Research on optimal supply decision of manufacturers considering futures purchase under deferred payment

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Abstract. Taking the three-level supply chain consisting of a single supplier, a manufacturer and a strong brand as the research object, the manufacturer's optimal supply decision model is constructed, which considers the futures purchase under the deferred payment of the brand, and it is proved that the optimal solution not only exists but also is unique. Then, the influence of parameter variation on the optimal ordering strategy was analyzed by numerical example simulation and sensitivity analysis. The results show that when the period of deferred payment to the manufacturer is extended and the price discount is strengthened, the manufacturer can purchase from the futures market to lock the price and make up for its income loss through the supplier's delayed payment. Production rate should also be controlled, too high or too low production rate will affect the manufacturer's production cycle.

1 Introduction

Brand parties represented by well-known real estate companies occupy a dominant position in the supply chain. Faced with this situation, manufacturers often use price discounts and deferred payments to attract brand buyers, reduce costs through mass production, and optimize inventory through futures purchasing.

In combination with supply chain purchasing strategies, Zhai[1] et al. studied multiple procurement scenarios in which both retailers and manufacturers could derive benefits from the production time hedging strategy. Keynes[2]proposed the earliest hedging theory (futures risk avoidance theory), which believed that the hedges could effectively transfer the price risk in the market. Kouvelis[3] found that cash hedging technology could realize cost hedging under the condition of minimizing the hedging cost. The above studies are mostly based on the hedging theory, but they do not take into account the cash flow pressure brought by the strong brand in the supply chain to the manufacturer, and how to deal with the cash flow pressure. In this regard, the current representative achievements in the field of inventory optimization under deferred payment can be used for reference. Goyal[4]first proposed the deferred payment EOQ model, and Chu[5] et al. extended Goyal's model by studying the situation of spoiled products. Sundararajan[6] established EOQ model on the condition of

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profit maximization per unit time, and discussed the impact of delayed payment and inflation on the model. Through the review of the representative results related to deferred payment and futures purchase, the current research mainly involves: the brand's optimal order quantity model with price discount under the deferred payment terms, does not combine the deferred payment and futures purchase strategy from the perspective of the manufacturer. In view of this, this paper constructs a three-level supply chain model of manufacturer futures procurement under deferred payment, and proposes the optimal strategy of the manufacturer. The influence of futures purchase price and price discount on the manufacturer's optimal strategy is analyzed by sensitivity.

2 Problem description and parameter description

2.1 Problem Description

Examine the three-tier supply chain system consisting of a single supplier, manufacturer and strong brand. When purchasing, the brand will ask for delayed payment and price discount. In order to ensure their own interests, manufacturers will reduce costs through mass production, deferred payments to upstream suppliers, and futures procurement. In the case of deferred payment, the manufacturer needs to adjust the deferred payment period and price discount coefficient to ensure stable operation. Supply chain structure is shown in figure 1:

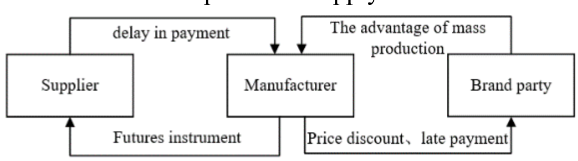

Fig. 1. Supply chain structure diagram.

2.2 Notations and assumptions

Notations used in the model:

D The total demand rate for the product, M Deferred payment period provided by raw material supplier, P Manufacturer's productivity, N Allow the brand party to delay the payment period, h Finished product storage costs, F_t Unit futures purchase price, m The number of times raw materials are delivered, A_{raw} Transportation costs of raw materials, B Storage costs of raw materials, H_a Carrying cost of finished goods, T The manufacturer's production cycle, w The purchase price of the brand, I_q Rate of return per unit time, I_p Interest rate paid per unit of time

Assumptions: No shortages allowed. The productivity and demand rate of manufacturers are constant values, and the productivity is greater than the demand rate. The deferred payment period given by the supplier to the manufacturer is no less than the deferred payment period given by the manufacturer to the brand. The futures contract of raw material commodities purchased by the manufacturer in the futures market can be delivered smoothly according to the provisions. Before the term M, the manufacturer deposits the income at interest rate I_q to earn interest. At the end of the term M, if the product in stock has not been sold, the manufacturer pays interest at interest rate I_p to the supplier.

