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Comparing various GARCH-type models in the estimation and forecasts of volatility of S&P 500 returns during Global Finance Crisis of 2008 and COVID-19 financial crisis

Xuanyu Chen

King's Business School, King's College London, London, WC2R 2LS, UK

Abstract: In this study, we utilize various GARCH-type models to estimate and forecast volatility on S&P 500 returns and compare the results between the two financial crises, the GFC of 2008 (Global Financial Crisis of 2008) and the COVID-19 financial crisis. These two financial crises are different from the forming reasons by whether mainly caused by the financial factors. This study also makes the evaluations on the performance of these GARCH-type models in estimating and forecasting volatility, which may provide the efficient models for reference for the research of the volatility of the future potential financial crisis. We find that as for the AIC/BIC assessments on the estimation of volatility, the GJR-GARCH model performs better during the GFC of 2008, while the EGARCH model has the better performance during the COVID-19 financial crisis. With respect to the QLIKE loss function evaluation on the forecasting ability of volatility, the GJR-GARCH model performs better during the GFC of 2008, while symmetric GARCH model has better volatility forecasting ability during the COVID-19 financial crisis.

Key words: Volatility, Financial Crisis, ARCH, GARCH, EGARCH, GJR-GARCH.

1. Introduction

Volatility is interpreted to reflect the uncertainty in financial market, which is considered as one of the determinants on investment decisions and portfolio creations (Poon and Granger, 2003). Thus, it is significant to effectively estimate and forecast the volatility of the stock returns, which may contribute to mitigating the uncertainty in the financial market. The degree of impact on stock market volatility is seen as a good indicator to measure the ability of stock market to resist the financial crisis (Rastogi, 2014). The financial crisis triggered by the COVID-19 pandemic has caused the most serious negative impacts on the real economy and financial markets all over the world since the Great Recession of 2008 and the GFC of 2008 (Global Financial Crisis of 2008). Many studies have suggested that the Great Recession of 2008 was the inevitable result of deregulation of financial markets and derivatives, especially the CDS (credit default swap), dramatic growth of the housing bubbles, and high default rates on rising sub-prime mortgages. The GFC of 2008 was a typical financial crisis caused by the imperfection financial system, implying that the financial factors were the main drivers of the GFC of 2008. The COVID-19 financial crisis was unexpected and really different from the GFC of 2008 on the main drivers. In essence, the COVID-19 crisis should not only be viewed as a health crisis, but also as an economic or financial crisis, which means that the severe impacts of COVID-19 pandemic on people's health and normal social system have further turned into a financial and economic crisis because of supply and demand (Goldstein, Koijen & Mueller, 2021).

There are several similarities and differences between the GFC of 2008 and the COVID-19 financial crisis. These two financial crises both resulted in a great recession on the world economy, huge uncertainty, and increased volatility in the financial markets. However, they had different forming reasons that the GFC was caused by the U.S. housing bubble burst, while the COVID-19 crisis arose from the outbreak of a global pandemic of infectious disease, which has no direct connection with the economic or financial factors. Setiawan et al. (2021) compared these two financial crises in the area of stock markets and found that the financial crisis caused by the COVID-19 pandemic may have a relatively higher volatility on stock market returns than the GFC of 2008, although these two financial crises both encountered very high volatility on stock markets.

Thus, it is worthy researching and analyzing the differences between these two financial crises mainly on the volatility of the stock market returns. This study will investigate and evaluate the comparison between these two financial crises on the volatility of the S&P 500 returns based on various GARCH-type models, mainly for the purpose of obtaining the conclusions about which

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GARCH-type models are more effective and efficient for the different types of financial crises. In the meanwhile, this study will also evaluate the performance of these GARCH-type models in estimating and forecasting volatility by some assessment criterion, AIC/BIC on assessing the estimation of volatility and Quasi-Likelihood loss function on assessing the forecasting ability of volatility, which may find out the most appropriate models for different periods.

2. Literature Review

Basic ARCH/GARCH models and the asymmetric GARCH models both are researched and compared by many studies in different area stock markets. Brailsford and Faff (1996) argued that it is difficult to make the volatility forecasting. In their study, the ARCH-type models were proved to generate superior predictive ability on volatility in the case of daily Australia data. It is still indicated that the ranking of models used displays to be sensitive to error statistics for evaluating the accuracy of forecasting (Balaban, Bayar & Faff, 2006). Varma (1999) researched daily data between 1990 and 1998 of India stock index and concluded that GARCH(1,1) outperformed the other variants of models. Angabini and Wasiuzzaman (2010) analyzed the Malaysian stock market volatility based on various types of GARCH models and found that the GARCH(1,1) performed best among all models based on the AIC criteria. Gokcan (2000) made a comparison between linear (GARCH(1,1)) and non-linear (EGARCH) models by measuring the monthly stock market return in the emerging financial markets. The result of his study was that the GARCH(1,1) had a better performance than the EGARCH in the aspect of forecasting the volatility of time series data, implying that the GARCH(1,1) was more appropriate in the emerging stock markets.

