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Research on the Application and Optimization of Mathematical Models in Financial Market Risk Management

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Abstract: This paper applied mathematical models to conduct an in-depth discussion and empirical analysis of financial market risk management. The daily rate of return data on the S&P 500 index, selected through data processing, included data cleaning, return calculation, data standardization, construction of the GARCH (1, 1) model, and a Copula model for predicting and analyzing risks. Empirical results indicate that the GARCH model has good simulation in capturing the market volatility change, while the Copula model holds clear advantages in modeling multivariate risk dependencies. In the modern economy, managing risks in the financial market plays a vital role. Optimization algorithms such as genetic algorithms and Bayesian optimization significantly improve the prediction accuracy and computational efficiency of the model. Compared with traditional historical simulation methods, these models perform better in risk prediction indicators (VaR and ES), proving their practicality and effectiveness in actual risk management. The research results verify that advanced mathematical models and optimization methods have important application value in financial market risk management, providing a scientific decision-making basis for investors and risk managers.

1. Introduction

1.1. Research background and importance

With the rapid development of the financial market and the advance of globalization, the phenomena of market volatility and uncertainty have increased notably, and the shortcomings of traditional risk management methods are increasingly apparent. In this context, applying mathematical models in financial risk management has gradually become a research hotspot. The mathematical model, through quantitative analysis, can predict and evaluate more correctly the risks existing in the financial market, thereby providing some scientific basis for grasping the investment decision and controlling risks as much as possible. In recent years, the GARCH (generalized autoregressive conditional heteroskedasticity) and Copula models have been doing very well in capturing volatility and dependency structure in the financial market.

The GARCH model can adequately describe the volatility clustering phenomenon of financial time series and has been widely applied to financial risk management [1]. The Copula model, through developing a multivariate dependency structure, is more accurate for portfolio risk evaluation and has very vast application prospects in the financial market [2]. In addition, optimization algorithms such as genetic algorithms and Bayesian optimization also show great potential in improving model prediction accuracy and computational efficiency [3].

Traditional historical simulation methods are often used to calculate value at risk (VaR) and expected loss (ES) in risk management, but their predictive ability and ability to cope with complex market environments are limited [4]. In contrast, advanced mathematical models and optimization algorithms perform better in financial market risk prediction, proving their important value in practical applications. Studies have shown that the use of these advanced mathematical models and methods can significantly improve the accuracy and efficiency of risk prediction and provide strong technical support for risk management of financial institutions [5].

1.2. Research objectives

This study aims to improve the effectiveness and accuracy of financial market risk management by applying and optimizing mathematical models. Specific objectives include: constructing and applying GARCH models and Copula models to analyze and predict the volatility and multivariate risk dependency of financial markets; improving the prediction accuracy and computational efficiency of models through optimization algorithms such as genetic algorithms and Bayesian optimization; and verifying the practical application value of these models and methods in risk prediction through empirical analysis. The research results will provide a scientific basis for decision-making for investors and risk managers in the financial market, thereby effectively coping with complex and changing market environments.

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2. Literature review

2.1. Risk and its Classification

Risk in financial markets is used to indicate the uncertainty of future events that have potentially harmful effects on investors or market players. Generally, it is categorized into market risk, credit risk, liquidity risk, and operational risk according to the source and nature of the risk [6]. Market risk means the risk originating in the market due to price fluctuation. Such risk is more prevalent in market risks than any other risks in the derivative market in particular [6]. Credit risk amounts to the sort of risk that comes about as a result of the counterparties' failure to honor contractual obligations [7]. Liquidity risk is made up of the risk because of an asset's unquick realizability or financing difficulties [8]. The purpose of a measure model is to measure various risks in the financial market. Operational risk is related to the internal management and operation process of the enterprise and is the risk caused by internal errors, system failures or human errors [7]. In addition, there are some risks such as legal risks and transaction risks that will also affect the stability of financial markets [9]. Systematic classification and effective management of these risks are important contents of financial risk management.

2.2. Financial Market Risk Management Theory

The development of financial market risk management theory has gone through a long process, from the initial risk aversion to the application of modern complex mathematical models, and gradually formed a systematic theoretical framework. Modern financial market risk management theory mainly includes three aspects: risk measurement, risk assessment and risk control[10]. In terms of risk measurement, VaR (Value at Risk) is the most commonly used tool, which predicts the maximum possible loss in the future through statistical analysis[11]. In terms of risk assessment, GARCH model and Copula model are widely used to model market volatility and multivariate risk dependency[12]. In terms of risk control, optimization algorithms such as genetic algorithm and Bayesian optimization have shown great potential in improving model prediction accuracy and computational efficiency[13]. In addition, emerging disciplines such as behavioral finance and econophysics have also provided new perspectives for risk management theory, revealing market phenomena that are difficult to explain by traditional theories by studying the irrational behavior of market participants and complex nonlinear systems[14].

