

# Countermeasures of Financial Internal Supervision Institutions of Administrative Institutions Based on the Double Game

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**Abstract.** The state audit in the new era focuses on strengthening the restriction and supervision of public power, and it will inevitably require strengthening the audit and supervision of administrative institutions, which leads to the discussion of the efficiency of the internal supervision institutions in administrative institutions. In order to effectively control the finance internal supervision risk, a moral hazard monitoring model with a two-tier game mechanism is proposed. In the model, it establishes an upper-level game and a lower-level game analysis framework for the "lazy" and "Free-riding" behaviors in moral hazard. Combining the equilibrium conditions to explain the game results, and put forward the overall risk monitoring countermeasures for the two behaviors.

## 1 Introduction

As an important part of the supervision system, the national audit in the new era must strengthen the restriction and supervision of public power, that is, focus on realizing the full audit coverage of administrative institutions (hereinafter referred to as "units"). In this context, it is more and more important to improve the unit's internal control mechanism and implement the unit's financial internal supervision and management. Therefore, how to ensure the efficient operation of the financial internal supervision organization of the unit is a problem worthy of in-depth discussion [1-3]. Under the condition of asymmetric information, the internal financial supervision organization of the unit may not act in full accordance with the wishes of external audit (National Audit), and there is the possibility of "moral hazard", which is manifested in the specific behavior of "laziness [4]" or "free ride [5]". Therefore, strengthening the guidance of external audit supervision and effectively reducing the probability of moral hazard [6] is an effective measure to improve the operation efficiency of the internal financial supervision organization of the unit.

Based on game theory and principal-agent theory, this paper establishes a moral hazard monitoring model with two-tier game mechanism [7-10]. Under the constraints of external audit, this paper discusses the countermeasures to establish the financial internal supervision mechanism of administrative institutions and scientifically avoid the moral hazard of internal supervision.

## 2. Upper level game of "laziness" between external audit department and internal supervision organization

### 2.1 Description of basic assumptions

(1) Establish the basic framework of the upper game, and define the two sides of the Bureau as the external audit department and the internal supervision organization respectively. The game is a repeated game under asymmetric information. The national audit department is the principal and the internal supervision organization is the agent.

(2) The audit department performs an audit task on a unit. The internal supervision organization of the unit carries out daily financial supervision with effort level  $u$ .  $u \in U$ ,  $U$  represents all optional action combinations of the supervision organization. The value range of  $u$  is  $0 < u < 1$ ,  $\phi$  is the exogenous random variable affecting daily financial supervision, and the value range of  $\phi$  is  $\Phi$ .

(3) The audit department judges the performance of the supervision organization by observing the audit results.  $x(u, \phi)$ . The financial supervision of the supervision organization improves the efficiency of the use of financial funds, and the total value created is  $v(u, \phi)$ . It is assumed that  $v(u, \phi)$  is a strictly increasing concave function of  $u$ .

(4)  $v$  is determined according to the observed performance  $x(u, \phi)$ . Set  $v = v(x) = \mu x$ ,  $\mu$  as the performance benefit conversion coefficient. For ease of

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explanation, it is assumed to be 1. In the process of supervision, the audit department rewards and punishes the internal supervision institutions according to the observed value  $x(u, \phi)$ . Generally, effort level  $u$  and performance  $x$  should be in direct proportion, that is, the harder the internal oversight body works, the easier it is to be observed to perform its duties. Here, it is assumed that:

$$x(u, \phi) = u + \phi_x \quad (1)$$

Where,  $\phi_x \sim N(0, \sigma_x^2)$  follows the normal distribution, and  $\phi_x$  represents the influence of exogenous variables on  $x$ .