On the other hand, there also are many studies arguing that asymmetric GARCH models can perform better in forecasting the volatility of stock market returns. Hansen and Lunde (2005) compared 330 ARCH-type models in volatility forecasting ability based on the out-of-sample analysis in DM–$ exchange rate data and IBM return data, and they argued that there was no evidence to claim that the GARCH(1,1) could have a better performance than the other models in the exchange rate data, while in IBM return data, due to the consideration of a leverage effect, the GARCH(1,1) was apparently not good as the asymmetric GARCH models in volatility forecasting. Shamiri and Hassan (2007) compared 3 types of volatility models, including the GARCH(1,1), EGARCH and GJR-GARCH, in the Malaysian and Singaporean stock market, and it was indicated that the AR(1)-GJR model was considered as the best out-of-sample tool in forecasting the data in Malaysian stock market whereas the AR(1)-EGARCH model was more excellent in forecasting volatility for Singaporean stock market. Floros (2008) utilized the symmetric and asymmetric GARCH-type models to model the volatility in daily data from CMA General Index and TASE-100 Index, and the result of the empirical evidence illustrated that the asymmetric GARCH models performed much better than the symmetric GARCH models, since the empirical evidence was found that these two indexes were impacted significantly by the asymmetric effect. Guidi (2009) found that the GARCH family models have asymmetric effects and these asymmetric GARCH models performed best in volatility forecasting, through the empirical evidence of comparing several GARCH family models in modelling and forecasting volatility in Germany, Switzerland, and the UK stock market indexes. Thus, it is worthy analyzing the utilization and comparison of various GARCH-type models in different periods of the financial crisis.

Although many studies on GARCH-type models cover the background discussion of single financial crisis, the above research does not involve a comparative discussion of financial crises caused by different reasons. This study will fill this gap, which may further contribute to the comparison of the applicability of GARCH-type models in estimation and forecasts during different crisis periods.

3. Methodology & Data

3.1 GARCH-type models

3.1.1 ARCH model

One of the simplest versions of the Autoregressive conditional heteroskedasticity models (ARCH model) was proposed by Engle (1982), which can be used to model volatility in the financial market. In the ARCH model, it is required to define the conditional variance of a random variable by u_t. σ_t^2 is defined as the conditional variance of u_t. The difference between conditional and unconditional variance is considered to be identical to the difference between conditional mean and unconditional mean (Brooks, 2019). In addition, the values of α_i should be less than 1 since it should satisfy the characteristic of stationarity in variance.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-1}^2 \quad (1)$$

3.1.2 GARCH model

Bollerslev (1986) proposed the GARCH model (Generalized ARCH model), which generalizes the ARCH model to allow the conditional variance of error term to depend also on its previous own lags. In the formula (2), $\alpha_1 u_{t-1}^2$ is considered to be the ARCH model component while $\beta\sigma_{t-1}^2$ is the GARCH model component. In the GARCH model, $(\alpha_1 + \beta)$ should be less than 1.

$$GARCH\ (p,q): \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (2)$$

3.1.3 EGARCH model

Nelson (1991) proposed the exponential GARCH model (EGARCH model). When $\alpha_1<0$ and $f(u_{t-1})>=0$, $\ln(\sigma_t^2)$ may decline, while when $\alpha_1<0$ and $f(u_{t-1})<=0$, $\ln(\sigma_t^2)$ tends to increase, which means for the same absolute value of f(ut-

$_1$), the negative value of $f(u_{t-1})$ may result in a larger $\ln(\sigma_t^2)$ compared with the positive value of $f(u_{t-1})$.

$$\ln(\sigma_t^2) = \alpha_0 + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha_1 \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \quad (3)$$

$$with\ f(u_{t-1}) = \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]$$

3.1.4 GJR-GARCH model

The GJR-GARCH model, proposed by Glosten, Jagannathan and Runkle (1993) is another version of extension of the symmetric GARCH model, which can also reflect the leverage effect. I_{t-1} is the indicative function to show $u_{t-1}<0$, implying that if u_{t-1} is less than 0, I_{t-1} is equal to 1, and otherwise, I_{t-1} is equal to 0, which can be used to capture the possible asymmetries ($\alpha_0>0$, $\alpha_1>0$, $\beta>=0$, $(\alpha_1+\gamma)>=0$).

$$AIC = -2\ln(L) + 2k$$
$$BIC = -2\ln(L) + \ln(n) * k \quad (4)$$

3.2 The AIC and BIC approaches & The QLIKE loss function

AIC, proposed by Akaike (1974) and BIC, proposed by Schwarz (1978) are the classical standards to measure the goodness of fit in the statistical model.

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \quad (5)$$

Where L is the maximum likelihood in the model, n is the number of data and k is the number of variables in the model.

The function of AIC and BIC is to figure out the relative amount of information lost for the statistical models. According to the amount of information lost, the performance of the statistical models can be evaluated, and the less information is lost, the better statistical model performs. In this study, AIC and BIC approaches are utilized to measure the performance of GARCH-type models on the volatility estimation, and the model which has the lowest value of AIC and BIC may be the most appropriate model in volatility estimation.

