The Utilization of The Case Teaching Method in The Blended Instruction of Probability Theory and Mathematical Statistics

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Abstract: Probability theory and mathematical statistics is a subject with strong application background and application prospect, and it is an indispensable and important knowledge in the training of innovative talents. However, students often appear in the study of "the theory does not understand, the reality will not use, the ability does not improve, the achievement is not ideal" of low quality learning state. Based on the objective needs of The Times for the cultivation of outstanding talents with international competitiveness and innovative spirit, and against the background of blended learning becoming the future development trend, this paper discusses the teaching reform path that integrates case teaching method and blended teaching mode, and fully demonstrates the important supporting role of education informatization on education modernization and national economic and social development.

1.Introduction

The introduction of the "Education Informatization 2.0 Action Plan", which aims to drive the modernization of education through educational informatization, and the emphasis on the deep integration of modern information technology and education in the "China Education Modernization 2035" initiative, has brought online and offline blended teaching to the forefront of higher education reform [1]. Blended teaching, a teaching approach that combines the strengths of online and traditional methods, represents an "online + offline" instructional model, seamlessly merging these two educational formats [2]. The sudden emergence of the COVID-19 pandemic in 2020 expedited the normalization of online and offline blended teaching. It is evident that blended learning is the future of education, with approximately 70 percent of higher education leaders expecting to incorporate 40 percent of blended courses into their daily teaching routines [3]. Blended learning constitutes a significant breakthrough in ongoing higher education teaching reforms. Researchers have conducted in-depth analyses and practical experiments, yielding positive teaching outcomes.

For instance, Gao Yanwei et al. conducted an analysis and study of the advantages and challenges encountered in the implementation of mixed teaching methods in engineering mathematics within the context of new engineering education [4]. Zhang Qianwei et al. investigated and assessed the current status and obstacles facing Chinese college teachers in terms of readiness for blended teaching, offering valuable recommendations [5]. Jinshi et al. critically examined the implementation of online and offline blended teaching, proposing strategic optimizations across various areas, including innovative teaching design, the creation of conducive teaching environments, the restructuring of teaching evaluation methods, and enhancements in teaching support [1].

After years of exploration and practice, college teachers have developed a solid foundation in information technology literacy. However, their capacity for innovation in information-based teaching remains somewhat inadequate, and there is a need to strengthen their ability to deeply integrate information technology with subject teaching [3]. The evolution of educational informatization is an ongoing process, necessitating alignment with the ever-changing demands of the era and society [6]. Consequently, the fusion of information technology and specific teaching content requires educators to maintain a spirit of continuous exploration. They must consistently identify emerging scenarios and tackle novel challenges during their explorations, ultimately shaping a mature, innovative instructional model[7].

The basic mathematics courses of non-mathematics majors in colleges and universities are collectively referred to as "university mathematics courses", which mainly include higher mathematics, linear algebra, probability theory and mathematical statistics. The teaching scope covers all undergraduate majors of economics, management and science and technology, and has the characteristics of large quantity and wide coverage. The reform of college mathematics teaching often has a leading, overall and basic positionin the reform of college education and teaching[8]. This paper aims at the background of education informatization with Chinese characteristics, taking the practical experience of mixed

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teaching reform of Probability Theory and Mathematical Statistics as an example, to explore the implementation path of informatization in higher education teaching reform, in order to cultivate more high-quality talents with both virtue and talent.

2.The necessity of applying case teaching method in the course of Probability Theory and Mathematical Statistics

Probability theory and mathematical statistics is a branch of mathematics that studies random phenomena and their statistical laws. This course is an indispensable compulsory course to improve college students' mathematical quality and mathematical application ability. It can provide college students with essential data processing methods, and is the basis for them to learn other courses and conduct quantitative analysis and research on experimental data. At the same time, this course has a special and important role in cultivating students' dialectical materialist view, statistical view, logical thinking ability, analytical judgment ability, innovation ability and application ability[7]. However, as a public basic mathematics course in college, it is generally regarded as "abstract, boring and useless" by students. In the learning process, there are often phenomena such as "not understanding the theory, not satisfactory grades, not using the reality, and not improving the ability". There are many problems in traditional classroom teaching, as illustrated in Table 1.

