

# Application of metabolic GM (1,1,t<sup>h</sup>,p) power model in fresh e-commerce prediction

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**Abstract.** Aiming at the prediction problem of fresh e-commerce industry, this paper attempts to combine the metabolic theory with the improved GM (1,1) power model. The article introduces the traditional GM (1,1) power model to enhance the adaptability of the model to the data series, and at the same time, a new parameter is introduced when constructing the background value, the background value is represented as a linear function of adjacent sequence points, and the form and solution method of the two specific models are given, with the goal of minimizing the average relative error, and the power exponential and new parameters are co-optimized by genetic algorithm. The optimal modeling dimension is determined through model testing, and on this basis, the metabolic GM (1,1,t<sup>h</sup>,p) power model is established, and the model is applied to the prediction of the transaction scale of the fresh e-commerce industry. The results show that the improved GM (1,1) power model can significantly improve the fitting accuracy compared with the original model, and the metabolic GM(1,1,t<sup>h</sup>,p) power model has good performance in the prediction problem of fresh e-commerce.

## 1 Introduction

Fresh e-commerce is currently in a more complex and changeable environment, although favorable policies and customer demand provide a better development environment for it, but how to accurately grasp the development direction and growth trend of fresh food e-commerce has become a problem that plagues the development of fresh food e-commerce enterprises.

Among the existing forecasting studies in the field of fresh e-commerce, more scholars focus on the forecast research of fresh product demand and cold chain logistics demand. Zhang Y L analyzed and predicted the sales volume of four fruits on the Ganfuyuan platform<sup>[1]</sup>. Ma J Y constructed the ARIMA-SVM combination prediction model based on the Sharp value weight allocation method, and analyzed the sales volume of fresh e-commerce agricultural products<sup>[2]</sup>. Li K used time series theory to establish several potato demand fluctuation forecasting models<sup>[3]</sup>. Du X F forecasts demand for perishable agricultural products based on the support vector machine (SVM) method<sup>[4]</sup>. Huang L uses

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the GM (1,1) model and the BP neural network model to simulate and predict logistics demand<sup>[5]</sup>. Li Y provided appropriate sales and supply solutions for the fresh food e-commerce platform<sup>[6]</sup>. As the "last blue ocean" in the e-commerce field, fresh e-commerce is very important to predict its industry transaction scale, but most of the existing predictions on industry transaction scale come from industry analysis reports. GM(1,1) power model can achieve better fitting of data series due to the flexibility of power exponential value, and has been widely used in many fields, so this paper attempts to use GM(1,1) power model for the first time to predict the industry transaction scale and user scale of fresh e-commerce.

## 2 Traditional GM(1,1) power model

Let the non-negative original sequence be  $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(N))$ , whose primary accumulation generates a sequence of  $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(N))$ , where  $x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$ ,  $k = 1, 2, \dots, N$ , noting the background values  $z(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1))$ ,  $k = 2, 3, \dots, N$

**Definition 1** Set  $X^{(0)}$ ,  $X^{(1)}$  and  $z(k)$  as defined above, the grey differential equation for the GM(1,1) power model is

$$x^{(0)}(k) + az(k) = b(z(k))^r, \quad k = 2, 3, \dots, N$$

Where  $r$  is the power exponent,  $a$  and  $b$  are constants.

**Definition 2** The whitening equation for the GM(1,1) power model is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b(x^{(1)}(t))^r$$

When  $\hat{x}^{(1)}(1) = x^{(1)}(1) = x^{(0)}(1)$ , the time response function is

$$\hat{x}^{(1)}(t) = \left\{ \frac{b}{a} + [x^{(1)}(1)^{(1-r)} - \frac{b}{a}] e^{a(r-1)(t-1)} \right\}^{\frac{1}{r-1}} \tag{1}$$