(5) The audit department cannot directly observe the effort level  $u$  of the supervision organization, but can only introduce the audit supervision signal  $Q$  to supervise the effort level  $u$  of the financial internal supervision organization of the unit. Obviously, there is an obvious correlation between audit supervision signal  $Q$  and effort level  $u$ . Note that  $Q$  is a function of  $u$ :

$$Q(u, \phi_Q) = \eta u + \phi_Q \quad (2)$$

Where,  $\phi_Q \sim N(0, \sigma_Q^2)$  follows the normal distribution,  $\phi_Q$  represents the influence of exogenous variables on  $Q$ , and  $Var(\phi_Q) = \sigma_Q^2$  reflects the accuracy of audit supervision signals. When  $\phi_x$  and  $\phi_Q$  are independent of each other, there is covariance  $Cov(\phi_x, \phi_Q) = 0$ . In order to be close to reality, let the expectation of  $\eta \sim N(1, \sigma_\eta^2)$ , then  $\eta$  is  $E\{\eta\} = 1$ ,  $E\{x(u, \phi_x)\} = u = E\{Q(u, \phi_Q)\}$ , so  $Q$  is an unbiased estimate of  $x$ .

(6) The audit department supervises the "lazy" behavior of the supervision organization, and sets the payment function given to the supervision organization as

$$Y(x, u, Q) = D + nx + \alpha Q \quad (3)$$

In the formula,  $D$  is the fixed expenditure cost of the audit department for the audit supervision of the supervision organization, and  $n$  is the sharing and transformation coefficient of the total value  $x$  obtained by the supervision organization after performing its duties. It can be understood as the savings of supervision resources caused by the internal supervision organization playing the role of financial supervision and forming an efficient and standardized virtuous circle system of financial management, which is obviously  $0 \leq n \leq 1$ .  $\alpha$  reflects the remuneration paid to the supervision organization according to the supervision signal.

(7) In the game process, it is assumed that the audit department is risk neutral and the internal supervision organization is risk averse. The risk cost of the person in the bureau is

$$C = \rho w \quad (4)$$

Where,  $w$  is the actual acquired value of the players, and at this time  $w = \frac{1}{2}(n^2\sigma_x^2 + \alpha^2\sigma_Q^2)$ ,  $\rho > 0$  is the risk aversion of the players.

## 2.2 Establishment and analysis of "lazy" behavior supervision model

According to the marginal cost theory and principal-agent theory, the agent's effort cost is positively correlated with the degree of effort, and the greater the degree of effort, the greater the increase of the required cost. Therefore, it is assumed that the financial supervision cost of the internal supervision organization is  $f = bu^2$ , where  $b > 0$  is the cost conversion coefficient. The larger  $b$  is, the higher the cost of the same effort  $u$  is. Accordingly, the payment function given by the audit department to the internal supervision organization is

$$\begin{aligned} Y(x, u, Q) &= D + nx + \alpha Q \\ &= D + n(u + \phi_x) + \alpha(\eta u + \phi_Q) \end{aligned} \quad (5)$$

From equation (5), the deterministic income of the internal supervision organization is

$$\begin{aligned} K_n(Y, u) &= E\{Y(x, u, Q)\} - f(u) - C \\ &= D + nu + \alpha\eta u - bu^2 - \frac{1}{2}\rho(n^2\sigma_x^2 + \alpha^2\sigma_Q^2) \end{aligned} \quad (6)$$

According to the game theory, the Pareto optimal condition is that the marginal cost is equal to the marginal income, that is, equations (1) and  $f = bu^2$  obtain the partial derivative of effort level  $u$  respectively

$$u^p = \frac{\partial x}{\partial u} = \frac{\partial f}{\partial u} = \frac{1}{2b} \quad (7)$$

At this time, the deterministic income of the audit department is

$$K_s(D, n, \alpha) = E\{x - Y\} \quad (8)$$

Obviously, according to hypothesis  $b > 0$ ,  $\Rightarrow \partial x / \partial u > 0$ ,  $\partial f / \partial u > 0$ , it shows that the audit department wants the internal supervision organization to work harder and not "lazy", while the internal supervision organization wants the audit department to give sufficient incentives, otherwise it will lack the motivation to work hard and choose "lazy" behavior. In order to achieve Pareto optimality, a "lazy" behavior supervision model is established, and the audit department supervises the performance of the supervision organization, in the form of:

$$\begin{cases} \max K_s(D, n, \alpha) = E\{x - Y\} = E\{x - (D + nu + \alpha n u)\} \\ \text{IR } K_n(Y, u) = E\{x, u, Q\} - f(u) - C \\ \quad = E\{D + nu + \alpha n u - bu^2 - \frac{1}{2} \rho(n^2 \sigma_x^2 + \alpha^2 \sigma_0^2)\} \geq K_0 \\ \text{IC } u \in \arg \max \left\{ D + nu + \alpha n u - bu^2 - \frac{1}{2} \rho(n^2 \sigma_x^2 + \alpha^2 \sigma_0^2) \right\} \\ K_s(D, n, \alpha) - K_s(D, n) \geq \alpha n u \end{cases} \quad (9)$$