Table 1. The main problems existing in traditional teaching and their specific manifestations

Main problems	Specific performance
Emphasizing Theory Over Practice	In mathematics courses, there is often a focus on the rigor of logical reasoning and the completeness of theoretical systems, while practical application and real-world relevance are neglected. This approach can make it challenging for students to fully engage with the material and feel a sense of achievement through practical application[9].
Emphasizing Intellectual Education and Neglecting Moral Education	The teaching process tends to prioritize knowledge transfer, overlooking the deeper guidance of students' emotions, attitudes, and values. Classroom instruction may concentrate solely on developing rational thinking, disregarding the humanistic aspects embedded in the results of such thinking. This disconnect can leave students questioning the purpose of their studies and lacking motivation.
Emphasizing Teaching and Neglecting Interaction	Many universities employ large class sizes for public basic courses, making it difficult for teachers to cater to individual student needs. The pressure to reduce class hours further compounds the issue. This traditional teaching approach often results in one-way communication, where teachers dominate the classroom, diminishing opportunities for student interaction and stifling students' initiative and non-intellectual factors like creativity and interest.
Value the Result Over the Process	Traditional teaching evaluation primarily focuses on final assessments and relies heavily on predetermined "standard answers". This approach places a strong emphasis on knowledge accuracy but neglects the development of students' creativity and problem-solving abilities. Consequently, it fails to provide comprehensive and effective feedback on teaching and learning outcomes.

However, the case teaching method, rooted in real-world cases, empowers educators to assume the roles of designers and motivators in the teaching process. It encourages students to actively engage in classroom instruction and fosters independent thinking. This approach not only instills students with the capacity for self-directed study but also redirects their focus from mere knowledge acquisition to skill development. Moreover, it places a significant emphasis on fostering two-way communication in the classroom [10][11].

Given that Probability Theory and Mathematical Statistics is a discipline with a substantial application-oriented background and promising prospects, the selected practical cases span a spectrum of traditional domains, such as medical diagnosis, financial investment, geology, and meteorology, as well as cutting-edge technologies like big data, artificial intelligence, and machine learning. This approach kindles students' enthusiasm for the subject, nurturing a spirit of inquiry and innovation.

Drawing inspiration from Dewey's educational theory, which posits that knowledge and experience emerge from the interaction between individuals and their environment [12], a multifaceted, interactive teaching environment is created using modern information technology. This environment guides students in actively constructing knowledge. In the integration of actual case teaching into the curriculum, basic mathematical concepts are initially linked to practical scenarios, and real-world problems are presented to pique students' interest. This process unfolds through content introduction and application segments.

As shown in Figure 1, through immersive experiences in genuine and vivid scenarios, active exploration driven by problem orientation, and participatory discussions within a relaxed and harmonious atmosphere, students evolve into problem researchers, explorers, and discoverers of new knowledge during their educational journey. This methodology kindles authentic motivation and enthusiasm for learning within their hearts. Furthermore, it assists students in comprehending the evolutionary trajectory of the course's theories and methodologies, and, more importantly, stimulates their innovation capabilities while enhancing their scientific literacy.

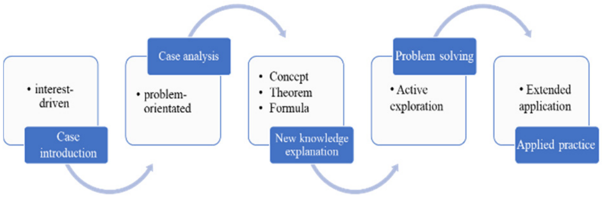

Fig. 1. Main Design Links of the Onsite Teaching Method.

3.Feasibility of Implementing the Case Teaching Method in a Blended Teaching Mode

Through this multifaceted approach, the aim is to nurture exceptional talents possessing international competitiveness and an innovative spirit. However, the challenges posed by large class sizes and reduced instructional hours often make it difficult, within the confines of traditional teaching, to strike a balance between imparting essential foundational knowledge and integrating practical real-world cases. The emergence of blended teaching modes offers an avenue for optimization.

