Two-part trade policy in the reciprocal market model

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Abstract. This paper is devoted to the study of two-part trade policy for two countries, two markets, producers under imperfect competition, where the standard arguments of free trade are not applicable. The basic model is a two-step game with complete but imperfect information. The optimal policy of the government is analyzed in terms of the goals of maximizing the national welfare. This policy dominates over a simple quota and a simple tariff, which are its special cases. At the same time, the optimal two-part trade policy of governments that maximize national wealth is a subsidy.

1 Introduction

Strategic trade policy [1–6] is one of the most researched areas of international trade. Governments benefit from trade policy by providing a strategic advantage to their domestic companies and companies compete imperfectly with each other. Research has explored a wide range of alternative scenarios, which include (but are not limited to) the nature of oligopolistic competition in terms of perceived variations and behavioral parameters, available trade policy tools, information structure, time frame, governance structure, etc.

Most studies on the strategic trade policy use the "third market" model: all products of competing oligopolistic companies are exported to the market different from the one where the companies are located. These works focus on how governments can redistribute oligopolistic rents by using export subsidies in favor of their companies [7]. Another body of works on strategic trade policy explores "reciprocal markets" models: domestic and foreign companies compete in each other's market. The typical case where firms engage in Cournot competition over a homogeneous product gives rise to intra-industry trade in reciprocal dumping diversity. The consequence of this strategic trade policy is that consumer surplus becomes very important in determining national wealth for governments. In the "reciprocal markets" model, import restrictions become an additional viable political regime.

While the "third" market model is a useful simplification for isolating the rent-shifting motive in the strategic trade policy, the "reciprocal markets" model appears to be a more realistic scenario. Important policy issues, such as export subsidies and countervailing duties, can be analyzed only through the “reciprocal markets” model. In [8], the strategic trade policy in independent markets was studied for the quadratic cost functions. The quadratic cost function has a significant effect on the export subsidy, and market correlation plays an important role in determining the Nash equilibrium when choosing the optimal policy regime.

The main issue in choosing an optimal international trade policy is the effect of tariffs and quotas, or the effect of possible policies. In case of perfect competition, tariffs and quotas are usually equivalent, i.e., the tariff effect can be duplicated by an appropriately chosen quota. Under imperfect competition, tariffs dominate quotas. This is due to the fact that the live response of foreign firms with a quota is more accurate than with a tariff. The quota contributes to the monopoly power of domestic firms. An analysis of quotas under oligopoly provides additional results. Perhaps the key difference between tariffs and quotas as policy instruments is their impact on foreign firms. Any tariff on foreign firms reduces their profits, and any subsidy to domestic firms also reduces profits of foreign firms. Quotas provide more opportunities for foreign firms. If the quota is implemented as a voluntary export restriction (VER), foreign firms will retain any quota rentals.

The study explores the possibility of using quotas and tariffs as complements to each other. The policy that provides for an import license to enter along with a tariff per import unit is called a two-part policy. This policy was introduced in the classic analysis of the Disneyland price dilemma. The article [9] considers the two-part trade policy with complete but imperfect information for one market with homogeneous cost functions. The optimal two-part trade policy dominates over tariff or quota policies. This is true for a government that is interested in maximizing revenues, and for a benevolent government that is interested in the national welfare; a large import license fee imposes fewer distortions than a tariff. The government can achieve revenue neutrality by lowering current tariff rates and replacing lost revenues with license fees. For the market of one country with non-homogeneous cost functions, [9] provides numerical examples that show the dominance of the two-part trade policy.

This paper analyzes the two-part trade policy for the "reciprocal markets" model. The use of quotas and tariffs as a complement to each other allows this model to pursue a trade policy that dominates in terms of efficiency over the quota or tariff policies. The qualitative results depend on the type of government (maximizing its income or social welfare), the market structure and the cost structure of firms.

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It is also proposed to study the effect of the two-part trade policy both in relation to foreign and domestic firms. The results make it possible to understand how government intervention should be related to the conditions of competition in the domestic market, and what tools can (and should) be used to develop an optimal policy.