This study uses the in-sample and out-of-sample analysis in forecasting volatility. The first sub-sample can be considered as the base sample, which may contribute to the volatility forecasting in the next sub-sample. We can figure out which models can perform best in volatility forecasting by comparing the forecasted volatility with the real volatility in the same period. QLIKE (Quasi-likelihood) loss functions are also widely used as a measure in volatility forecasting ranking, since they satisfy the necessary condition that the loss functions are too robust to noise in the volatility proxy (Patton, 2011). The intuition behind QLIKE loss function refers to finding the model minimizing the difference between the forecast and the real data, and the model which shows the lowest maximum can be considered as that minimization.

$$QLIKE: L(\sigma^2, h) = \log(h) + \frac{\sigma^2}{h} \quad (6)$$

3.3 Data selection

This study uses the data of the S&P 500 index from Yahoo Finance (2022). As for the GFC of 2008, the whole period covers the range between 2002 and 2010, where the first sub-period is from 1st October 2002 to 1st August 2007, and the second sub-period is between 1st October 2002 and 1st March 2010. When it comes to the COVID-19 financial crisis, the whole period is between 2012 and 2020, where the first sub-period is from 3rd January 2012 to 21st January 2020, and the second sub-period is from 3rd January 2012 to 31st December 2020. The daily closed price will be used to inspect the impact on volatility. The influence of the financial crisis of 2007-2008 will not be considered in the first sub-period (2002-2007) but the second sub-period (2002-2010), while the influence of the COVID-19 financial crisis will not be considered in the first sub-period but the second sub-period as well, which can effectively represent the scenario of no impacts of financial crisis and the scenario of under the impacts of financial crisis.

With respect to the analysis between the first sub-period (1st October 2002 - 1st August 2007) during the first main period (2002-2010), this is a control group, without any impacts of financial crisis, while as for the second sub-period (1st October 2002 - 1st March 2010), it is intuitive to witness whether there are any possible different impacts on volatility of stock market returns, because the second sub-period covers the impacts of the financial crisis. It is the same for the COVID-19 financial crisis during 2012-2020, implying that the sub-period between 3rd January 2012 - 21st January 2020 is the control group, which can be used to make the comparison with the second sub-period, 3rd January 2012 - 31st December 2020, to analyze the impacts of financial crisis on volatility.

4. Empirical Results

4.1 Descriptive statistics

At first, it is required to generate the returns of the S&P 500 index because this study concentrates on the volatility of the stock market returns. Daily stock market returns can be generated by the function of daily adjusted closed price of S&P 500 index.

$$Returns(R_t) = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (7)$$

Where R_t represents the daily returns of S&P 500 index, and P_t represents the daily adjusted closed price of S&P 500 index.

Table 1. Descriptive statistics of returns of S&P 500 index

Descriptive statistics of returns of S&P 500 index		
Statistics	Series Periods	
	2002-2007	2002-2010
Mean	0.0004502	0.0001472
Std. dev.	0.0084846	0.0137469
Min	-0.0358672	-0.0946951
Max	0.0462494	0.1095719
Skewness	0.1357044	-0.211993
Kurtosis	5.318495	13.55563
Descriptive statistics of returns of S&P 500 index		
Statistics	Series Periods	
	2012-2020.01	2012-2020.12
Mean	0.0004722	0.0004765
Std. dev.	0.0080697	0.0105467
Min	-0.0418429	-0.1276522
Max	0.0484033	0.0896831
Skewness	-0.4515601	-1.021902
Kurtosis	6.284071	25.41335

Table 1 illustrates the descriptive statistics of returns of S&P 500 index for these two financial crises. It shows that the mean for the four sub-periods tend to be zero, which satisfies the trend of a time series of returns. It is witnessed that there is a relatively large difference between the maximum and minimum values of returns, while the returns of S&P 500 index tend to show a higher level of fluctuation for the second sub-periods since the standard deviation is higher with more observations. The negative skewness during 2002-2010 and 2012-2020.12 means that there are asymmetric tails, implying that the increase of volatility may be affected more significantly by the financial time series with a negative shock compared with the same-level positive shock (Brooks, 2019). As for the kurtosis, the results for these four sub-periods show the characteristic of fat-tail since the kurtosis for those are larger than 3.