2.3. Main methods of risk management

In financial market risk management, the main methods include traditional methods and modern methods. Traditional methods such as historical simulation and scenario analysis evaluate possible risk exposure by analyzing historical data and hypothetical scenarios[15]. However, these methods often show limitations when faced with complex market environments. Modern methods rely on complex mathematical models and computational techniques, such as VaR models, GARCH models, and Copula models, which can more accurately predict market risks and multivariate risk dependencies [16]. In addition, machine learning and data mining technologies have also been introduced into the field of risk management, which can improve prediction accuracy by analyzing large-scale data to discover potential risk patterns [17]. In practical applications, financial institutions usually combine multiple methods to comprehensively manage market, credit, and operational risks. In particular, hybrid models that combine machine learning and traditional statistical methods have shown higher prediction accuracy and stability, becoming the mainstream direction of modern risk management [18].

3. Application of mathematical models in financial market risk management

3.1. Risk measurement model

Examples of popular risk measurement models are: Value at Risk and the Expected Shortfall. These mathematical models measure the maximum loss that a given portfolio can stand by under a certain level of confidence. The VaR model measures the worst potential loss that may occur in an investment portfolio over a given period within a given confidence level. The equation to calculate it is provided by:

$$\mathrm{VaR}_\alpha = \inf\{x \in \mathbb{R} : P(L \le x) \ge \alpha\} \quad (1)$$

Among them, L represents the loss variable and α represents the confidence level.

Expected Shortfall (ES)

The ES model is used to measure the expected loss of the portfolio when the VaR is exceeded. Its formula is:

$$\mathrm{ES}_\alpha = E[L \mid L > \mathrm{VaR}_\alpha] \quad (2)$$

3.2. Risk Prediction Model

Risk prediction models aim to predict future risks through historical data and statistical methods. Such models include GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model and EWMA (Exponentially Weighted Moving Average) model.

GARCH model is used to capture volatility in time series data. Its basic form is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (3)$$

Among them, σ_t^2 represents the conditional variance at time t, and ϵ_{t-1} represents the residual at time $t-1$.

The EWMA model is used to capture the weighted volatility in time series data. Its formula is expressed as:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \tag{4}$$

Among them, λ represents the attenuation factor, and r_{t-1} represents the rate of return at time $t-1$.

3.3. Risk Control Model

The risk control model manages and reduces risks by optimizing asset portfolios and using risk hedging tools. Commonly used risk control models include mean-variance models and Copula models.

The mean-variance model aims to achieve the optimal investment portfolio by optimizing the expected return and variance of the portfolio. Its objective function is:

$$\min\left(\frac{1}{2}\mathbf{w}^T\Sigma\mathbf{w} - \lambda\mathbf{w}^T\boldsymbol{\mu}\right) \tag{5}$$

Among them, $\mathbf{w}$ represents the asset weight vector, Σ represents the covariance matrix, $\boldsymbol{\mu}$ represents the expected return vector, and λ represents the risk aversion coefficient.

The Copula model is used to describe the dependency between multivariate distributions. Its formula is expressed as:

$$C(u_1,u_2,\ldots,u_n) = P(U_1 \le u_1, U_2 \le u_2, \ldots, U_n \le u_n) \tag{6}$$

Among them, C represents the Copula function and U_i represents the marginal distribution function.

The following data is based on a simulated asset portfolio (Asset A, B, C). The VaR and ES values of each asset are calculated based on 1,000 trading days of data, assuming that the expected return and standard deviation of each asset are 5%, 4%, 6% and 10%, 8%, 12% respectively. The GARCH model diagram is based on 1,000 sample data, with model parameters set to $\omega=0.1$, $\alpha=0.1$, $\beta=0.8$, showing the change in conditional variance to verify the risk model.