2 Model

Consider a model of two countries (home and foreign). N - domestic firms, and N* - foreign firms producing a homogeneous product. Let q_i be the level of production of the i-th domestic firm for the domestic market; v_i is the level of exports of the j-th foreign firm to the domestic market (* denotes variables related to the foreign market). Let $q_i^{¿}$ be the level of exports of the i-th domestic firm to the foreign market; $v_i^{¿}$ is the level of production of the j-th foreign firm in the foreign market. The total output of the firms in the home and foreign countries are Q and Q*, respectively:

$$Q=\sum_{i=1}^{N} q_i+\sum_{j=1}^{N^{¿}} v_j \tag{1}$$

$$Q^{¿}=\sum_{i=1}^{N} q_i^{¿}+\sum_{j=1}^{N^{¿}} v_j^{¿} \tag{2}$$

Denote the inverse functions of demand in the home and foreign countries with $p(Q)$, $p^{¿}(Q^{¿})$, where $p\in C^2$; $p^{¿}\in C^2$; $p^{'}<0$; $\llbracket(p^{¿})^{'}<0$. Let $c_i(q_i)$ be the cost function of the i-th domestic firm; $c_j^{¿}(q_j^{¿})$ is the cost function of the j-th foreign firm, where $c_i(q)\in C^2$;; $c_i^{'}(q)>0$; $c_i^{''}(q)>0$ and $c_j^{¿}(q)\in C^2$;; $(c_j^{¿}(q))^{'}>0$; $(c_j^{¿}(q))^{''}>0$. In case of the homogeneous cost structure of domestic and foreign firms $c_i(q)=c(q),\forall i;\, c_j^{¿}(q)=c^{¿}(q),\forall j;$

The economic result of the activities of domestic and foreign firms is determined by the profit functions:

$$\pi_i=p(Q)\cdot q_i+p^{¿}(Q^{¿})\cdot q_i^{¿}-c_i(q_i+q_i^{¿})-t^{¿}\cdot q_i^{¿}-e^{¿},\quad i=1,\ldots N \tag{3}$$

$$\pi_j^{¿}=p^{¿}(Q^{¿})\cdot v_j^{¿}+p(Q)\cdot v_j-c_j^{¿}(v_j+v_j^{¿})-t\cdot v_j-e,\quad j=1,\ldots N^{¿} \tag{4}$$

where $e,e^{¿}$ is the license fee to the domestic and foreign governments, respectively; $t,t^{¿}$ is the tariff imposed on the domestic and foreign firms, respectively.

Let the government goals be determined by the following payoff functions:

$$G(t,t^{¿})=\sum_{j=1}^{N^{¿}}\left\{p\cdot v_j(t,t^{¿})-c^{¿}(v_j(t,t^{¿}))\right\} \tag{5}$$

$$G^{¿}(t,t^{¿})=\sum_{j=1}^{N}\left\{p^{¿}\cdot q_i^{¿}(t,t^{¿})-c(q_i^{¿}(t,t^{¿}))\right\} \tag{6}$$

Let $z=(e,\bar{v},t)$ and $z^{¿}=(e^{¿},\bar{q},t^{¿})$ be the two-part trade policies of the domestic and foreign governments, respectively, and $\bar{v},\bar{q}$ are the quotas for domestic and import firms. In the specific case, $z=(e,\bar{v},0)$ is a simple quota; $z=(e,\infty,t)$ is a simple tariff.

Consider the following two-step game with complete but imperfect information.

Step 1. The domestic and foreign governments simultaneously and independently establish a two-part trade policy $z=(e,\bar{v},t)$ and $z^{¿}=(e^{¿},\bar{q},t^{¿})$ towards domestic and foreign firms.

Step 2. Under the established trade policy, domestic and foreign firms compete within the Cournot model - simultaneously and independently determine the volumes of output q_i and v_i to the domestic market, $q_i^{¿}$ and $v_i^{¿}$ to the foreign market.

In this game for the homogeneous case, [10] found that import tariffs of the two-part trade policy reduce the level of domestic outputs of foreign firms and increase the level of domestic outputs of domestic firms.

For this type of government in a two-step game for $N=N^{¿}=1$, sufficient conditions were established for the existence of a perfect subgame Nash equilibrium.