3 Model establishment and solution

3.1 Mathematical models

The manufacturer starts to consume raw materials at $t=0$, the moment t_0 production is complete, the finished product reaches the highest level. With the sale to the brand, the inventory goes to zero at time T, at which point an order cycle ends, and the whole process repeats. Assume that the demand rate is D, the inventory cycle length of finished products is $T^{'}(T^{'}=T-t_0)$, T is the duration of the entire production cycle, and the productivity of the manufacturer is $P(t)=P(P>D)$, then the storage level $I(t)$ of finished products satisfies the equation and can be derived:

$$\begin{cases} \dfrac{dI(t)}{dt}=P(t)-D & 0\le t\le t_0 \\ \dfrac{dI(t)}{dt}=-D & t_0\le t\le T \end{cases} \qquad I(t)=\begin{cases} (P-D)t & 0\le t\le t_0 \\ -Dt+DT & t_0\le t\le T \end{cases}$$

Total inventory of finished products in time period $[0,T]$: $L(t)=\int_0^{t_0}[P(t)-D](t_0-t)dt+\int_0^{T}(t-t_0)Ddt$

Finished product inventory storage costs per cycle are: $C_h=hL(t)=h\left[\int_0^{t_0}(P-D)(t_0-t)dt+\int_0^{T}(t-t_0)Ddt\right]$ According to $\int_0^{t_0}Pdt=\int_{t_0}^{T}Ddt$, it can be concluded that: $t_0=\dfrac{DT}{P+D}$.

Through the futures purchase of financial derivatives, the risks caused by price fluctuations can be avoided to a certain extent. In addition, considering that the strong brand has the advantage of order volume, it requires more price discounts from the manufacturer during the purchase. It is assumed that the discount rate is $\tau(0<\tau<1)$, w_0 is the benchmark price without discount. Then the wholesale price of the brand party is: $w=\tau w_0$.According to the above assumptions, the profit function of the manufacturer in an order cycle is composed of the following parts:

Manufacturers' sales revenue per cycle can be divided into three situations:

1) $T^{'}\le N$ .The brand must pay the rest at the end of term N, so the manufacturer's sales revenue: $S_r=wDT^{'}$ .2) $N\le T^{'}\le M.S_r=wDN+wD(T^{'}-N)$. 3) $M\le T^{'}.S_r=wDN+wD(T^{'}-N)$

Within each cycle, the manufacturer realizes scale effect by organizing mass production and reduces a series of additional costs caused by line change of product models and specifications, including equipment factor cost, quality factor cost, energy consumption factor cost, personnel factor cost and raw material consumption cost. These costs are represented by the following symbols. It can be seen that the indirect income obtained by the manufacturer is as follows: $\sum c_{f-x}=c_{f-mac}+c_{f-qua}+c_{f-ener}+c_{f-hum}+c_{f-mater}$.

The interest on the manufacturer's sales revenue per cycle exists in the following three situations: 1) $T^{'}\le N$. Brand Party shall make full payment at the end of term N, from time $N\sim M$. Therefore, interest earned by the manufacturer: $Y_a=wDI_qT^{'}(M-N).2)N\le T^{'}\le M.Y_a=wDI_q$ $\left[\int_N^{T^{'}}tdt+\int_{T^{'}}^{M}T^{'}dt\right].3)M\le T^{'}.Y_a=wDI_q\int_N^{M}tdt$. Manufacturers' raw material consumption costs mainly include raw material purchase cost, transportation cost and inventory holding cost.

Where, if the storage level of raw materials is satisfied $dI_1(t)/dt=-P$, it can be derived $I_1(t)=-Pt+K$ (K is any constant value). As can be known from the initial conditions $I_1(t_0)=0$, $K=Pt_0$, then the quantity of raw materials obtained by the manufacturer after delivery in the futures market is $Q^{'}=Pt_0$,the inventory curve of raw materials is $I_1(t)=P(t_0-t)$, the inventory of raw materials in time $[0,t_0]$ is $L_B=\int_0^{t_0}P(t_0-t)=\frac{1}{2}Pt_0^2$, the storage cost of raw materials in each cycle is $W_B=\frac{1}{2}BPt_0^2$, and the total cost of raw materials consumed in each cycle is: $TC_{raw}=F_tPt_0+A_{raw}+\frac{1}{2}BPt_0^2$ .In addition, interest paid by the manufacturer on the amount of goods not sold after the expiration of each period of deferred payment is divided into three categories:1) $T^{'}\le N$. The manufacturer allows the brand party to delay the payment period n is less than the delay payment period and then the interest payable to the manufacturer is: Y_b=0. 2)$N\le T^{'}\le M$.Y_b=0. 3)$M\le T^{'}$.$Y_b=F_tI_P\int_M^{T^{'}}I(t)dt$. To sum up, the manufacturer's net profit function in each cycle can be expressed as:

$$\Pi_{mi}=S_r+\sum c_{f-x}+Y_a-C_h-TC_{raw}-Y_b$$

After simplification, we can get:

$$
\begin{aligned}
&(1)T^{'}\le N\\
&\Pi_{m1}=wD(T-\frac{DT}{P+D})+\sum c_{f-x}+wDI_q(T-\frac{DT}{P+D})(M-N)-\frac{1}{2}h(P-D)(\frac{DT}{P+D})^2\\
&-D\left[\frac{1}{2}T^2-T(\frac{DT}{P+D})\right]-F_tP(\frac{DT}{P+D})-A_{raw}-\frac{1}{2}BP(\frac{DT}{P+D})^2 \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&(2)N\le T^{'}\le M\\
&\Pi_{m2}=wD(T-\frac{DT}{P+D})+\sum c_{f-x}+wDI_q\left[(T-\frac{DT}{P+D})M-\frac{1}{2}N^2-\frac{1}{2}(T-\frac{DT}{P+D})^2\right]-\frac{1}{2}h\left(P-D\right)\\
&(\frac{DT}{P+D})^2-D(\frac{1}{2}T^2-\frac{DT^2}{P+D})-F_tP(\frac{DT}{P+D})-A_{raw}-\frac{1}{2}BP(\frac{DT}{P+D})^2 \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&(3)M\le T^{'}\\
&\Pi_{m3}=wD(T-\frac{DT}{P+D})+\sum c_{f-x}+\frac{wDI_q}{2}(M^2-N^2)-\frac{h}{2}(P-D)(\frac{DT}{P+D})^2-D\left(\frac{1}{2}T^2-\frac{DT^2}{P+D}\right)-F_tP(\frac{DT}{P+D})\\
&-A_{raw}-\frac{BP}{2}(\frac{DT}{P+D})^2-F_tI_pD\left\{T\left[(T-\frac{DT}{P+D})-M\right]+\frac{1}{2}\left[M^2-(T-\frac{DT}{P+D})^2\right]\right\} \qquad (3)
\end{aligned}
$$

It is easy to verify: $\Pi_{m1}(N)=\Pi_{m2}(N),\Pi_{m2}(M)=\Pi_{m3}(M)$ So $\Pi_{mi}(T)$ is continuous on the interval $(0,+\infty)$.

3.2 Model solution

According to the necessary conditions for the existence of extreme values, the optimal value of T must satisfy: $\frac{d\Pi_{mi}(T)}{dT}=0$ $i=1,2,3$ .Thus, the following can be obtained:

$$\frac{d\Pi_{m1}(T)}{dT}=\frac{wPD}{P+D}+\frac{(P+D)(wDI_qM-wDI_qN)}{P+D}+\frac{D^2T}{P+D}-\frac{D+PDT+PDF_t}{P+D}-\frac{BPD^2T+hD^2(P-D)T}{(P+D)^2} \quad (4)$$

$$\frac{d\Pi_{m2}(T)}{dT}=\frac{wPD}{P+D}+\frac{wPDI_q(M-T)}{P+D}-\frac{PDT-D^2T}{P+D}-\frac{PDF_t}{P+D}-\frac{BPD^2T}{(P+D)^2}-\frac{hD^2(P-D)T}{(P+D)^2} \quad (5)$$

$$\frac{d\Pi_{m3}(T)}{dT}=\frac{wPD}{P+D}-\frac{BPD^2T}{(P+D)^2}-\frac{hD^2(P-D)T}{(P+D)^2}-\frac{PDF_t}{P+D}-\frac{PDT-D^2T}{P+D}-\frac{F_tI_pD(P^2T+2PDT-M(P+D)^2)}{(P+D)^2} \quad (6)$$

Firstly, the property theorem about the optimal solution of function $\Pi_{mi}(T)$ is given: **Theorem 1** (1)If $d^2\Pi_{m1}/dT^2<0$, then $\Pi_{m1}(T)$ increases strictly in the interval $(0,T_1^*)$ and decreases strictly in the interval $(T_1^*,+\infty)$, that is, $\Pi_{m1}(T)$ reaches its maximum value at T_1^* within $(0,+\infty)$.(2)(3)The same can be obtained. According to theorem 1, The optimal value T^* of $\Pi_{mi}(T)$ has been obtained. According to equations (4)~(6), $f_i(T)=d\Pi_{mi}/dT$ can be obtained: $f_1(N)=f_2(N)\, f_2(M)=f_3(M)$ For ease of expression, let $\varphi_1=f_1(N)=f_2(N)\, \varphi_2=f_2(M)=f_3(M)$, while $f_2(T)$ strictly monotonically decreases within $(0,+\infty)$, so $\varphi_1\geq\varphi_2$. The following theorem can find the global optimal solution T^*.**Theorem 2** If $\varphi_1<0\,\&\,\varphi_2<0,\Pi_{mi}(T^*)=\Pi_{mi}(T_1^*)$, therefore $T^*=T_1^*$.(2)(3)*The same can be obtained*.

