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Examining interaction in mathematical activities from different theoretical perspectives: The registers of semiotic representations and Habermas' construct of rationality

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Abstract. This study aims to examine and compare how Duval's theory of registers of semiotic representations and Habermas' construct of rationality approach the concept of interaction in mathematics classrooms. Duval defines conceptual understanding as the construction and use of representations of mathematical objects and states that students interact through representations in mathematical activities. Researchers adapting Habermas' construct of rationality into mathematics education explain the interaction students engage in during mathematical activities by examining their tendencies to use representations of mathematical objects correctly and purposefully, and to communicate this process in a comprehensible way to others. It is noteworthy that both theories address student interaction with peers and teachers in mathematical activities but approach this concept from different perspectives. This study is focused on how these two theories can be used together based on the networking of theories and strategies to analyze interaction in mathematics classrooms. The results indicate that there are critical connections between these theories and that using them together to analyze the interaction students establish through the construction and use of representations can yield more detailed insights for mathematics teachers and researchers.

1 Introduction

Understanding mathematical objects is a multifaceted phenomenon that cannot be fully defined, understood, or explained by a single theory. Hence, mathematics learning and teaching necessitates the use of diverse theories. Over time, in mathematics education, different theories have emerged in various cultural contexts [1]. English and Sriraman [2] mention the importance of evaluations of diverse mathematical theories and point out the need for guidelines on how to use these theories. This requirement has become even more explicit due to the following problems caused by the differences between diverse theoretical perspectives [3-5]:

- Language problem

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- Connectivity problem

Different theoretical perspectives might utilize the same phrases and/or terminology in diverse ways, or different theoretical perspectives might employ different phrases and/or terms to describe the same or similar situations [6]. Hence, the language problem arises in the theoretical research literature. The connectivity issue pertains to how findings from various theoretical perspectives in literature can be integrated to address the same or similar research problems [5]. Thus, it is essential to employ scientific strategies to address a variety of theories. The concept of networking theories is proposed as an approach for this purpose [7].

Creating a network of theories means establishing a systematic link between theories. Theory networking aims to create a coherent framework for research [5]. The theory networking consists of understanding different theories used to analyze the same or similar mathematical concept or phenomenon, analyzing the same phenomenon or concept from these theoretical perspectives, evaluating the employment of these theories, comparing the nature of these theories, exploring the possibilities of connecting theories, and building a linkage between theories if it is possible considering their identity [7].

Our study aims to suggest initial steps for a possible network between Duval's [8] theory of registers of semiotic representations and Habermas' [9] construct of rationality, which basically addresses the concept of interaction in mathematics classrooms from different perspectives. Representations are used as mathematical language and communication tools during interaction in mathematical activities. Duval [8] states that students perform in mathematical activities based on their conceptual understanding and defines conceptual understanding as transforming within the same register or between different registers of the representations for the relevant mathematical concept. The cognitive gap that arises during the transformation within the same register or between different registers of representations may cause students to have difficulties in mathematical activities. Habermas' [9] construct of rationality analyses the difficulties students experienced during constructing and using representations in the context of being conscious and purposeful and communicating with others in mathematics classrooms. We think that Duval's theory and Habermas' construct of rationality can be used together to explain these difficulties and to reveal the causes of these difficulties. Considering Prediger et al.'s [4] framework of strategies for networking theories, our study focused on understanding and explaining these two theories, identifying their similarities and differences, and examining the possibility of using them together to analyze the interaction in mathematics classrooms. In line with this aim, we focus on the following research problem: "How do Duval's theory of registers of semiotic representations and Habermas' construct of rationality approach to the interaction concept in mathematical activities? Does the combined use of these theories lead to results that can be compared, contrasted, and combined in the context of interaction analysis?"

2 Method

Prediger et al. [4] identified the strategies for building connections between theories, including "understanding theories and making them understandable", "comparing" and "contrasting", "combining" and "coordinating", "integrating locally" and "synthesizing". As Prediger et al. [4] explain, the aim of "comparing" is related to find similarities and differences, while "contrasting" emphasizes specifically the differences between the theories. "Comparing" and "contrasting" between theories can always be performed because identifying similarities and/or differences between two theories is generally possible. However, "coordinating", "integrating locally" or "synthesizing" theories are challenging networking strategies. In "coordinating", elements from the theories are chosen to investigate a specific research problem, and these elements are brought together in a strictly compatible manner. In "combining" theories, the chosen elements from the theories may not necessarily require having the compatibility. The theories can be combined together to analyze the same or similar mathematical concept(s) or situation(s) based on their similarities and differences between the theories.