In equation (9),  $K_0$  is the retention benefit of the internal supervision organization, and  $K_s(D, n)$  is the deterministic benefit when the audit department adopts the "no audit" supervision strategy. The constraints satisfied are briefly described as follows:

① Satisfy agent rational constraint (IR). Only when condition  $K_n(Y, u) \geq K_0$  is met can the internal supervision organization have the motivation to strive for financial supervision activities.

② Satisfy the incentive compatibility constraint (IC). Only when the effort level selected by the internal supervision organization is within the scope of feasibility set, the internal supervision organization will perform the financial supervision function according to the wishes of the audit department.

③ Audit supervision signals introduce constraints. Only when the audit department can ensure its deterministic income growth after introducing the audit supervision signal can it implement the signal supervision.

According to the partial derivative of  $u$  obtained by the constraint condition IC in equation (9), it can be seen that the maximum effort of the internal supervision organization to perform its duties is

$$u^* = \frac{n + \alpha \eta}{2b} \quad (10)$$

At this time, compare the Pareto optimal effort with the maximum effort of the internal supervision organization to perform the supervision responsibilities. If  $n + \alpha \eta < 1$ , it indicates that the internal supervision organization has insufficient efforts and "lazy" behavior. The audit department needs to strengthen audit supervision and promote the internal supervision organization to work hard. If  $n + \alpha \eta = 1$ , the Pareto optimal condition is satisfied, and there is no "lazy" behavior in the internal supervision organization.

When the constraint conditions of the audit supervision signal are satisfied, according to the Kuhn Tucker condition, make the agent rationality constraint IR tight, and substitute equation (10) into the objective function of equation (9):

$$\begin{cases} \max K_s(D, n, \alpha) \\ = E \left\{ \frac{n + \alpha \eta}{2b} + \phi_x - K_0 - b \frac{(n + \alpha \eta)^2}{4b^2} - \frac{1}{2} \rho(n^2 \sigma_x^2 + \alpha^2 \sigma_0^2) \right\} \end{cases} \quad (11)$$

Calculate the first-order partial derivative of  $\alpha$  of equation (11), there is

$$1 = n + \alpha^* [E(\eta^2) + 2b\rho\sigma_0^2] \quad (12)$$

According to the variance formula

$$\begin{aligned} Var(\eta) &= E(\eta^2) - E^2(\eta), \\ \Rightarrow E(\eta^2) &= 1 + \sigma_\eta^2, \end{aligned}$$

The best  $\alpha^*$  is

$$\alpha^* = \frac{1 - n}{1 + \sigma_\eta^2 + 2b\rho\sigma_0^2} \quad (13)$$

Calculate the first-order partial derivative of  $n$  in equation (11), that is, the optimal  $n^*$  is

$$n^* = \frac{1 - \alpha}{1 + 2b\rho\sigma_x^2} \quad (14)$$

Substituting equation (14) into equation (13), there is

$$\alpha^* = \frac{2b\rho\sigma_x^2}{\sigma_\eta^2 + 2b\rho\sigma_0^2 + 2b\rho\sigma_x^2 + 2b\rho\sigma_x^2\sigma_\eta^2 + 4b^2\rho^2\sigma_x^2\sigma_0^2} \quad (15)$$

Substituting equation (15) into equation (14), there is

$$n^* = \frac{\sigma_\eta^2 + 2b\rho\sigma_0^2 + 2b\rho\sigma_x^2\sigma_\eta^2 + 4b^2\rho^2\sigma_x^2\sigma_0^2}{(\sigma_\eta^2 + 2b\rho\sigma_0^2 + 2b\rho\sigma_x^2 + 2b\rho\sigma_x^2\sigma_\eta^2 + 4b^2\rho^2\sigma_x^2\sigma_0^2)(1 + 2b\rho\sigma_x^2)} \quad (16)$$