Following the least squares method for the grey differential equation, we obtain  $(a, b)^T = (B^T B)^{-1} B^T Y$ ,

of which

$$B = \begin{bmatrix} -z(2) & (z(2))^r \\ -z(3) & (z(3))^r \\ \vdots & \vdots \\ -z(N) & (z(N))^r \end{bmatrix} \quad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(N) \end{bmatrix}$$

Power exponent  $r$ , which is generally found by optimization methods. Combining  $a$ ,  $b$ , and  $r$  substitute into formula (1). The simulated value of the original sequence is obtained by  $\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$ ,  $k = 2, 3, \dots, N$ . The predicted value of the q-step

of the original non-negative sequence is obtained by  $\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$ ,  $k = N+1, N+2, \dots, N+q$ .

### 3 Metabolic GM (1,1,t<sup>h</sup>,p) power model

#### 3.1 GM(1,1,t<sup>h</sup>,p)power model

**Definition 3** Set  $X^{(0)}$ ,  $X^{(1)}$  and  $z(k)$  are defined above, then the grey differential equation for the GM(1,1,t<sup>h</sup>,p) power model is

$$x^{(0)}(k) + az(k) = (b_0 + b_1t + b_2t^2 + \dots + b_h t^h)(z(k))^r, \quad k = 2, 3, \dots, N.$$

**Definition 4** The whitening equation for the GM (1,1,th,p) power model is

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = (b_0 + b_1t + b_2t^2 + \dots + b_h t^h)(x^{(1)}(t))^r \quad (2)$$

When  $h = 0$ , formula (2) is the traditional GM(1,1) power model.

When  $h = 1$

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = (b_0 + b_1t)(x^{(1)}(t))^r \quad (3)$$

is the whitening equation for the GM(1,1,t,p) power model.

Let  $y = (x^{(1)}(t))^{1-r}$ , then

$$\frac{dy}{dt} + a(1-r)y = (b_0 + b_1t)(1-r) \quad (4)$$

When  $\hat{x}^{(1)}(1) = x^{(1)}(1) = x^{(0)}(1)$ , the ordinary differential equation tells us

$$y(t) = e^{\int_1^t -a(1-r)d\theta} [(x^{(1)}(1))^{1-r} + \int_1^t e^{\int_1^\theta a(1-r)d\theta} (1-r)(b_0 + b_1\theta)d\theta] \quad (5)$$

Further

$$y(t) = e^{-a(1-r)(t-1)} \{ (x^{(1)}(1))^{1-r} + (1-r)e^{-a(1-r)} \left[ \frac{b_0 e^{a(1-r)t}}{a(1-r)} - \frac{b_0 e^{a(1-r)}}{a(1-r)} + \frac{b_1 t e^{a(1-r)t}}{a(1-r)} - \frac{b_1 e^{a(1-r)t}}{a^2(1-r)^2} - \frac{b_1 e^{a(1-r)}}{a^2(1-r)^2} + \frac{b_1 e^{a(1-r)t}}{a(1-r)} - \frac{b_1 e^{a(1-r)}}{a(1-r)} \right] \} \quad (6)$$

The time response function is

$$\hat{x}^{(1)}(t) = \{ e^{-a(1-r)(t-1)} [(x^{(1)}(1))^{(1-r)} + (1-r)e^{-a(1-r)} g(t)] \}^{\frac{1}{1-r}}$$

of which

$$g(t) = \frac{b_0 e^{at(1-r)}}{a(1-r)} - \frac{b_0 e^{at(1-r)}}{a(1-r)} + \frac{b_1 t e^{at(1-r)r}}{a(1-r)} - \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} + \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} - \frac{b_1 e^{at(1-r)}}{a(1-r)}$$

Integrating formula (3) on the basis of  $[k-1, k]$  get

$$\int_{k-1}^k \frac{dx^{(1)}(t)}{dt} dt + \int_{k-1}^k ax^{(1)}(t) dt = \int_{k-1}^k (b_0 + b_1 t)(x^{(1)}(t))^r dt$$