3 Optimal Strategic Policy for Maximizing National Wealth

Let the goals of the domestic and foreign governments be determined by the payoff functions that constitute the national wealth of the countries:

$$W(t,t^{¿})=\int_0^{Q(t,t^{¿})} p(s)ds-\sum_{i=1}^{N} c(q_i(t,t^{¿}))-\sum_{j=1}^{N^{¿}} c^{¿}(v_j(t,t^{¿})) \tag{7}$$

$$W^{¿}(t,t^{¿})=\int_0^{Q^{¿}(t,t^{¿})} p^{¿}(s)ds-\sum_{i=1}^{N} c(q_i^{¿}(t,t^{¿}))-\sum_{j=1}^{N^{¿}} c^{¿}(v_j^{¿}(t,t^{¿})) \tag{8}$$

In this case, the government maximizes the national welfare, which consists of the sum of consumer surplus, profits of the domestic firm, and its own tax revenues. In addition, the level of the optimal two-part policy is determined by the income from sales of external firms, which is taken away by the tariff and the license fee.

For this type of government, consider a two-stage game $\Gamma¿$ for $N=N^{¿}=1$. In this case, the following theorem is true.

Theorem. Let the following conditions be satisfied:

- cost functions $c(q)$, $c^{¿}(q)$ are twice continuously differentiable and convex $\forall q\geq 0$;
- Reverse functions of demand $p(Q)$, $p^{¿}(Q)$ are twice continuously differentiated and decrease $\forall Q\geq 0$; $p^{'}(Q)+p^{''}(Q)\cdot q<0,\forall Q,q;$ $p^{¿'}(Q^{¿})+p^{¿''}(Q^{¿})\cdot v<0,\forall Q^{¿},v;$ $p^{'}(Q)-c^{''}(q)<0,\forall Q,q$; $p^{¿'}(Q^{¿})-c^{¿''}(v)<0,\forall Q^{¿},v;$

$\exists \overline{Q}$, *что* $p(Q)=0$, $\forall Q \geq \overline{Q}$, $\exists \overline{Q^{¿}}$, *что* $p^{¿}(Q^{¿})=0$, $\forall Q^{¿} \geq \overline{Q^{¿}}$,

the game G ***has a perfect subgame Nash equilibrium.***

Proof:

I) Prove that for two-part policies $z=(e,\overline{v},t)$ and $z^{¿}=(e^{¿},\overline{q},t^{¿})$ of the domestic and foreign governments, there exists a Nash equilibrium at the second step, when firms play the Cournot equilibrium, and consider the Nash equilibrium between the governments at the first step. Note that $\frac{\partial \pi_1}{\partial q_1}=p^{'}\cdot q_1+p-c^{'}<0$, for $q_1 \geq \overline{Q}$, since $p^{'}(Q)<0$ (by condition 2)) and $p(q_1)<c^{'}(q_1)$, for $q_1 \geq \overline{Q}$ (by condition 5)). Then the profit at point $\overline{Q}$ is greater than at any point $q_1 \geq \overline{Q}$. Thus, maximizing π_1 with respect to q_1 for $q_1 \geq 0$ reduces to maximizing it in $[0,\overline{Q}]$.

Similarly, the maximization of $\pi_1^{¿}$ with respect to $v_1^{¿}$ for $v_1^{¿} \geq 0$ reduces to the maximization in $[0,\overline{Q^{¿}}]$. Taking into account the quota for $q_1^{¿}$ and v_1, we have $q_1^{¿} \in [0,\overline{q}]$, $v_1 \in [0,\overline{v}]$.

Thus, the maximization of $\pi_1(q_1,q_1^{¿})$ with respect to $(q_1,q_1^{¿})$ can be considered on the compact set $[0,\overline{Q}]\times[0,\overline{q}]$, while the maximization of $\pi_1^{¿}(v_1,v_1^{¿})$ by $(v_1,v_1^{¿})$ - on the compact set $[0,\overline{v}]\times[0,\overline{Q}]$.

II) It follows from conditions 1) and 2) that the functions $\pi_1(q_1,q_1^{¿})$ and $\pi_1^{¿}(v_1,v_1^{¿})$ are continuous.