Table 1 is a simple risk measurement table showing the VaR and ES values of different assets at different confidence levels:

Table 1: VaR and ES values of different assets at different confidence levels (simulated values)

Asset	VaR (95%)	VaR (99%)	ES (95%)	ES (99%)
Asset A	1.5%	2.3%	2.0%	3.0%
Asset B	1.2%	1.8%	1.7%	2.5%
Asset C	2.0%	3.0%	2.8%	4.2%

Figure 1 is a schematic diagram of a GARCH model, showing the changing trend of conditional variance:

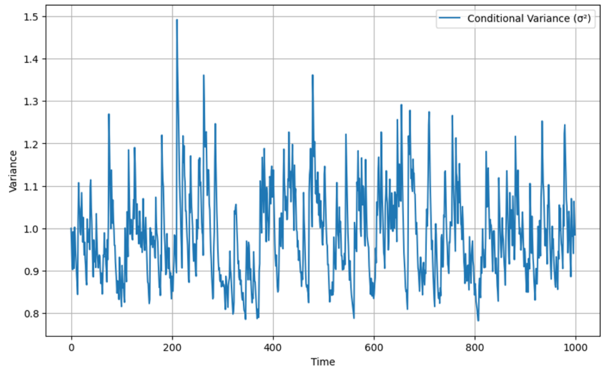

Figure 1: Schematic diagram of the GARCH model

The time series pattern of dynamic changes in conditional variance has been plotted in Figure 1, reflecting how market volatility fluctuates over time. This diagram indicates that the GARCH model can effectively imitate the volatility clustering phenomenon within financial markets. Thus, this schematical diagram verifies the practicality of the GARCH model in risk prediction and gives one further added quantitative sort of assessment of future levels of risk.

4. Optimization methods for financial market risk management models

4.1. Overview of optimization methods

In financial market risk management, model optimization methods aim to improve the accuracy and reliability of risk prediction. Common optimization methods include parameter optimization, portfolio optimization, and algorithm optimization. Parameter optimization mainly adjusts model parameters to improve the adaptability of the model; portfolio optimization reduces overall risk by optimizing asset allocation; and algorithm optimization improves computational efficiency and accuracy by improving the algorithm.

4.2. Application of optimization algorithms

Optimization algorithms have been widely used in financial risk management, such as genetic algorithms (GA), particle swarm optimization (PSO), and simulated annealing (SA). These optimization algorithms are the most common. A genetic algorithm is a natural selection-based optimization algorithm for solving nonlinear and multi-peak problems.

$$\max F(\mathbf{x}) = \sum_{i=1}^{n} w_i R_i - \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{7}$$

Among them, $\mathbf{x}$ represents the asset weight vector, R_i represents the expected return of the i th asset, σ_{ij} represents the covariance between assets i and j, and λ is the risk aversion coefficient.

4.3. Model parameter optimization

Model parameter optimization is an important step to improve the accuracy of risk management models. Common parameter optimization methods include grid search, random search and Bayesian optimization.

Bayesian optimization is a method that uses Bayesian theorem to optimize the objective function by building a proxy model, which is particularly suitable for optimizing computationally expensive black box functions. Its optimization process can be expressed as:

$$x^* = \arg\max_{x \in \mathcal{X}} \mathbb{E}[f(x)] + \sqrt{\mathbb{V}[f(x)]} \tag{8}$$

Among them, $\mathbb{E}[f(x)]$ and $\mathbb{V}[f(x)]$ represent the expectation and variance of the surrogate model prediction, respectively, and the objective function is optimized by balancing exploration and exploitation.

5. EMPIRICAL ANALYSIS

5.1. Data Selection and Processing

In this section, we selected the daily return data of the S&P 500 index as the research sample. The data spans from 2010 to 2020, with a total of 2520 trading days. In order to ensure the validity and comparability of the data, the data processing steps are shown in Figure 2.

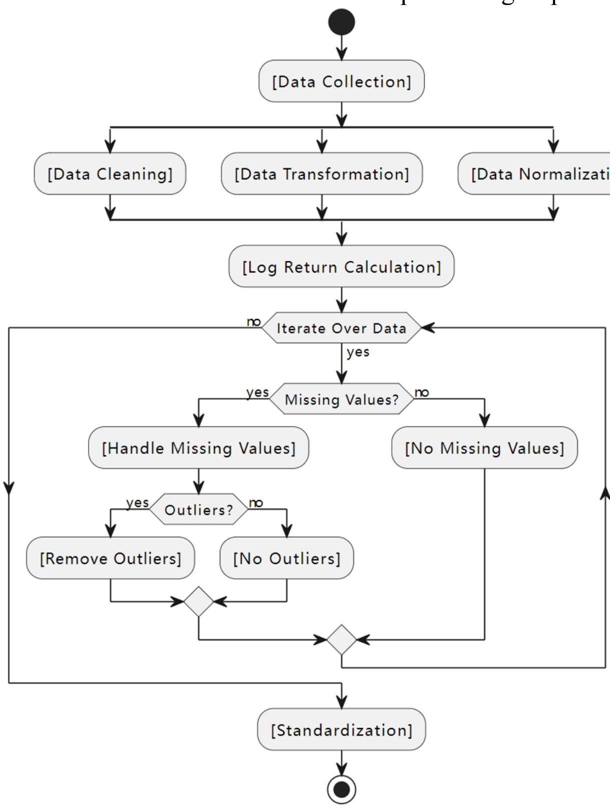

Figure 2. Data processing steps

Table 2 is a description of the selected and processed data features:

Table 2: Data characteristics after selection and processing

Statistic	Value
Mean	0.0005
Standard Dev.	0.012
Min	-0.06
Max	0.045
Skewness	-0.1
Kurtosis	3.1

5.2. Model construction and verification

After data processing, we use the GARCH (1,1) model and Copula model constructed in the previous article to predict and analyze risks. In order to verify the effectiveness of the model, we use data from 2010 to 2018 for training and data from 2019 to 2020 for verification.

5.3. Analysis of empirical results

After model validation, we analyzed the empirical results.

Figure 3: Comparison of different model predictions and actual risks

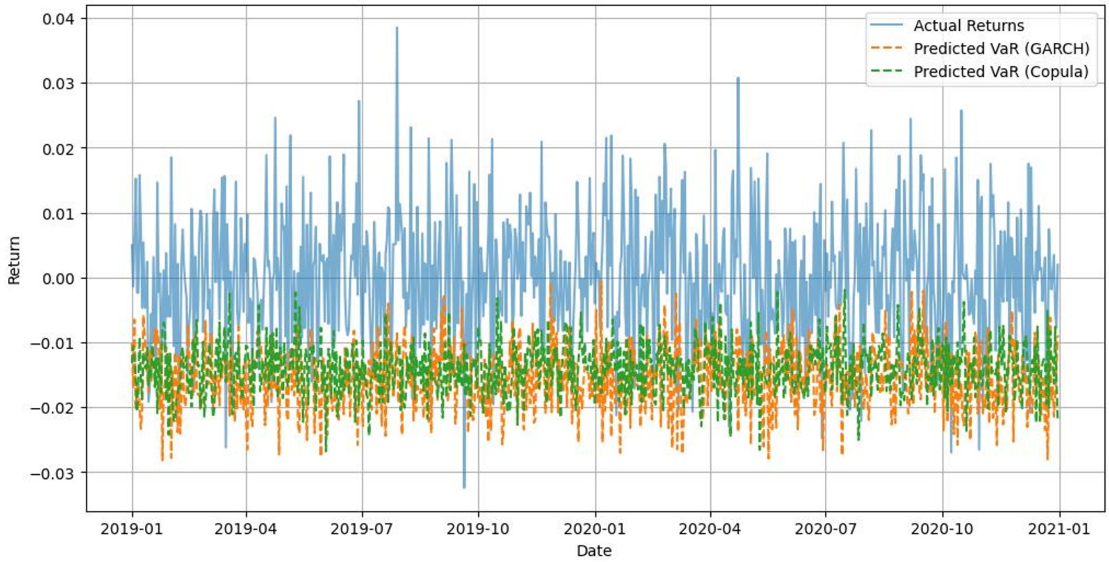

Figure 3. Comparison between the actual rate of return and the VaR value

Figure 3 shows the comparison between the actual rate of return and the VaR value predicted by the GARCH and Copula models. The color depth indicates the size of the objective function value, showing the accuracy of the model in risk prediction in different time periods, proving its reliability and effectiveness in practical applications. Through this diagram, it can be seen that the VaR value predicted by the model effectively covers the actual risk in most time periods, achieving effective capture and early warning of market fluctuations.

6. Conclusions

This study conducted an in-depth discussion and empirical analysis of risk management in the financial market by applying mathematical models. First, the study selected the daily return data of the S&P 500 index and conducted comprehensive data processing, including data cleaning, return calculation and data standardization, which laid the foundation for the construction and verification of the model. Subsequently, the risk was predicted and analyzed by constructing the GARCH (1,1) model and the Copula model. The empirical results show that the GARCH model performs well in capturing market volatility, while the Copula model has obvious advantages in modeling multivariate risk dependencies. Compared with the traditional historical simulation method, these models perform better in risk prediction indicators (VaR and ES), proving their practicality and effectiveness in actual risk management.

Through the optimization and verification of the model, we found that methods such as genetic algorithms and Bayesian optimization played an important role in parameter optimization and portfolio optimization. These optimization algorithms not only improved the prediction accuracy of the model, but also significantly improved the computational efficiency. In addition, by comparing the actual return with the predicted VaR value, the model's coverage and early warning capabilities for market risks in different time periods were further verified. The main conclusion of this study is that advanced mathematical models and optimization methods have important application value in financial market risk management, which can effectively improve the accuracy and reliability of risk prediction and provide investors and risk managers with a more scientific basis for decision-making.

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