Substituting equation (13) into the IR condition of equation (9), it is found that the optimal fixed expenditure cost of the audit department is

$$D = K_0 + \frac{1}{2} \rho(n^2 \sigma_x^2 + \alpha^2 \sigma_0^2) - \frac{[n^2 + 2n\alpha + \alpha^2(1 + \sigma_\eta^2)]}{4b} \quad (17)$$

### 2.3 game result analysis of "laziness" behavior supervision

Analyze the results of the "lazy" behavior supervision game in equations (13) to (17), and the basic conclusions are as follows:

First,  $\alpha^*$  and  $n$  are negatively correlated. If  $n$  approaches 0, then  $\alpha^*$  approaches 1. On the contrary, if  $n$  approaches 1, then  $\alpha^*$  approaches 0. The two have an alternative relationship. This shows that the higher the accuracy of the audit supervision signal, the more the audit department can truly detect the effort level of the internal supervision organization. Therefore, the more sufficient the basis for the audit department to accurately pay the internal supervision organization (obtain effective audit supervision opinions) according to the audit supervision signal.

Second,  $\alpha^*$  is inversely proportional to  $\sigma_\eta^2$ . specifically, with the improvement of the accuracy of audit supervision signal, the variance  $\sigma_\eta^2$  decreases and  $\alpha^*$  increases; On the contrary, if the variance  $\sigma_\eta^2$  increases,  $\alpha^*$  decreases. When  $\sigma_\eta^2 \rightarrow 0$ , then  $\alpha^* \rightarrow 1 - m$ ; When  $\sigma_\eta^2 \rightarrow \infty$ , then  $\alpha^* \rightarrow 0$ . This shows that in the process of long-term game, the more stable the state of audit supervision signal is, the higher the reliability of the conclusion of detecting whether there is "laziness" behavior of internal supervision institutions, and the audit department will rely more on audit supervision signal for payment, that is, increase the payment weight.

Third,  $\alpha^*$  is inversely proportional to  $\sigma_Q^2$ . When the external uncertain factors become larger,  $\sigma_Q^2$  increases and  $\alpha^*$  decreases. At this time, the monitoring of the effort level of the internal supervision organization by the supervision signal is inaccurate, and the payment to the internal supervision organization according to the supervision signal should be reduced. Otherwise, the payment will be made according to the supervision signal.

Fourth,  $\alpha^*$  is inversely proportional to  $b$ , and  $D$  is negatively correlated with  $b$ , which indicates that when the financial supervision cost of the internal supervision organization is too high, the "lazy" behavior will probably occur. At this time, the signal fails, and the audit department is no longer willing to pay according to the audit supervision signal. If we want to ensure that the internal supervision institutions will not be "lazy", we can only improve the fixed payment cost of the audit department under the condition of constant cost conversion efficiency, that is, increase the frequency and intensity of audit supervision.

### 3 lower level game of "free ride" behavior between external audit department and internal supervisors

#### 3.1 description of supplementary assumptions

(1) Establish the basic framework of the lower level game, and define the two sides of the Bureau as the national audit department and the internal supervisors of the unit respectively. The audit department is the principal and the internal supervisors are the common agents.

(2) The internal supervision organization of an administrative institution has a staff of  $m$  to jointly perform financial supervision. The internal supervisors are risk averse, choose the level of effort  $u_i \in [0, \infty)$  and are independent of each other, with  $i = 1, 2, \dots, m$ . The audit department cannot directly observe the effort level of each supervisor, nor can it calculate the work efficiency of each supervisor separately.

(3) The financial supervision cost of each internal supervisor is  $f(u_i) = \frac{1}{2} b_i u_i^2$ , where  $b_i > 0$  is the cost conversion coefficient.