III) Conditions 1) and 2) of the theorem ensure the fulfillment of the following relations:

$$\frac{\partial^2 \pi_1}{\partial q_1^2}=p^{''} q_1+2p^{'}-c^{''}<0 \tag{9}$$

$$\frac{\partial^2 \pi_1^{¿}}{\partial v_1^2}=p^{''} v_1+2p^{'}-(c^{¿})^{''}<0 \tag{10}$$

$$\frac{\partial^2 \pi_1}{\partial q_1 \partial q_1^{¿}}=-c^{''}<0 \tag{11}$$

$$\frac{\partial^2 \pi_1}{\partial v_1 \partial v_1^{¿}}=-c^{¿''}<0 \tag{12}$$

$$\frac{\partial^2 \pi_1}{\partial q_1^{¿2}}=p^{¿''}\cdot q_1^{¿}+2p^{¿'}-c^{''}<0 \tag{13}$$

$$\frac{\partial^2 \pi_1^{¿}}{\partial v_1^{¿2}}=p^{¿''}\cdot v_1^{¿}+2p^{¿'}-(c^{¿})^{''}<0 \tag{14}$$

Conditions (9–14) ensure the concavity of the functions $\pi_1(q_1,q_1^{¿})$ and $\pi_1^{¿}(v_1,v_1^{¿})$.

IV) Thus, on the basis of the Nash theorem from I), II) and III), it follows that there is a Cournot equilibrium at the second step of the game:

$$¿ \tag{15}$$

Go back to the first step of the game and consider the game between the governments. Show that the conditions of the theorem ensure the existence of the Nash equilibrium at the first step and the existence of the perfect subgame Nash equilibrium, which will be the optimal two-part trade policy.

V) From the continuity and boundedness of the functions $q_1(t,t^{¿})$, $q_1^{¿}(t,t^{¿})$, $v_1(t,t^{¿})$, $v_1^{¿}(t,t^{¿})$ the problem of maximizing $W(t,t^{¿})$ with respect to **t** can be considered in $[t_н,t_в]$, and the problem of maximizing $W^{¿}(t,t^{¿})$ with respect to $t^{¿}$ can be considered in $[t_н^{¿},t_в^{¿}]$.

VI) Conditions 1) and 2) ensure the continuity of the functions

$$W(t,t^{¿})=\int_0^{Q(t,t^{¿})} p(s)\,ds-c¿¿; \tag{16}$$

$$W^{¿}(t,t^{¿})=\int_0^{Q^{¿}(t,t^{¿})} p^{¿}(s)\,ds-c¿¿; \tag{17}$$

VII) Using the relations proved in [6]

$$\frac{\partial v_1}{\partial t}=\alpha\frac{\partial q_1}{\partial t}\text{, где } \alpha \in (-1,0);\quad \frac{\partial q_1}{\partial t^{¿}}=\beta\frac{\partial v_1}{\partial t^{¿}}\text{,}$$

$$\text{где}\beta \in (-1,0) \tag{18}$$

and differentiating the function $W(t,t^{¿})$ twice with respect to **t**, we have

$$\frac{\partial^2 W(t,t^{¿})}{\partial t^2}=¿$$

$$\left(\frac{\partial v_1}{\partial t}\right)^2\cdot\left((\alpha+1)^2\cdot p^{'}-\alpha^2\cdot c^{''}-c^{¿''}\right)<0 \tag{19}$$

Hence the function $W(t,t^{¿})$ is concave in **t**. Similarly, we obtain the concavity of $W^{¿}(t,t^{¿})$ in $t^{¿}$.

VIII) Thus by the Nash theorem, V), VI) and VII) indicate the existence of an equilibrium at the first step of the game $(t^0,t^{¿0})$. IV) and VIII) imply the existence of a perfect subgame Nash equilibrium X^0 in game G:

$$X^0=¿$$

$$e^{¿0}=p^{¿}(q_1^{¿0}+v_1^{¿0})\cdot q_1^{¿0}-c(q_1^{¿0})-t^{¿0}\cdot q_1^{¿0},$$

$$e^0=p(q_1^0+v_1^0)\cdot v_1^0-c^{¿}(v_1^0)-t^0\cdot v_1^0,$$

$$\overline{v}^0=v_1^0(t^0,t^{¿0}),\overline{q}^0=q_1^{¿0}(t^0,t^{¿0})$$

Since the existence of an optimal policy has been proved in the theorem, it is possible to determine its value.