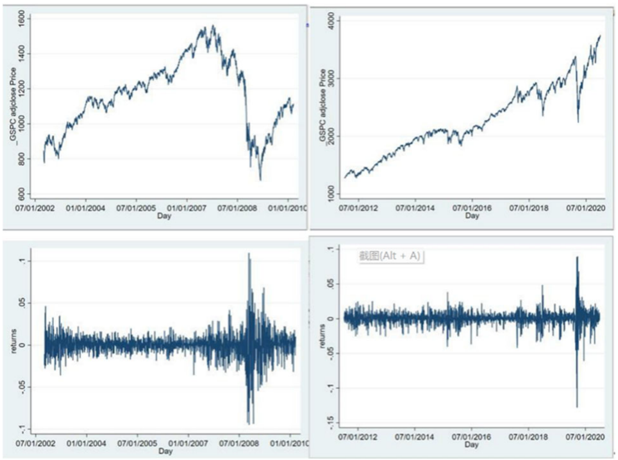

Figure 1. S&P 500 prices and returns (top left: prices during 2002-2010, top right: prices during 2012-2020.12, bottom left: returns during 2002-2010, bottom right: returns during 2012-2020.12)

In Figure 1, it is indicated that the S&P 500 prices changed smoothly without financial crises, during 2002-2007 and 2012-2020.01, while declined sharply during the periods with the financial crises. During 2007-2008, the S&P 500 prices fell by as much as 50% while during 2020.01-2020.04, the S&P 500 prices fell by as much as 30% (Yahoo Finance, 2022). The drastic changes of the S&P 500 prices resulted in the increase in volatility of S&P 500 returns during these two financial crises. According to the diagrams of returns, it is indicated that the trends of S&P 500 returns in two overall periods satisfy the volatility clustering that large changes are followed by large changes and small changes are followed by small changes.

4.2 ADF test and ARCH-LM tests

At first, this study utilizes the unit root test, augmented Dickey Fuller test (ADF test), to obtain the preliminary evidence of stationarity of series. Based on the zero p-values, we should reject the null hypothesis and select the alternative hypothesis, which means the series of returns is stationary.

Table 2. Unit Root Test (ADF test)

Unit Root Test (ADF test)		
Series Period	t-statistics	p-value
1st Oct 2002-1st Aug 2007	-38.234	0.0000
1st Oct 2002-1st Mar 2010	-48.998	0.0000
Unit Root Test (ADF test)		
Series Period	t-statistics	p-value
3rd Jan 2012-21st Jan 2020	-45.763	0.0000
3rd Jan 2012-31st Dec 2020	-56.722	0.0000

Table 3. ARCH-LM Test for the first overall period (2007-2008 financial crisis) and for the second overall period (COVID-19 financial crisis)

GARCH models								
	no financial crisis (2002-2007)				include financial crisis (2002-2010)			
Coefficients	ARCH	GARCH	EGARCH	GJR-GARCH	ARCH	GARCH	EGARCH	GJR-GARCH
α_0	0.0000614	1.33E-06	(-0.1726238)	1.05E-06	0.0001205	9.85E-07	(-0.0945911)	8.30E-07
α_1	0.1398447 (0.001)	0.0545302 (0.000)	0.0953885 (0.000)	0.1018578 (0.000)	0.4337005 (0.000)	0.0658489 (0.000)	0.0891226 (0.000)	0.0998076 (0.000)
β		0.9238276 (0.000)	0.9819763 (0.000)	0.9344266 (0.000)		0.9251784 (0.000)	0.9895939 (0.000)	0.9491758 (0.000)
γ			(-0.0913255) (0.000)	(-0.1028597) (0.000)			(-0.1069032) (0.000)	(-0.1160316) (0.000)

The Table 3 illustrates the ARCH-LM tests for two different overall periods, 2002-2010 and 2012-2020.12 respectively. In these two different overall periods, they have two sub-periods respectively, including or excluding the impacts of financial crisis. Based on the results of the ARCH-LM tests with using lags from 1 to 5, it is indicated that we reject the null hypothesis of no ARCH effects and there is clear evidence of the presence of the ARCH effects since the p-values for all lags are zero for these four sub-periods, implying that all statistics are statistically significant at 1%, 5%, and 10% confidence level. This result is consistent with the implication of the diagram of S&P 500 returns, which means that for these four sub-periods, the existence of ARCH effects or volatility clustering is proved by the ARCH-LM tests. Thus, for the existence of ARCH effects, this study will utilize various GARCH-type models for capturing the ARCH/GARCH effects to reveal the impacts of financial crises on the volatility estimation and forecasts.

4.3 GARCH-type models analysis and News impact curve analysis

This study utilizes the Stata to model, estimate and forecast the volatility of S&P 500 returns by the ARCH, GARCH, EGARCH, GJR-GARCH models. For simplicity, the lag order (1,1) is considered sufficient to cover the characteristic of volatility clustering (Brooks, 2019).