(4) When the internal supervisors perform financial supervision, the common output benefit function is  $v(u, \varepsilon)$ ,  $u = (u_1, u_2, \dots, u_m)$ ,  $v(u, \varepsilon) = \lambda \sum u_i + \varepsilon$ ,  $\lambda > 0$  is the benefit conversion coefficient, and  $u = 0$ ,  $v(0, \varepsilon) = 0$ ;  $\varepsilon$  is an exogenous random variable,  $\varepsilon \sim N(0, \sigma^2)$  follows a normal distribution, the cumulative distribution function of the benefit function is  $G(v, u)$ , and the density function is  $g(v, u)$ .

Suppose the first partial derivative  $\frac{\partial G(v, u)}{\partial u} \leq 0$ , the second partial derivative  $\frac{\partial^2 G(v, u)}{\partial u^2} \geq 0$ .

(5) Select a post "model" from the internal supervisors, which is represented by subscript  $j$ ; The audit department shall set up supervision and evaluation standards  $\bar{z}$  (such as fund supervision, asset supervision, fund supervision, account management supervision, supervision system construction, etc.);  $\bar{v}$  is the output benefit function corresponding to the evaluation standard  $\bar{z}$ .

(6) If the internal supervisors fail to meet the standard  $\bar{z}$  set by the audit department, they will be punished by  $h_i > 0$ . If they exceed the standard  $\bar{z}$ , they will be rewarded  $K_i(v)$ .

#### 3.2 establishment and analysis of "free ride" behavior supervision model

In order to effectively control the "free ride" behavior of internal supervisors, the audit department shall establish the following supervision mechanism:

If  $i \neq j$ ,

$$K_i(v) = \begin{cases} D_i + \alpha_i v(u, \varepsilon) - h_i, & \text{if } v(u, \varepsilon) < \bar{v} \\ D_i + \alpha_i v(u, \varepsilon), & \text{if } v(u, \varepsilon) \geq \bar{v} \end{cases} \quad (18)$$

If  $i = j$ ,

$$K_i(v) = \begin{cases} D_i + \alpha_i v(u, \varepsilon) - h_i, & \text{if } v(u, \varepsilon) < \bar{v} \\ D_i + \alpha_i v(u, \varepsilon), & \text{if } v(u, \varepsilon) = \bar{v} \\ D_i + \alpha_i v(u, \varepsilon) + m_0, & \text{if } v(u, \varepsilon) > \bar{v} \end{cases} \quad (19)$$

$$s.t. \quad m_0 = v(u, \varepsilon) - \bar{v} \quad (20)$$

$$\sum_{i=1}^m \alpha_i = 1 \quad (21)$$

Where,  $D_i$  is the fixed payment given by the audit department to the internal supervisor  $i$ ;  $\alpha_i$  is the benefit sharing rate brought by personnel  $i$  and other internal supervisors to jointly perform financial supervision;  $m_0$  and  $h_i$  are quantifiable rewards and punishments respectively, and  $m_0 > 0$ ,  $h_i > 0$ .

According to the assumptions, the common expected return of  $m$  internal supervisors is

$$E\{K(v)\} = E\{v(u, \varepsilon)\} - \sum_{i=1}^m f(u_i) \quad (22)$$

According to Pareto optimal conditions:

$$\frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} - b_i u_i = 0 \quad (23)$$

The Pareto optimal effort level is

$$u_i^p = \frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} \frac{1}{b_i} = \frac{\lambda}{b_i} \quad (24)$$

At the same time, the deterministic income of internal supervisors  $i$  is

$$K_i(v) = \begin{cases} D_i + \alpha_i E\{v(u, \varepsilon)\} - h_i G(\bar{v}, u) - \frac{1}{2} b_i u_i^2 - \frac{1}{2} \rho_i \alpha_i^2 \sigma^2, & \text{if } i \neq j \\ D_i + \alpha_i E\{v(u, \varepsilon)\} - h_i G(\bar{v}, u) - m_0 [1 - G(\bar{v}, u)] - \frac{1}{2} b_i u_i^2 - \frac{1}{2} \rho_i \alpha_i^2 \sigma^2, & \text{if } i = j \end{cases} \quad (25)$$

In equation (23) ~ equation (25),  $E$  is the symbol of expected value.