For instance, by harnessing modern digital teaching tools such as Rain Classroom and Learning Pass, online and offline components can be seamlessly interconnected, following a “pre-class preparation, in-class interaction, and post-class analysis” approach (as shown in Figure 2). During the online learning phase, high-quality open courses and relevant literature related to key knowledge points are primarily disseminated before class. This segment focuses on delivering low-level factual and conceptual knowledge that necessitates memorization, cultivating students’ research awareness, reinforcing foundational principles, and extending the breadth of class discussions. In the offline component, the emphasis shifts toward guiding students in applying, analyzing, and engaging in higher-order cognitive training. This phase underscores practical application while delving deeper into the subject matter. The following two points should be considered when integrating practical case teaching into the curriculum.

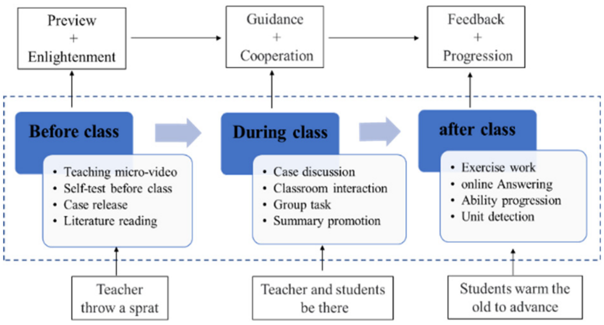

Fig. 2. Flowchart of Blended Teaching Mode.

3.1.Optimize Teaching Content

In light of the rapid economic development and ongoing societal progress, mathematics has emerged as a fundamental competency within contemporary society. As shown in Figure 3, this involves delving into the foundational content of traditional teaching while simultaneously expanding the practical applications to higher levels. It entails deepening and broadening the scope of the teaching material.

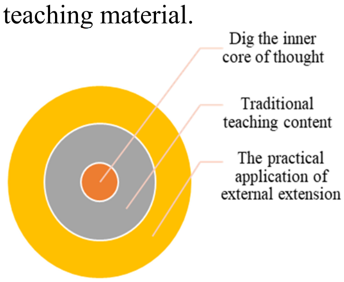

Fig. 3. Optimizing Teaching Content

3.2.Enhance Teaching Design

Embrace an integrated teaching approach, combining “probability and statistics” with specialized courses, fostering synergistic learning across multiple subjects. As shown in Figure 4, this approach facilitates a connected learning experience where related courses are linked rather than treated as isolated, standalone subjects. Utilize a flipped classroom format that combines discussions with reports or theses, emphasizing the significance of highlighting key knowledge points, conducting comprehensive case analyses, and fostering profound student understanding.

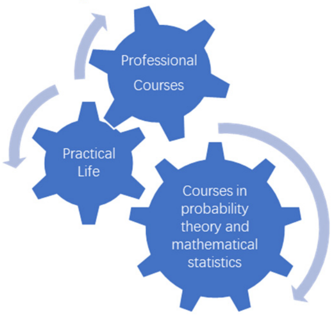

Fig. 4. Optimizing Teaching Design

4.Application Examples of the Case Teaching Method in Blended Teaching Mode

In alignment with the teaching objectives, which emphasize knowledge transfer, ability cultivation, and quality enhancement, the course group has implemented case teaching in pivotal chapters of probability theory and mathematical statistics. The following highlights the application of conditional probability in the Monty Hall Paradox and the use of two-dimensional normal distribution in image blur processing. These examples illustrate practical methods for integrating case studies into the core content of probability theory and mathematical statistics, with the aim of enhancing the learning interest of the majority of students, elucidating the practical applications within their majors, and broadening their knowledge base.