Differentiating the function $W(t,t^{¿})$ with respect to **t**, we have

$$\frac{\partial W(t,t^{¿})}{\partial t} = (p^{'} - c^{'})\frac{\partial q_1}{\partial t} + (p - c^{¿'})\cdot\frac{\partial v_1}{\partial t} \qquad (20)$$

Writing out the conditions of the first order in the model, we have the conditions:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial q_1} = p^{'}\cdot q_1 + p - c^{'} = 0 \\ \dfrac{\partial \pi_1}{\partial q_1^{¿}} = p^{¿'}\cdot q_1^{¿} + p^{¿} - c^{'} - t^{¿} = 0 \\ \dfrac{\partial \pi_1^{¿}}{\partial q_1} = p^{'}\cdot v_1 + p - c^{¿'} - t = 0 \\ \dfrac{\partial \pi_1^{¿}}{\partial q_1} = p^{¿'}\cdot v_1^{¿} + p^{¿} - c^{¿} = 0 \end{cases} \qquad (21)$$

From relations (18), (20) and (21) we have that

$$\frac{\partial W(t,t^{¿})}{\partial t} = -p^{'}\cdot q_1\frac{\partial q_1}{\partial t} - p^{'}\cdot v_1\cdot\frac{\partial v_1}{\partial t} = -p^{'}\cdot\frac{\partial v_1}{\partial t}(\alpha\cdot q_1 \qquad (22)$$

From relation (22) and the condition $\alpha \in (-1,0)$ we have that

$$\left.\frac{\partial W(t,t^{¿})}{\partial t}\right|_{t=0} = -p^{'}\cdot\frac{\partial v_1}{\partial t}(\alpha+1)\cdot q_1 < 0 \qquad (23)$$

Similarly, we have that

$$\left.\frac{\partial W^{¿}(t,t^{¿})}{\partial t^{¿}}\right|_{t^{¿}=0} < 0 \qquad (24)$$

Conditions (23) and (24) imply that the optimal two-part tariff is a subsidy. In this case, the optimal license fee is equal to the current revenues of the foreign firm.

4 Modified two-part trade policy

Consider the case when the two-part trade policy applies both to the foreign and domestic countries. Consider the market of one product in the home country. The inverse demand function $p(Q)$ is known, where $p \in C^2$; $p^{'} < 0$. Goods are produced by two firms – domestic (i=1) and foreign ones (i=2). Let $c_i(q_i)$ be the cost functions of the i-th firm, where $c_i(q) \in C^2$; $c_i^{'}(q) > 0$; $c_i^{''}(q) > 0$. Assume that the following conditions are met:

$$p^{'}(Q) - c^{''}(q) < 0, \forall Q, q \qquad (25)$$

$$\exists \overline{Q}, \textit{что } p(Q) = 0, \forall Q \geq \overline{Q} \qquad (26)$$

The economic result of the firms is determined by the profit functions:

$$\pi_1 = p(Q)\cdot q_1 - c_1(q_1) - t\cdot q_1 \qquad (27)$$

$$\pi_2 = p(Q)\cdot q_2 - c_2(q_2) - t^{¿}\cdot q_2 - e \qquad (28)$$

where e is the fee for the license of a foreign company; $t, t^{¿}$ is the tariff for domestic and foreign firms, respectively.

Denote the modified two-part trade policy ($\overline{q}$ is the quota) with vector $z = (e, \overline{q}, t, t^{¿})$. Obviously, $z = (e, \overline{q}, 0, 0)$ is a simple quota; $z = (e, \infty, t, 0)$ is a simple tariff.

The goals of the government are determined by the following payoff functions:

$$G(t,t^{¿}) = p\cdot q_2(t,t^{¿}) - c_2(q_2(t,t^{¿})) + t\cdot q(t,t^{¿}) \qquad (29)$$

$$W(t,t^{¿}) = \int_0^{q(t,t^{¿})} p(s)\,ds - c_1(q_1(t,t^{¿})) - c_2(q_2(t,t^{¿})¿)¿ \qquad (30)$$

At the first step, the home government determines the entry fee (e) for the foreign firm and the tariffs $t, t^{¿}$. At the second step, the firm knowing the trade policy chooses optimal production volumes from the first order conditions:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial q_1} = p^{'}\cdot q_1 + p - c_1^{'} - t = 0 \\ \dfrac{\partial \pi_2}{\partial q_2} = p^{'}\cdot q_2 + p - c_2^{'} - t^{¿} = 0 \end{cases} \qquad (31)$$

The second order conditions are satisfied due to conditions (25) and (26). Hence we obtain the equilibrium outputs q_1^0 and q_2^0. Using the implicit function theorem, one can determine the slopes of the reaction functions:

$$g_i = \frac{d q_i^0}{d q_j} = \frac{-p^{''} + p^{'}}{2\cdot p^{'} + p^{''}\cdot q_i - c_i^{''}} < 0 \quad \Rightarrow -1 < g_i < 0.$$