Let  $\frac{\partial K_i(v)}{\partial u_i} = 0$  the necessary and sufficient

condition of Nash equilibrium is:

If  $i \neq j$ ,

$$\alpha_i \frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} - h_i \frac{\partial G(\bar{v}, u)}{\partial u_i} - b_i u_i = 0 \quad (26)$$

If  $i = j$ ,

$$\alpha_i \frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} - (h_i + m_0) \frac{\partial G(\bar{v}, u)}{\partial u_i} - b_i u_i = 0 \quad (27)$$

Remember that the optimal effort level after partial derivation is  $u_i^*$ , then

$$u_i^* = \frac{1}{b_i} \left[ \alpha_i \frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} - h_i \frac{\partial G(\bar{v}, u)}{\partial u_i} \right] \quad (28)$$

$$= \frac{1}{b_i} \left[ \alpha_i \lambda - h_i \frac{\partial G(\bar{v}, u)}{\partial u_i} \right], \quad \text{if } i \neq j$$

$$u_i^* = \frac{1}{b_i} \left[ \alpha_i \frac{\partial E\{v(u, \varepsilon)\}}{\partial u_i} - (h_i + m_0) \frac{\partial G(\bar{v}, u)}{\partial u_i} \right] \quad (29)$$

$$= \frac{1}{b_i} \left[ \alpha_i \lambda - (h_i + m_0) \frac{\partial G(\bar{v}, u)}{\partial u_i} \right], \quad \text{if } i = j$$

### 3.3 game result analysis of "free ride" behavior supervision

Combining equations (24), (28) and (29) to analyze the game results, whether the internal supervisors choose the "free ride" behavior depends on the size of  $u_i^*$  and  $u_i^p$ .

When  $u_i^* < u_i^p$ , it indicates that the efforts of internal supervisors are not enough, there is a "free ride" behavior risk, and measures and mechanisms need to be taken for audit supervision. On the contrary, it shows that the supervisors have no motivation to adopt the "free ride" behavior.

Due to  $0 \leq \alpha_i \leq 1$ ,  $m_0 > 0$ ,  $\frac{\partial G(\bar{v}, u)}{\partial u_i} \leq 0$ , the

audit department can ensure that the  $u_i^* = u_i^p$  condition is met by reasonably setting the evaluation standard  $\bar{v}$ , punishment  $h_i$  and reward  $m_0$ , that is, it is completely

feasible to realize the Pareto optimal effort level of internal supervisors. Specifically, when  $i \neq j$ ,  $u_i^* = u_i^p$

can be realized if  $\alpha_i \lambda - h_i \frac{\partial G(\bar{v}, u)}{\partial u_i} = \lambda$ ,

$\frac{\partial G(\bar{v}, u)}{\partial u_i} = -\frac{\lambda(1-\alpha_i)}{h_i}$  is satisfied. When  $i = j$ ,  $u_i^* = u_i^p$

can be realized if  $\alpha_i \lambda - (h_i + m_0) \frac{\partial G(\bar{v}, u)}{\partial u_i} = \lambda$ ,

$\frac{\partial G(\bar{v}, u)}{\partial u_i} = -\frac{\lambda(1-\alpha_i)}{(h_i + m_0)}$  is satisfied. At this time, it means

to effectively avoid the "free ride" behavior of internal supervisors.

## 4.conclusions and Countermeasures

The above analysis proves that it is feasible in theory to improve the financial internal supervision of administrative institutions and improve the probability of avoiding "laziness" and "free ride". On this basis, the countermeasures are put forward as follows:

(1) Establish a standardized working paradigm of financial internal supervision of units. The conclusion of the game is that by optimizing the content of the supervision signal and reducing the signal error, the stability and credibility of the supervision signal can be improved, so as to ensure that the audit department can rely on the signal to correctly judge the behavior of the internal supervision institutions and personnel, accurately identify the two behaviors in the process of financial internal supervision, and effectively reduce the risk of misjudgment. Therefore, through the form of laws and regulations, we should clarify the functions of the financial internal supervision institutions of administrative institutions, clarify the responsibilities of supervisors, and build a set of observable, quantifiable and standardized explicit financial supervision and evaluation indicators as the supervision signal.