4.1.Monty Hall Paradox and Conditional Probability

The Total Probability Formula stands as a fundamental and significant formula in the section on conditional probability in this course. It enables the resolution of complex probability problems arising under various causes or situations. By introducing the renowned “Monty Hall Paradox”, also known as the “Three Doors Problem”, the teaching team created an interactive learning environment to apply this knowledge. The teaching process unfolds as follows:

4.1.1 Pre-Class Preparation

Through the online teaching platform, topic discussion cases related to the Monty Hall problem are disseminated before class. The Monty Hall problem originates from an American game show called “Let's Make a Deal”. In this game, there are three doors: behind one is a car, and behind the other two are goats. The host initially asks the contestant to choose one of the doors. Suppose the contestant selects door number 1. Instead of immediately revealing what's behind door number 1, the host deliberately opens another door, say door number 2, which reveals a goat. The contestant is then given the option to change their original choice to door number 3. The question posed is whether the contestant should change their selection at this point.

4.1.2 In-Class Discussion

The class engages in a discussion about whether changing doors is advantageous, focusing on the probability of winning the car after making the switch.

4.1.3 Problem Solving

The problem is resolved by defining events as follows: A = {the initial choice is a car}, $\overline{A}$ ={ the initial choice is a goat}, B = {the car is behind the door after changing chosen }. Therefore, the probability of event B can be calculated from the following total probability formula.

$$P(B) = P(A)P(B \mid A) + P(\overline{A})P(B \mid \overline{A}) = \frac{1}{3}\times 0 + \frac{2}{3}\times 1 = \frac{2}{3}.$$

Indeed, with a 1/3 probability of the car being behind one of the initially chosen doors by the contestant, and a 2/3 probability of it being behind one of the remaining doors, it is statistically more advantageous for the contestant to change doors.

4.1.4 Post-Class Enrichment

Drawing from the Law of Large Numbers, which states that “when the sample size is sufficiently large, the frequency of an event approaches its probability”, Monte Carlo simulation is employed. Computer software programming is used to simulate the game process, and results are statistically analyzed to verify the frequency of winning. Within the blended teaching mode, teachers guide students with spare learning capacity to independently or collaboratively complete the simulation portion omitted during class. This approach effectively enhances students’ ability to analyze and solve practical problems utilizing their knowledge of probability theory and mathematical statistics. To facilitate beginners in emulating this learning, a network learning platform can be established. This platform can disseminate relevant literature on actual cases, modeling processes, program code, and encourage students to continually refine and upload their algorithms and program code to the platform.

4.2.Image Blurring with Two-Dimensional Normal Distribution

When teaching the topic of “two-dimensional continuous random variables and their distribution”, the concept of two-dimensional normal distribution takes center stage in students’ learning. However, grasping the depth of the normal distribution concept and comprehending its practical problem-solving applications can be challenging for students. To bridge this gap, the teaching team introduced a familiar real-world scenario involving the “beauty function” commonly found in camera software.

Fig. 5. Comparison Before and After Image Blurring Filtering

4.2.1 Pre-Class Preparation

Before class, topic discussion cases and relevant literature are shared through the online teaching platform. Students are prompted to consider how common photo retouching software or camera beauty functions blur imperfections such as freckles on people's faces, as shown in Figure 5. They are encouraged to explore literature related to the widely used Gaussian filter blurring technique in image processing and its connection to probability and mathematical statistics[13].

4.2.2 In-Class Discussion

During the lesson, it is revealed that Gaussian filter blurring primarily leverages the unique shape and properties of the two-dimensional normal distribution. Let the probability density function of two-dimensional random variables (X,Y) be

$$f(x,y)=\frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}\exp\{-\frac{1}{2(1-\rho^2)}[\frac{(x-\mu_1)^2}{\sigma_1^2}-2\rho\frac{(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}+\frac{(y-\mu_2)^2}{\sigma_2^2}]\}$$

Where $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$ are all constants, and $\sigma_1 > 0, \sigma_2 > 0, -1 < \rho < 1$. In this way, the random variable (X,Y) obeys to the two-dimensional normal distribution of the parameter $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$, denoted by $(X,Y) \sim N(\mu_1, \mu_2; \sigma_1^2, \sigma_2^2; \rho)$.