Note  $z^{(1)}(k) = \int_{k-1}^k x^{(1)}(t) dt$ ,  $z^{(2)}(k) = \int_{k-1}^k (x^{(1)}(t))^r dt$ ,  $z^{(3)}(k) = \int_{k-1}^k t(x^{(1)}(t))^r dt$

Then

$$z^{(1)}(k) = \int_{k-1}^k x^{(1)}(t) dt = px^{(1)}(k) + (1-p)x^{(1)}(k-1)$$

$$z^{(2)}(k) = \int_{k-1}^k (x^{(1)}(t))^r dt = p(x^{(1)}(k))^r + (1-p)(x^{(1)}(k-1))^r$$

$$z^{(3)}(k) = \int_{k-1}^k t(x^{(1)}(t))^r dt = pk(x^{(1)}(k))^r + (1-p)(k-1)(x^{(1)}(k-1))^r$$

Therefore

$$\hat{x}^{(0)}(k) + az^{(1)}(k) = b_0 z^{(2)}(k) + b_1 z^{(3)}(k)$$

For the given  $p$  and  $r$ , following the least squares method, we get

$(a, b_0, b_1)^T = (G^T G)^{-1} G^T H$ , of which

$$G = \begin{bmatrix} -z^{(1)}(2) & z^{(2)}(2) & z^{(3)}(2) \\ -z^{(1)}(3) & z^{(2)}(3) & z^{(3)}(3) \\ \vdots & \vdots & \vdots \\ -z^{(1)}(N) & z^{(2)}(N) & z^{(3)}(N) \end{bmatrix} \quad H = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(N) \end{bmatrix}$$

$$\min MAPE = \frac{1}{N-1} \sum_{i=2}^N \left| \frac{x^{(0)}(t) - \hat{x}^{(0)}(t)}{x^{(0)}(t)} \right| \times 100\%$$

$$s.t. \begin{cases} \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1) \\ \hat{x}^{(1)}(t) = \{e^{-a(1-r)(t-1)} [(x^{(1)}(1))^{1-r} + (1-r)e^{-a(1-r)} g(t)]\}^{\frac{1}{1-r}} \\ g(t) = \frac{b_1 t e^{at(1-r)r}}{a(1-r)} - \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} + \frac{b_0 e^{at(1-r)}}{a(1-r)} + \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} - \frac{b_1 e^{at(1-r)}}{a(1-r)} - \frac{b_0 e^{at(1-r)}}{a(1-r)} \\ (a, b_0, b_1)^T = (G^T G)^{-1} G^T H \\ r \neq 0, 1 \end{cases} \quad (7)$$

When  $h = 2$ ,

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = (b_0 + b_1 t + b_2 t^2)(x^{(1)}(t))^r \quad (8)$$

is called the whitening equation for the GM(1,1,t<sup>2</sup>,p) power model. The same can be proved that its time response function is

$$\hat{x}^{(1)}(t) = \{e^{-a(1-r)(t-1)}[(x^{(1)}(1))^{1-r} + (1-r)e^{-a(1-r)}f(t)]\}^{\frac{1}{1-r}}$$

Where

$$f(t) = \frac{b_2 t^2 e^{at(1-r)}}{a(1-r)} - \frac{2b_2 t e^{at(1-r)}}{a^2(1-r)^2} + \frac{b_1 t e^{at(1-r)}}{a(1-r)} + \frac{2b_2 e^{at(1-r)}}{a^3(1-r)^3} - \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} + \frac{b_0 e^{at(1-r)}}{a(1-r)} - \frac{2b_2 e^{at(1-r)}}{a^3(1-r)^3} + \frac{2b_2 e^{at(1-r)}}{a^2(1-r)^2} - \frac{b_2 e^{at(1-r)}}{a(1-r)} + \frac{b_1 e^{at(1-r)}}{a^2(1-r)^2} - \frac{b_1 e^{at(1-r)}}{a(1-r)} - \frac{b_0 e^{at(1-r)}}{a(1-r)}$$

### 4 Example analysis

The transaction scale of the fresh e-commerce industry is very important to its industry development trend, so this paper models and analyzes the transaction scale of the fresh e-commerce industry to further promote the development of the fresh e-commerce industry.