(2) Actively explore technical means and scientific methods to improve the efficiency of financial internal supervision. The upper game results show that when the financial supervision cost of the internal supervision organization is too high, if we want to ensure that the internal supervision organization will not be "lazy", under the condition of constant cost conversion efficiency, we can only improve the fixed payment cost  $D$  of the audit department, that is, increase the frequency and intensity of audit supervision. Obviously, this will further increase the burden of audit work, so reducing the cost conversion coefficient  $b$  of financial supervision efforts has become the preferred way to improve work efficiency. Therefore, the audit department should provide necessary business guidance for the internal supervision institutions, actively carry out the exchange of audit business skills, explore technologies and methods to improve the efficiency of internal supervision, improve the business skills of the financial internal supervision institutions and personnel,

and promote the improvement of cost conversion efficiency coefficient.

(3) Establish a rating system and "black and white list" system for financial internal supervision and assessment of the unit. From the perspective of the upper game, the supervision quality assessment is the final recognition of the audit department's work evaluation of the internal supervision organization, and it is also the basis for the audit department to make actual payment based on the supervision signal. From the perspective of lower level game, in order to reduce the risk of "free riding", it is necessary to determine the penalty  $h_i$  and reward  $m_0$  under the Pareto optimal effort level of internal supervisors. Therefore, it is necessary to establish standardized and complete quality assessment business procedures and establish five grade evaluation rating systems of a, B, C, D and E. A is the best and E is the worst. For the supervision institutions rated as grade A, they shall be commended and incorporated into the "white list" of inspection exemption for the next year. For the supervision institutions rated as grade D and E, they shall be criticized, tracked their later rectification and included in the "blacklist" of inspection required for the next year. The supervision institutions listed in files B and C shall conduct sampling inspection as appropriate.

(4) Establish a flexible and adjustable reward and punishment system for internal financial supervision of the unit. This is because in the game between the upper and lower levels, the confirmation of the payment function of the audit department is based on the conclusion that the reward and punishment effect is credible. When the effect of rewards and punishments is unbelievable or the confidence level changes, the intensity and mode of rewards and punishments shall be adjusted. Generally, rational economic people are more sensitive to positive incentives, and negative incentives will bring more resistance. At this time, it is unbelievable for the audit department to increase the punishment measures. Therefore, it is suggested to focus on the way of "high reward and low punishment", which is easier to stimulate the work enthusiasm of internal supervisors.

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## References

1. Wang Meiying, Zeng Changli, Liu Fang. Research on national audit, internal governance and risk-taking of state-owned enterprises [J]. *Audit research*, 2019 (5): 15-22.
2. Wang Meiying, Guo Honglian Supervision strategy of government audit outsourcing under incomplete

- information game [J]. *China's circulation economy*, 2018 (10): 93-101.
3. Xiang Renxiang, Chen Heping, Wu Haiying. Strengthening scientific management of military finance [J]. *Military finance*, 2013 (11): 48-49.
4. Yao Ping, Jiang Dali, Li de. research on the supervision mechanism of "laziness" behavior of local logistics suppliers [J]. *Journal of college of logistics engineering*, 2012, 28 (1): 71-75.
5. Huang Yuancui. "Free riding" behavior: analysis of motivation, influencing factors and avoidance measures [J]. *Management observation*, 2019 (21): 35-36.
6. Zhao Xi, Zhang Shipeng. National audit risk analysis based on game theory [J]. *Audit research*, 2015 (6): 31-36.
7. Wang Bo, Xiao Gang, Xiao Xiao. Research on civil aviation industry chain based on two-tier game model [J]. *Operations research and management*, 2019, 28 (3): 7-12.
8. Tan Zhongfu, Tan Caixia, Pu Lei, Yang Jiacheng. Double level game model of virtual power plant based on Collaborative immune quantum particle swarm optimization algorithm [J]. *Power construction*, 2020, 41 (6): 9-17.
9. Hu Bangan, Liu youbo, Xu Lixiong, et al. Solution of reliability value-added service based on two-level game model considering distribution system reconfiguration strategy[J]. *Proceedings of the CSEE*, 2022, 31: 1-19.
10. Kong Qingfeng, Li Ao. The US Trade War with China from the Perspective of Double Layer Game Theory[J]. *Issues of Contemporary World Socialism*, 2020 (3) : 147-157.