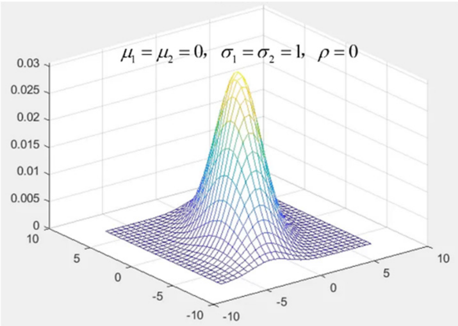

Fig. 6. Two-dimensional normal distribution probability density function with different parameters

When observing the image, the function values of the bell-shaped surface with two-dimensional normal distribution exhibit an interesting pattern: the closer to the center point, the greater the value, and conversely, the farther away from the center point, the smaller the value, as shown in Figure 6. This characteristic is exploited in image processing filter blur technology to create varying degrees of image blur, producing a soft and fuzzy effect.

To achieve image blurring, the first step is to convert the analog signal of the image into a digital signal, where the pixel values of the image are represented by a matrix. Blurring a point involves taking the average pixel values of the surrounding points. However, to ensure a smooth transition in the blurred image, it is crucial to assign weights to the pixels within the "surrounding" points in the blurring radius. The idea is that the pixels on the outer edge of the blur radius should have lower weights, exerting a smaller impact on the pixel being blurred. Conversely, the pixels closer to the center of the blur radius should have higher weights, exerting a more significant impact. The characteristics of the two-dimensional normal distribution probability density function align perfectly with this weight distribution requirement, making it an ideal choice.

4.2.3 Problem Solving

In practice, this translates to creating a pixel square matrix with the center point (0,0) as the core, and the entire blurring process is controlled by determining the scope of the "surrounding" points involved. Initially, let's consider a fuzzy range with a radius of 2. Suppose the coordinate matrix for the center point (0,0) and its surrounding points is:

$$\begin{pmatrix} (-2,2) & (-1,2) & (0,2) & (1,2) & (2,2) \\ (-2,1) & (-1,1) & (0,1) & (1,1) & (2,1) \\ (-2,0) & (-1,0) & (0,0) & (1,0) & (2,0) \\ (-2,-1) & (-1,-1) & (0,-1) & (1,-1) & (2,-1) \\ (-2,-2) & (-1,-2) & (0,-2) & (1,-2) & (2,-2) \end{pmatrix}$$

Next, let's calculate the weight matrix. For the sake of model simplicity, let's assume the parameter values of the two-dimensional normal distribution are $\mu_1 = \mu_2 = 0, \ \sigma_1 = \sigma_2 = 1, \ \rho = 0$. Therefore the weight of point (2,2) is the probability density function value $f(2,2) = \frac{1}{2\pi} e^{-\frac{2^2+2^2}{2}} = \frac{1}{2\pi} e^{-4}$. Similarly, we can get the weight matrix for all the points. Finally, we can compute the weighted average value at the center of the pixel matrix.

4.2.4 Post-Class Enrichment

Encouraging students with an interest in image processing and programming capabilities, teachers invite them to experiment with image blurring independently after class. Students can then share their modeling processes and program code on the network learning platform, fostering collaborative learning and knowledge exchange.

5.Conclusion

Teaching may have methods, but there is no single definitive method. Education reform should be a continuous process that evolves in alignment with the changing times and societal developments. In November 2021, the Cyberspace Administration of China introduced the "Outline of Action to Enhance Digital Literacy and Skills for All", marking China's first comprehensive national policy on digital literacy[14]. The outline sets a clear goal: "By 2035, China will essentially become a nation of digital talent, with the entire population attaining a higher level of digital literacy and skills". Therefore, the implementation of blended teaching must be an ongoing exploration and adaptation to meet the new requirements of the era.

Education seeks educators who possess wisdom, a deep understanding of humanity, and creativity in the classroom, rather than those who are simply too busy to contemplate. The journey ahead is long, but I am committed to continued growth and improvement.

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