Table 4. GARCH-type models during the first overall period (2002-2010)

GARCH models								
	no impacts of COVID (2012-2020.01)				include the impact of COVID (2012-2020.12)			
Coefficients	ARCH	GARCH	EGARCH	GJR-GARCH	ARCH	GARCH	EGARCH	GJR-GARCH
α_0	0.0000468	4.77E-06	(-0.7008692)	3.97E-06	0.0000518	4.55E-06	(-0.6365595)	4.46E-06
α_1	0.2832461 (0.000)	0.1915708 (0.000)	0.1505102 (0.000)	0.2922935 (0.000)	0.5324662 (0.000)	0.2184474 (0.000)	0.3017765 (0.000)	0.3667005 (0.000)
β		0.7414525 (0.000)	0.9288173 (0.000)	0.807104 (0.000)		0.7373518 (0.000)	0.9332604 (0.000)	0.739282 (0.000)
γ			(-0.2453586) (0.000)	(-0.3237448) (0.000)			(-0.1773335) (0.000)	(-0.2923334) (0.000)

Table 5. GARCH-type models during the Second overall period (2012-2020)

	Exclude GFC of 2008 (2002-2007)					Include GFC of 2008 (2002-2010)				
No. of lags	1	2	3	4	5	1	2	3	4	5
chi-square	40.994	131.067	155.532	177.683	204.951	78.485	327.428	333.416	385.323	497.501
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
df	1	2	3	4	5	1	2	3	4	5
	Exclude COVID-19 financial crisis (2012-2020.01)					Include COVID-19 financial crisis (2012-2020.12)				
No. of lags	1	2	3	4	5	1	2	3	4	5
chi-square	125.017	165.535	197.014	214.564	214.650	571.791	850.105	851.982	852.391	886.553
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
df	1	2	3	4	5	1	2	3	4	5

Table 4 and 5 output the results of coefficients for different GARCH-type models in Stata for two overall periods. The four sub-periods are classified by whether including the impacts of financial crisis of 2007-2008 or COVID-19 financial crisis. The p-values for the coefficients are in the parentheses. Overall, all the p-values are zero, which means that all coefficients are statistically significant at 1%, 5%, and 10% confidence level. Among these models, if the sum of α_1 and β is less than 1, the model can be interpreted to be stationary since the value of (α_1+β) reflects the persistency of the whole financial series.

The leverage effect (asymmetric effect) can be examined by the EGARCH and GJR-GARCH models. In Table 4, the gammas (γ) for the EGARCH model are -0.091 during the first sub-period without including the impacts of GFC of 2008, and -0.107 during the second sub-period including the impacts of GFC of 2008. The negative value of the gamma in the EGARCH model means that the volatility of the S&P 500 returns is negatively correlated to the returns, and the volatility is affected by the leverage effect.

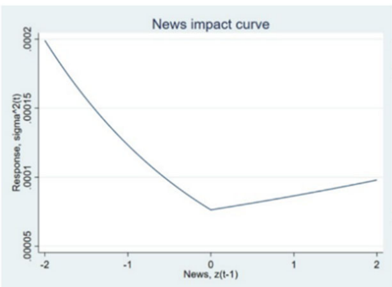

Figure 2. News impact curve during 2002-2010 (EGARCH)

According to Figure 2, it is obvious that the leverage effect measured by the EGARCH model is greatly significant. Compared with the same-level positive shocks, S&P 500 returns were reacting more sharply when being affected by the negative shocks.

In Table 4, as for the GJR-GARCH model, the gammas measured in Stata are negative (γ_{Stata}), -0.103 during the first sub-period without including the impacts of GFC of 2008, and -0.116 during the second sub-period including the impacts of GFC of 2008. The negative γ_{Stata} can be seen as the positive γ in the original GJR-GARCH model.

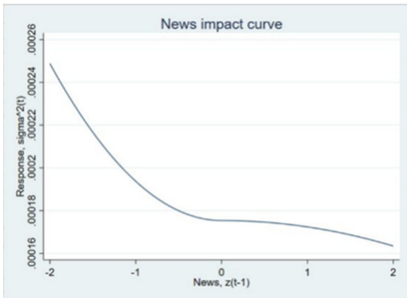

Figure 3. News impact curve during 2002-2010 (GJR-GARCH)

In Figure 3, the result is consistent with the explanation of particular gamma in Stata, and the news impact curve indicates the same result with the EGARCH model, indicating that the negative shocks arose a more intense response than the same-level positive shocks. This proves that the leverage effect exists in the volatility of S&P 500 returns during 2002-2010.

In Table 5, it is shown that the gammas (γ) for the EGARCH model are -0.245 during the first sub-period without including the impacts of COVID-19 financial crisis, and -0.177 during the second sub-period including the impacts of COVID-19 financial crisis. The negative values of gammas imply that there is the leverage effect on the volatility on S&P 500 returns during 2012-2020.12 measured by the EGARCH model.

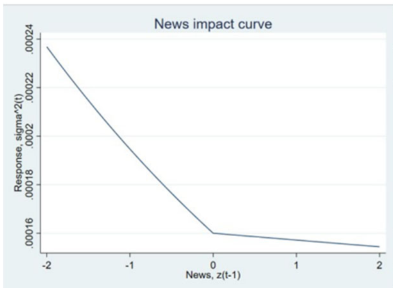

Figure 4. News impact curve during 2012-2020.12 (EGARCH)

In Figure 4, the news impact curve during 2012-2020.12 measured by the EGARCH model shows the same result of the negative values of gammas that there is the leverage effect, negative shocks may be more significant than positive shocks in affecting the volatility. Compared this curve with the news impact curve of the first overall period, the diagram shows that the leverage effect during 2002-2010 is more significant in both negative and positive shocks than that in the second overall period, 2012-2020.12, implying that the influence of news may be relatively diminished in the second overall period.