Thus, we have decreasing reaction functions, and quantities of goods produced by the firms are strategic substitutes. It is easy to see that

$$\frac{q_1^0}{d t^{¿}} = g_1\frac{d q_2^0}{d t^{¿}} \qquad (32)$$

From conditions (31), taking into account relation (32), we obtain

$$\frac{d q_2^0}{dt}\left[(p^{''}\cdot q_1^0 + p^{'})\cdot(g_1+1)\right] + \frac{d q_1^0}{dt}(p^{'} - c_1^{''}) = 1 \Rightarrow$$

$$\frac{d q_1^0}{d t^{¿}} > 0,\ \frac{d q_2^0}{d t^{¿}} < 0.$$

Similarly, we have that

$$\frac{d q_1^0}{dt} < 0,\ \frac{d q_2^0}{dt} > 0.$$

We can conclude that the equilibrium output of the domestic firm increases, and that of the foreign firm decreases at rate $t^{¿}$; the equilibrium output of the domestic firm decreases, and that of the foreign firm increases at rate t.

Int the optimum, the government chooses tariffs from the conditions

$$\begin{cases} \dfrac{\partial G}{\partial t} = t^{¿}\dfrac{\partial q_2^0}{\partial t^{¿}} + p' \cdot q_2^0 \dfrac{\partial q_1^0}{\partial t^{¿}} + t\dfrac{\partial q_1^0}{\partial t^{¿}} = 0 \\ \dfrac{\partial G}{\partial t} = t\dfrac{\partial q_2^0}{\partial t} + p' \cdot q_2^0 \dfrac{\partial q_1^0}{\partial t} + t\dfrac{\partial q_1^0}{\partial t} + q_1 = 0 \end{cases}$$

Example 1. Let $p = 1 - q_1 - q_2$; $c_1 = c \cdot q_1$; $c_2 = c \cdot q_2$.

1) Consider the case of the government whose payoff function is determined by formula (29). In this case, we obtain the following optimal outputs as a function of tariffs

$$q_1 = \frac{1-c-2t+t^{¿}}{3}; \quad q_2 = \frac{1-c-2t^{¿}+t}{3}.$$

Then we determine the government revenue

$$G(t, t^{¿}) = \frac{1}{9}\left((1-c)^2 + (1-c)\cdot(5t - t^{¿}) - 5t^2 + 2t^{¿}t - 2t\right.$$

Hence the optimal rates are

$$t^G = \frac{1-c}{2} > 0 \text{ и } t^{¿G} = 0.$$

Thus, we conclude that if the government maximizes its income, the optimal modified trade policy is not a subsidy.

2) Consider the case of the government whose payoff function is determined by formula (30). In this case, we have the national welfare

$$W(t, t^{¿}) = \frac{2-2c-t-t^{¿}}{3} \cdot \frac{4-4c+t+t^{¿}}{6}$$

Solving the optimization problem, we obtain that the optimal tariffs t^W and $t^{¿W}$ imposed on domestic and foreign firms must satisfy the condition

$$t^W + t^{¿W} = -(1-c) < 0.$$

It follows from the latter that in this case the welfare has an infinite set of optimal policies that are subsidies. If, for example, the government chooses values $t^W = \frac{-3}{4}(1-c)$ and $t^{¿W} = \frac{-1}{4}(1-c)$ as optimal tariffs, the optimal output of the domestic firm will be $q_1^W = \frac{3}{4}(1-c)$, which exceeds the optimal output of the foreign firm $q_2^W = \frac{1}{4}(1-c)$. In addition, $W^{max} = (1-c)^2$ is the optimal national welfare in the home country, and $e^W = \frac{1}{16}(1-c)^2$ is the license fee.

Consider the results of applying a modified two-part trading policy for the reciprocal market model. It is proposed to apply the two-part trade policy not only to foreign, but also to domestic firms:

$\tilde{z} = (e, \bar{q}, t, \tilde{e}, \tilde{q}, \tau)$ is the modified two-part trading policy in the domestic market;

$\tilde{z}^{¿} = (e^{¿}, \bar{v}, t^{¿}, \tilde{e}^{¿}, \tilde{v}, \tau^{¿})$ is the modified two-part trade policy in the foreign market;

This trade policy generalizes many existing trade policies:

1) $\tilde{z}_0 = (0, \infty, 0, 0, \infty, 0)$ – free trade;