**Table 1.** Transaction scale of fresh e-commerce industry from 2010 to 2019.

Unit: 100 million yuan

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Transaction scale	4.2	10.5	40.5	130.2	289.8	542	913.9	1402.8	1950	2554.5	3641.3	4658.1

Data source: Net Economics, Prospective Industry Research Institute

#### 4.1 Select the dimensions of the best predictive model

In order to improve the accuracy of the model as much as possible, the optimal data dimension must be determined, this paper aims to predict the transaction scale of the fresh e-commerce industry in 2019, establish GM (1, 1, t<sup>h</sup>, p) power model of different dimensions, and determine the most suitable data dimension by the size of the average error, the specific values are shown in Table 2:

**Table 2.** Comparison of accuracy of models with different dimensions.

Data dimension	Traditional GM(1,1) power model			GM(1,1,t <sup>h</sup> ,p) power model					
	Simulation error	forecast error	average error	GM(1,1,t,p) power model			GM(1,1,t <sup>2</sup> ,p) power model		
				simulation error	forecast error	average error	simulation error	forecast error	average error
9 (2010-2018)	10.5%	2.1%	6.3%	2.3%	5.6%	4.0%	5.5%	1.5%	3.5%
8 (2011-2018)	3.4%	0.5%	<u>2.0%</u>	1.1%	5.4%	3.3%	1.2%	3.1%	2.1%
7 (2012-2018)	2.1%	2.2%	2.1%	1.3%	4.2%	2.8%	0.8%	1.7%	1.2%
6 (2013-2018)	2.2%	2.7%	2.5%	1.1%	3.2%	2.1%	0.4%	0.4%	<u>0.4%</u>
5 (2014-2018)	2.4%	3.2%	2.8%	0.4%	1.7%	0.3%	0.2%	1.5%	0.9%
4 (2015-2018)	1.8%	4.7%	3.2%	0.0%	0.3%	<u>0.2%</u>	0.7%	2.6%	1.6%

Note: (1) Forecast error refers to the prediction error in 2019;

## **4.2 The average error is the average of the simulation error and the prediction error.**

It can be seen from Table 2 that the improved GM(1,1,t<sup>h</sup>,p) power model can significantly improve the fitting and prediction effect of the model compared with the traditional GM(1,1) power model, so in this case, the GM(1,1,t,p) power model is used to establish the model, and the data dimension is 4 dimensions.

## **4.3 Establish a metabolic GM (1,1,t,p) power model**

According to the results of Table 2, the data for the 2015 year is removed, the data for 2019 is added, and then the GM(1,1,t,p) power model is established, which is the first metabolic GM(1,1,t,p) power model, and so on for the second and third metabolic iterations. In 2022, the transaction scale of fresh e-commerce industry will reach 562.5 billion yuan.

## **5 Conclusion**

In order to be more in line with the principle of new information first, the metabolism theory is combined with the GM (1,1,t<sup>h</sup>,p) power model, and it is applied for the first time in the prediction of the transaction scale of the fresh e-commerce industry, and the examples show that the improved model can improve the prediction and simulation error well, and further expand the application scope of the power model, which has certain practical significance and application prospects. In addition, although China's fresh e-commerce is still in its infancy, with the popularization of the Internet and the development of e-commerce, the future fresh e-commerce has great development prospects, fresh e-commerce practitioners should follow the trend of the times, keep pace with the times, continue to innovate, and enhance the core competitiveness of the brand.

## **Reference**

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