In Table 5, with respect to the GJR-GARCH model, the gammas measured in Stata are negative (γ_{Stata}), -0.324 during the first sub-period without including the impacts of COVID-19 financial crisis, and -0.292 during the second sub-period including the impacts of COVID-19 financial crisis.

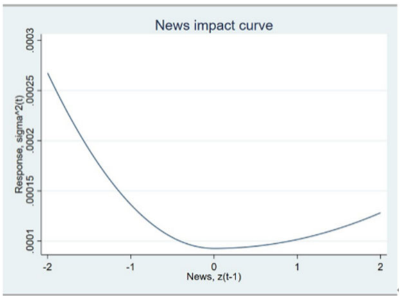

Figure 5. News impact curve during 2012-2020.12 (GJR-GARCH)

In Figure 5, the news impact curve during 2012-2020.12 measured by the GJR-GARCH model indicates the existence of the leverage effect. Compared with the curve during 2002-2010 measured by the same model, this curve shows that the influence of negative shocks is approximately the same, whereas the influence of positive shocks is beyond that in the former overall period.

Table 6. The differences and percentage differences of the coefficients

Difference (2002-2007 vs 2002-2010)					Percentage				
	α_0	α_1	β	γ		α_0	α_1	β	γ
ARCH	0.0000591	0.2938558			ARCH	96.25%	210.13%		
GARCH	-3.45E-07	0.0113187	0.0013508		GARCH	-25.94%	20.76%	0.15%	
EGARCH	0.0780327	-0.0062659	0.0076176	-0.0155777	EGARCH	45.20%	-6.57%	0.78%	-17.06%
GJR-GARCH	-2.20E-07	-0.0020502	0.0147492	-0.0131719	GJR-GARCH	-20.95%	-2.01%	1.58%	-12.81%
Difference (2012-2020.01 vs 2012-2020.12)					Percentage				
	α_0	α_1	β	γ		α_0	α_1	β	γ
ARCH	0.000005	0.2492201			ARCH	10.68%	87.99%		
GARCH	-2.20E-07	0.0268766	-0.0041007		GARCH	-4.61%	14.03%	-0.55%	
EGARCH	0.0643097	0.1512663	0.0044431	0.0680251	EGARCH	9.18%	100.50%	0.48%	27.72%
GJR-GARCH	4.90E-07	0.074407	-0.067822	0.0314114	GJR-GARCH	12.34%	25.46%	-8.40%	9.70%

Based on the coefficients in Table 4 and 5, in Table 6, the differences are calculated by subtracting the value of coefficients in the second sub-period (inclusion of financial crisis) by those in the first sub-period (exclusion of financial crisis), while the percentage differences are formulated by the value of differences over the value of coefficients in the first sub-period. The differences and percentage differences can intuitively show the impacts of the financial crises on the volatility estimation by analyzing the changes of each coefficient.

Firstly, we will analyze the differences and the implications behind them for the first overall period, 2002-2010 (affected by the GFC of 2008). The beta (β) is one of the coefficients, characterizing the correlation between σ_t^2 and σ_{t-1}^2. Table 6 indicates that when GARCH model has a 0.15% increase in β, the persistency in conditional variance increases by 0.15%, while the persistency in conditional variance increases by 0.78% and 1.58% in the EGARCH and GJR-GARCH models. The Alpha one (α_1) describes the rate of change of conditional variance, and Table 6 shows the rate of change of conditional variance increases by 210.13% and 20.76% in the ARCH and GARCH models, while decreases by 6.57% and 2.01% in the EGARCH and GJR-GARCH models. For the symmetric GARCH model, there is a 20.76% increase in volatility and a 0.15% increase in the persistency of volatility, implying that under the influence of the 2007-2008 financial crisis, volatility on S&P 500 returns has increased largely and persistently. The gamma (γ) measures the leverage effect, and there are 17.06% and 12.81% decline in the gamma for the EGARCH and GJR-GARCH models, which means that the financial crisis leads to the larger leverage effect, measured by the asymmetric GARCH models.

Secondly, the analysis on the differences and the implications for the second overall period, 2012-2020.12, affected by the COVID-19 financial crisis, is followed. As for the beta (β), there are 0.55% decline in the symmetric GARCH model, 0.48% increase in the EGARCH model, and 8.40% decline in the GJR-GARCH model respectively, which can be interpreted that the persistency of conditional variance increased by 0.48% in the EGARCH model while decreases by 0.55% and 8.40% in the GARCH and GJR-GARCH models. Compared with the former period, 2002-2010, in this period, the persistency of volatility tends to be weakened for these three models, especially for the GJR-GARCH model, implying that the COVID-19 financial crisis has less impacts on the persistent sharp changes in volatility. In other words, the COVID-19 financial crisis, not mainly caused by financial factors but an outbreak of epidemic disease, may have a relatively short-term impacts on the volatility of S&P 500 returns.