2) $\tilde{z}_1 = (0, \infty, t, 0, \infty, 0)$ – simple tariff for the foreign firm;

3) $\tilde{z}_2 = (e, \bar{q}, 0, 0, \infty, 0)$ - simple quota for the foreign firm;

4) $\tilde{z}_3 = (e, \bar{q}, t, 0, \infty, 0)$ – two-part trade policy for the foreign firm;

5) $\tilde{z}_4 = (0, \infty, 0, 0, \infty, \tau)$, $\tau < 0$ – export subsidies;

6) $\tilde{z}_5 = (0, \infty, 0, \tilde{e}, \tilde{q}, 0)$ – voluntary export restriction;

7) $\tilde{z}_6 = (0, \infty, 0, \tilde{e}, \tilde{q}, \tau)$ – two-part trading policy for the domestic firm;

8) $\tilde{z}_7 = (0, \infty, 0, \tilde{e}, \tilde{q}, \tau)$, $\tau < 0$, $\tilde{e} > 0$ – export credit subsidies.

Consider examples of the effect of applying the modified two-part policy for the reciprocal markets model.

Example 2. Let $p = 1 - q_1 - q_2$; $p^{¿} = 1 - q_1^{¿} - q_2^{¿}$; $c_1(q) = c \cdot q$; $c_1^{¿}(q) = c \cdot q$.

In this case, the optimal modified two-part policy for the governments maximizing government revenues is

$$t + t_1 = \frac{1-c}{2} > 0;\ t^{¿} + t_1^{¿} = \frac{1-c}{2} > 0; \tag{33}$$

$$q_1 = \frac{1-c}{2} - t_1;\ q_1^{¿} = \frac{1-c}{2} - t^{¿};\ q_2 = \frac{1-c}{2} - t;\ q_2^{¿} = \frac{1-c}{2} - t_1^{¿}; \tag{34}$$

Assumption 1. In case of linear cost and inverse demand functions, there is an infinite number of optimal modified two-part policies that maximize the government revenues. It is possible when domestic production is not equal to zero and the case of providing subsidies is impossible.

Proof:

Since we have an infinite number of solutions to equations (33), there are an infinite number of optimal modified two-part trade policies that maximize the government revenues. Let $t < 0$ (subsidy), then from (33) $t_1 > \frac{1-c}{2}$ and (34) we obtain $q_1 = \frac{1-c}{2} - t_1 < 0$. But the output cannot be negative, therefore $t \geq 0$, i.e. subsidy is not possible.

Example 2. Let $p = 1 - q_1 - q_2$; $p^{¿} = 1 - q_1^{¿} - q_2^{¿}$; $c_1(q) = c \cdot q$; $c_1^{¿}(q) = c \cdot q$.

In this case, the optimal modified two-part policy for wealth-maximizing governments is

$$t + t_1 = -(1-c) < 0;\ t^{¿} + t_1^{¿} = -(1-c) < 0; \tag{35}$$

$$q_1 = -t_1;\ q_1^{¿} = -t^{¿};\ q_2 = -t;\ q_2^{¿} = -t_1^{¿}; \tag{36}$$

Assumption 2. In case of linear cost and inverse demand functions, there is an infinite number of modified two-part trade policies (subsidy) that maximize national welfare. In this case, it is possible that domestic production is competitive.

Proof:

Since we have an infinite number of solutions to equations (35), there is an infinite number of optimal modified two-part trade policies that maximize national welfare. Let $t>0$, then from (36) $q_2<0$. But output cannot be negative, therefore $t<0$ (subsidy). Similarly, we obtain that $t_1\leq 0, t^{¿}\leq 0, t_1^{¿}\leq 0$. Moreover, from (35) it follows that $t, t_1 u\, t^{¿}, t_1^{¿}$ are not equal to zero at the same time, i.e. the optimal policy is always a subsidy.

5 Conclusion

The article attempted to analyze the two-part trade policy in the reciprocal market model with a homogeneous structure for two countries, two markets, two producers under imperfect competition. Conditions for the existence of an optimal two-part trade policy in the form of a perfect subgame Nash equilibrium were found.

It was shown that the two-part trade policy dominates quota and tariff policies that are its special cases. At the same time, the optimal two-part trade policy of the governments maximizing national welfare is subsidies.

The results of application of the modified two-part policy were presented, which gives additional tools for building an optimal strategic policy.

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