	0.6 ≤ k < 0.8
High (2)	Activity is fully conscious	0.8 ≤ k ≤ 1.0

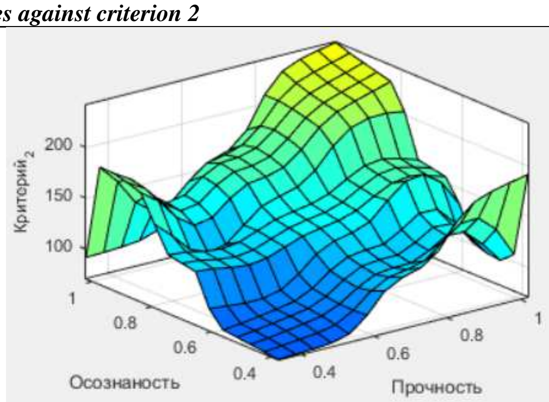

Fig.2. Three-dimensional graph of membership functions for indicator Y.

Having described the linguistic variables and their values, it is necessary to build and describe a system of relations that converts input variables into output variables, i.e., to describe the system of rules "if ... – then ...", based on expert qualitative and quantitative assessments.

The fuzzy logical equations composed in this way can assess the integral quality of mathematical competences for fixed values of selected indicators [17].

To determine the level of mathematical competences, the results for each criterion are taken.

To quantitatively compare the levels of proficiency, scores are presented for each level:

– 0 for low level;
– 1 for intermediate level;
– 2 for high level of mathematical competences.

Based on each indicator assessed in the group, data were collected, grouped, and presented in the table for each of the criteria (Table 5) and in the form of three-dimensional graphs (Fig. 1, Fig. 2), where k is the coefficients for determining the level of mathematical competences against the relevant indicators.

The result of the model showing the level of mathematical competences is denoted by F and within a particular mathematical discipline, it is enough to conduct a study on the compiled rating scale. For example: when $F_1 \leq 40\% \Rightarrow H$, when $40 \leq F_1 \leq 80\% \Rightarrow C$, when $F_1 \geq 80\% \Rightarrow B$.

The competence assessment system introduced herein will reduce subjectivity in assessing each student and produce an overall assessment of the whole group.

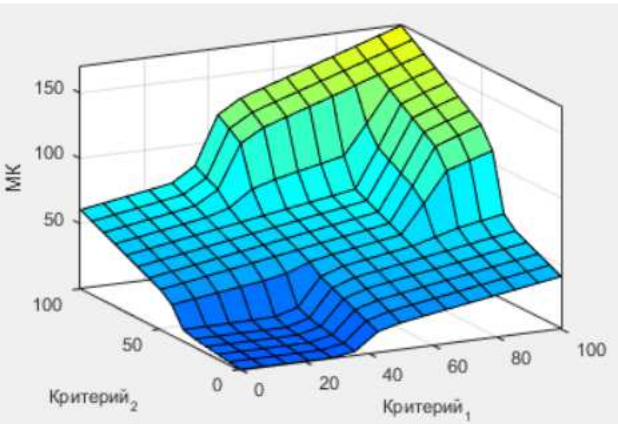

Fig. 3. Relationship graph of scorecard elements X, Y.

In education, no classical mathematical models are presented to deal with management tasks, so the use of linguistic variables to describe characteristics and indicators significantly expands the scope of management.

4 Conclusion

Transformations in the life of global community call for both changing values and new approaches to professional training. The language of competences is used to assess learning outcomes, to describe the levels of students' performance.

The model of quality assurance in higher education relies on the control of both its individual components and the entire educational process. The presented methodology is based on possible uses of new smart technologies designed to incorporate classical mathematical models for assessing the level of professional knowledge and skills. Through logical functions and linguistic rules, we can study dynamic changes, aligning them to new conditions and requirements, changing key factors under new effective projects. The corresponding model based on mathematical fuzzy apparatus further can provide mathematical modeling of the quality management system in higher education.

It is a true mathematical assessment model that can analyze the overall rating, both for an individual student and the university as such, and to predict further observation results to respond to the demand for highly qualified specialists.

With this model, we can achieve clear regulations and characteristics of all the criteria to be assessed, with due account for their interrelations, and influence the result.

References

1. Presidential Decree "On national goals and strategic objectives of the development of the Russian Federation for the period up to 2024" (2018) Retrieved from: http://kremlin.ru/events/president/news/57425
2. Passport of the priority project on Training of highly qualified specialists and working personnel, in line with modern standards and advanced technologies (approved by the Presidium of the Council under the President of the Russian Federation for Strategic Development and Priority Projects – protocol dated October 25, 2016, No. 9)
3. R.W. White, Motivation reconsidered: the concept of competence, Psychological Review **66(5)**, 297–333 (1959)
4. E. Zeer, E. Symanyuk, Competence-based approach to the modernization of vocational education, Higher Education in Russia **4**, 23–30 (2005)
5. S.A. Voronov, Competence and competency as a category of activity: similarities and differences in understanding, Historical and Socio-Educational Thoughts **9(6(1)**, 165–172 (2017)
6. R.E. Boyatzis, *The Competent Manager: A Model for Effective Performance* (John Wiley & Sons, 1991)
7. N.O. Vasetskaya, Cognitive competences of a graduate in the context of knowledge-digital economy, New Economy World **14(1)**, 101–107 (2020)

8. Federal state educational standard of higher education in the specialty 26.05.05 Navigation (specialist level) (approved by order of the Ministry of Education and Science of the Russian Federation of March 15, 2018, No. 191, Moscow)
9. T.A. Tabishev, Methodical System for Monitoring The Mathematical Training Of University Students: Doctoral dissertation (Astrakhan State University, 2010)
10. S.I. Kondratiev, A.L. Boran-Keshishyan, A.N. Tomilin, Conceptual foundations for the development of a bank of test tasks for the state final certification of graduates of marine educational organizations, Marine Intelligent Systems **1–2(43)**, 149–156 (2019)
11. S.A. Balyaeva, T.G. Khvingiya, S.A. Kalinina, Problems of modernization of educational process at maritime university, Bulletin of the Adyghe State University, Series 3: Pedagogy and Psychology **2(238)**, 15–21 (2019)
12. G.V. Tokmazov, S.I. Pankina, M.N. Lyutikova, Interdisciplinary links in organizing student research activities, World of Science, Culture, Education **5(78)**, 92–96 (2019)
13. V.A. Belikov, Target factor of mastering personality development, Modern Higher School: Innovative Aspect **8(4(34)**, 99–107 (2016)
14. P.Yu. Romanov, Theory and practice of the formation of research skills in mathematical training of students: PhD thesis (Magnitogorsk, 2002)
15. A.V. Usova, On the criteria and levels of students' cognitive skills, Soviet Pedagogy **2**, 45–48 (1980)
16. I.V. Veshneva, *Mathematical Models in the Quality Management System of Higher Education Using Fuzzy Logic*, p 187 (Saratov Istochnik Publishing House, Saratov, 2010)
17. S.I. Pankina, O.V. Tsepordey, A.I. Kravchenko, M.L. Somko, M.N. Lyutikova, Coastal ecosystem impact assessment model, IOP Conference Series: Earth and Environmental Science **872(1)**, 012016 (2021)