With respect to the Alpha one (α_1), the rate of change of conditional variance increases by 87.99%, 14.03%, 100.50% and 25.46% in the ARCH, GARCH, EGARCH and GJR-GARCH models respectively. Compared with the period of 2002-2010, in this period, the symmetric GARCH model and ARCH model show the less increase in volatility, while the asymmetric GARCH models show a significant increase in volatility, which implies that the volatility of S&P 500 returns may be more sensitive to the COVID-19 pandemic. For the symmetric GARCH model, there is a 14.03% increase in volatility and a 0.55% decrease in the persistency of volatility, implying that under the influence of the COVID-19 financial crisis, volatility of S&P 500 returns has increased largely but relatively short-term. When it comes to the gamma (γ), it witnesses a 27.72% increase in the EGARCH model and 9.70% increase in the GJR-GARCH model, which implies that there may be less asymmetric effects measured by the asymmetric GARCH models.

Overall, based on the comparison on the coefficients and the differences on them, it may be concluded that although the financial crisis caused by the COVID-19 pandemic can result in a huge volatility increase in a short period of time, its duration of this volatility increase is not as persistent as the duration for the 2007-2008 financial crisis.

4.4 Compare the performance of different GARCH-type models in volatility estimation

Table 7 is illustrated by the AIC and BIC for these four GARCH-type models during 2002-2007 (exclusion of financial crisis) and 2002-2010 (inclusion of financial crisis). The results in Table 7 indicate that the GJR-GARCH model performs better over the 4 models for both sub-periods because the GJR-GARCH model has the lowest AIC (-11945.02) and BIC (-11917.37), which means that the relative amount of information lost for the GJR-GARCH model may be less than the other models.

Table 7. AIC and BIC for the GARCH-type models in the first overall period (2007-2008 financial crisis)

2002-2007				
Model	No. of Obs	df	AIC	BIC
ARCH	1216	3	-8181.476	-8166.166
GARCH	1216	4	-8368.868	-8348.454
EGARCH	1216	5	-8387.631	-8362.115
GJR-GARCH	1216	5	-8391.653	-8366.137
2002-2010				
Model	No. of Obs	df	AIC	BIC
ARCH	1865	3	-10959.69	-10943.10
GARCH	1865	4	-11886.35	-11864.22
EGARCH	1865	5	-11936.14	-11908.48
GJR-GARCH	1865	5	-11945.02	-11917.37

Table 8 shows the AIC and BIC for these four GARCH-type models during 2012-2020.01 (exclusion of COVID-19 pandemic) and 2012-2020.12 (inclusion of COVID-19 pandemic). The results in Table 8 indicate that the EGARCH model performs better over the 4 models for both sub-periods because the EGARCH model has the lowest AIC (-155594.25) and BIC (-155565.63), which means that the relative amount of information lost for the EGARCH model may be less than the other models.

Table 8. AIC and BIC for the GARCH-type models in the second overall period (COVID-19 financial crisis)

2012-2020.01				
Model	No. of Obs	df	AIC	BIC
ARCH	2024	3	-13938.20	-13921.36
GARCH	2024	4	-14191.21	-14168.76
EGARCH	2024	5	-14353.21	-14325.14
GJR-GARCH	2024	5	-14301.47	-14273.40
2012-2020.12				
Model	No. of Obs	df	AIC	BIC
ARCH	2264	3	-14982.06	-14964.89
GARCH	2264	4	-15508.80	-15485.90
EGARCH	2264	5	-15594.25	-15565.63
GJR-GARCH	2264	5	-15578.97	-15550.35

For these two financial crises, asymmetric GARCH models have better performance than the symmetric ARCH/GARCH model, which proves the influence of the leverage effect on the volatility of S&P 500 returns and emphasizes the importance of consideration of the leverage effect in estimating volatility.

4.5 Compare the forecasting ability of volatility among different GARCH-type models

It is also important to compare these 4 GARCH-type models in measuring volatility forecasting ability. This study uses the in-sample and out-of-sample analysis in evaluating the forecasting ability of the models. For the first overall period (2002-2010), the base sample period is between 1st Oct 2002 and 1st Aug 2007, while the forecasted sample period is between 2nd Aug 2007 and 1st Mar 2010. For the second overall period (2012-2020.12), the base sample period is between 3rd Jan 2012 and 21st Jan 2020, while the forecasted sample period is between 22nd Jan 2020 and 31st Dec 2020.