4 Example analysis

In order to verify the feasibility of the model, specific parameters are set as follows: $M=20/360year, N=15/360year, I_p=0.15, I_q=0.09, F_t=24yuan/piece, P=7000piece/year, D=5000piece/year,$

$w_0=30yuan/piece$ $and\varphi_1=15609.26>0,\varphi_2=15317.90>0,\Pi_{m2}(M)=910.56,\Pi_{m3}(T_3^*)=6211.83,$

$B=4yuan/piece, h=6yuan/piece/year, \sum c_{f-x}=50yuan/time, A_{raw}=12yuan/piece,$ τ=0.98,

Through calculation, we can get: $d^2\Pi_{m1}/dT^2<0, d^2\Pi_{m2}/dT^2<0,$ $d^2\Pi_{m3}/dT^2<0$ According to Theorem 1, the optimal production cycle of the manufacturer's inventory system is $T^*=T_3^*=0.7394$, and the maximum profit is $\Pi_m^*=6211.83$. According to Theorem 1, the optimal production cycle of the manufacturer's inventory system is $T^*=T_3^*=0.7394$, and the maximum profit is $\Pi_m^*=6211.83$. In order to directly show the influence of brand party's dominance on the manufacturer, M、N、τ、F_t are adjusted and the profit changes of the manufacturer are observed, as shown in Table 1:

Table 1. Influence of changes in M、N、τ、F_t on manufacturers' supply strategy.

M	F_t	N	τ	T^*	Π_m^*
20/360	24	5/360	0.98	0.7394	6222.04
30/360	23.8	5/360	0.98	0.7910	7069.03
20/360	24	15/360	0.97	0.7008	5581.64
35/360	23.7	5/360	0.98	0.8169	7511.56
25/360	23.6	5/360	0.98	0.8099	7374.16

Table 2. Effects of productivity P changes on manufacturers' production cycle.

D	P	η	T^*
5000	5500	0.91	0.7638
5000	6000	0.83	0.7513
5000	8000	0.63	0.7374
5000	10000	0.5	0.7467
5000	12000	0.42	0.7631
5000	14000	0.36	0.7814

As can be seen from the first and second sets of data in the table, when the period of deferred payment to the manufacturer is extended to half a month (15 days) and the price discount coefficient is increased from 2% to 3% (group 1 and Group 3 data), the manufacturer's profit will decrease by 10.29%. At this time, manufacturers can lock the price in the futures market and delay the payment for goods to make up for their own income loss, achieving a profit increase of 13.61%. The results of the fourth and fifth groups were adjusted $MandF_t$. However, consider that in real life, manufacturers not only buy cheap ($F_t = 24$) futures locked prices, but also buy raw materials on the spot market. If the manufacturer buys the futures contract at a low price in the futures market before production ($t = 0$), stores the raw materials in the warehouse after delivery, and buys the same amount of raw materials in the spot market at the price of 24.5 half a month (15 days) later when the price of raw materials in the spot market rises, and the storage cost of raw materials is 4, then the cost of buying futures is 1.35% less than that of buying spot. Therefore, the purchase of futures for enterprises to reduce the cost of procurement are beneficial. In addition, by changing the size of supplier productivity P , where η represents the ratio of demand rate D to productivity P , observe its impact on manufacturers' profits, as shown in Table 2:

As can be seen from Table 5, with the decrease of productivity, the production cycle of manufacturers becomes longer. The last three sets of data suggest that high productivity also leads to longer production cycles. This means that the manufacturer needs to control the production rate. Both too high and too low production rate will affect the production cycle of the supply chain, thus affecting the inventory cost.

5 Conclusion

Strong brands compress manufacturers' profit margins through delayed payment and wholesale price discounts, which deeply affects the sustainable circulation of manufacturers' own funds and the stability of production and operation. In this paper, a supply chain consisting of a single supplier, a manufacturer and a brand is considered, and an optimal supply decision model is constructed for the manufacturer considering futures purchase under the condition of deferred payment by the brand. The influence of futures purchase price, deferred payment period, price discount coefficient and other factors on the supply strategy of the manufacturer is further analyzed. The effect of manufacturer productivity on the optimal production cycle is also discussed.

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