Table 9. Quasi-Likelihood loss function for these four GARCH-type models during the period of 2002-2010 and the period of 2012-2020.12

2002-2010					
Variable	No. of Obs	Mean	Std. dev.	Min	Max
QLvar_ARCH	649	1227.795	889.9918	0.5561256	2411.254
QLvar_GARCH	649	514.3458	284.4462	0.911767	988.5204
QLvar_EGARCH	649	503.063	283.2007	1.132216	974.8785
QLvar_GJR-GARCH	649	486.2838	269.4788	1.222178	943.2978
2012-2020.12					
Variable	No. of Obs	Mean	Std. dev.	Min	Max
QLvar_ARCH	240	446.3587	207.617	5.396011	695.9708
QLvar_GARCH	240	177.9816	96.34957	4.973493	337.4416
QLvar_EGARCH	240	292.859	199.7066	4.579447	585.8624
QLvar_GJR-GARCH	240	255.7145	171.3653	4.688829	502.7425

The intuition behind QLIKE loss function refers to finding the model minimizing the difference between the forecast and the real data, and the result of the QLIKE loss function which shows the lowest maximum can be considered as that minimization. According to the results generated from Stata in Table 9, it is indicated that the Quasi-likelihood variance statistics are 2411.254 for ARCH model, 988.5204 for GARCH model, 974.8785 for EGARCH model, and 943.2978 for GJR-GARCH model respectively during the period of 2002-2010. With the comparison of the maximum values of these 4 models, the GJR-GARCH model achieves to minimize the loss function Quasi-likelihood variance. In conclusion, the GJR-GARCH model is considered best-performed in volatility forecasting ability among these 4 GARCH-type models during the first overall period.

During 2012-2020.12, Table 9 shows that the Quasi-likelihood variance statistics are 695.9708 for ARCH model, 337.4416 for GARCH model, 585.8624 for EGARCH model, and 502.7425 for GJR-GARCH model respectively. In terms of the maximum values of these 4 models, the symmetric GARCH model minimizes the loss function Quasi-likelihood variance, which means that the symmetric GARCH model performs best in volatility forecasting ability among these 4 GARCH-type models during the second overall period.

In comparison with these two overall periods, 2002-2010 and 2012-2020.12, it is found that the GJR-GARCH model can perform better in volatility forecasting when suffering from the financial crisis caused by the financial factors, while symmetric GARCH model may have better volatility forecasting ability when encountering the financial crisis caused by the non-financial factors like the COVID-19 pandemic.

5. Conclusion

This study examines various GARCH-type models (the ARCH, GARCH, EGARCH and GJR-GARCH models) to analyze the performance of volatility changes on the U.S. stock market by the S&P 500 index during two overall periods, 2002-2010 and 2012-2020.12. Through the horizon and vertical comparison analysis of each coefficient of the GARCH-type models, there are several empirical results.

Firstly, by observing the news impact curves, it is found that during these two financial crises, the S&P 500 returns were both affected by the leverage effects. Secondly, with respect to the persistency of conditional variance, the volatility during the GFC of 2008 is relatively more persistent compared with the COVID-19 financial crisis, while with regards to the rate of change of conditional variance, among these 4 GARCH-type models, the COVID-19 financial crisis results in the relatively higher volatility than the GFC of 2008. This result means that the financial crisis mainly caused by non-financial factors often leads to the sharp fluctuations of the index, but the fluctuations are short-term compared with the financial crisis caused mainly by financial factors. Thirdly, the asymmetric effects in volatility have been analyzed theoretically and elaborated in the data analysis, figuring out the result that the GFC of 2008 leads to the larger leverage effects, while there may be less leverage effects measured by the asymmetric GARCH models during the period of financial crisis caused by the COVID-19 pandemic. This implies that the financial crisis caused mainly by the non-financial factors, such as COVID-19 pandemic, may have relatively less impacts of negative news in comparison with that mainly by the financial factors. The reasons behind this implication may be that the negative impacts on the financial market caused by the non-financial factors are indirect and require a transmission process. Therefore, the negative impacts may be slightly smaller than those of the financial crisis caused by the direct financial factors.

This study also utilizes the AIC and BIC to assess the performance of different GARCH-type models in volatility estimation and finds out that the GJR-GARCH model performs better in the period of the 2007-2008 financial crisis and EGARCH model has the better performance in the period of COVID-19 financial crisis. Overall, due to the existence of the leverage effects, the asymmetric GARCH-type models seem to have the better performance than the symmetric models in volatility estimation, no matter for any types of financial crises.

The Quasi-Likelihood loss function was used to compare the forecasting ability of volatility among different GARCH-type models, and the result is that GJR-GARCH model can perform better in volatility forecasting when suffering from the financial crisis caused by the financial factors, while symmetric GARCH model may have better volatility forecasting ability when encountering the financial crisis caused by the non-financial factors like the COVID-19 pandemic. This may be because during the COVID-19 financial crisis, the leverage effect tends to be less than without this financial crisis, and thus, this indirect financial crisis may be more appropriate for the symmetric GARCH model to forecast volatility.

With the in-depth researches related to the volatility (conditional variance), a huge number of GARCH-type and non-GARCH-type models are developed and improved in estimating the changes of volatility. More sophisticated models introduce more influencing factors, which can cover more of the real financial market variables to the greatest extent. This study utilizes the several GARCH-type models to research and analyze the financial crises of different forming reasons and provides the efficient models for reference for the study of the volatility of the future potential financial crisis.

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