This study focuses on making understanding Duval's [8] theory of registers of semiotic representations and Habermas' [9] construct of rationality and comparing and contrasting them as the first step of combining these theories. The concept of "interaction" from these two theories is selected to investigate our research problem. The choice of this specific concept considers the fact that interaction is treated as an important and fundamental concept in the nature of both theories [8-10]. First, we identified how both theories explain interaction in mathematical activities and how interaction is explicitly or implicitly conceptualized in each theory. Thus, we aimed to form a first idea for the possibility of combining these theories as mentioned in the literature [11, 12].

3 Results

3.1 Duval's theory of *registers of semiotic representations*

Mathematical objects become understandable through their representations. Verbal, numeric, graphical, geometric, algebraic, or numerical representations are used to understand a mathematical object. According to Duval [8], the semiotic register is a system of representations and relations that are governed by certain rules and are used to understand and explain a mathematical object, communicate, and produce new knowledge based on these rules. These rules can be categorized as follows:

1. Rules concerning the elements of a representation register,
2. Rules related to transformations within a representation register (treatment),
3. Rules related to transformations from one representation register into a different representation register (conversion).

The first rule is related to the content of the register. The other rules are related to the transformations of representations within the same register or between different registers. Transformation of representations refers to cognitive activities related to the conceptual understanding of the relevant mathematical object [8, 10]. Duval [8] defines the transformations of representations within the same register as treatment and the transformations of representations from one register into another register as conversion. In these cognitive activities, students are expected to know the content of the registers of representations for a mathematical object and use the elements of the content to reach the aim of a mathematical task. Furthermore, students are required to communicate with their peers and teachers in the context of the transformations of representations using the content of the registers in mathematics classrooms.

Duval [8] emphasized that the representations of mathematical objects differ, but the object itself remains the same and that it is necessary to coordinate various representations to understand the concept. Coordination requires the ability to convert the representations of a couple of registers in both directions [10]. Transforming representation between two different registers in both directions requires knowing the content of each register and choosing and using proper mathematical strategies to reach the aim in a mathematical activity. Duval [8, 10] states that the transformations of representations in coordination are the source of problems in mathematics learning, and that examining the transformations is crucial for identifying and analyzing the difficulties that students experience in understanding mathematics [10].

3.2 Habermas' construct of rationality

In mathematical activities, communication requires to use of representations of mathematical objects through speech, written symbols, drawings, or physical objects [13]. To communicate understandably and acceptably, the student needs to persuade others about the construction and/or transformation of representations. Conceptual understanding determines the building of the relationship between mathematical concepts and the use of their representations in mathematical activities [14]. Early studies analyzed conceptual understanding within the framework of the cognitive approach and paid attention to the mental structures and internal processes of the individual [13]. Recent studies have reported that socio-cultural factors are also related to conceptual understanding [15]. According to the socio-cultural approach, learning depends on social interaction-communication processes because they enable students to participate in the learning environment [16]. Sociological and philosophical theories have been adapted into mathematics education to analyze student performances in mathematical activities to conduct a socio-cultural analysis of conceptual understanding [17]. Habermas' construct rationality is one of these theories [18].

Habermas [9] defines rational behavior as performing a task by making conscious and purposeful choices in accordance with the criteria valid for that task and by caring about being in consensus with others. Habermas [9] argues that a rational individual acts to achieve a goal and that his/her choices can be explained by the validity criteria of the relevant context and the factors that restrict communication. Therefore, Habermas [9] identifies three interrelated components of rational behavior. The first component of the theory is epistemic rationality which refers to "knowing" something involves being aware of why the statements we make about what we know are true or false. Otherwise, knowledge becomes dogmatic. The second component of the theory is teleological rationality which is concerned with the "intentional acts" which shows the awareness of choosing and using appropriate means to achieve a goal. The third component of the theory is communicative rationality which includes the interaction among individuals within a community. In communication, the speaker aims to reach a consensus with the listener. For this consensus to be achieved, the discourse must be as understandable and acceptable as possible to the listener.

Morselli and Boero [19] adapted this theory to mathematics education and analyzed the behaviors of students interacting in mathematics classrooms based on the rationality components. In this context, epistemic rationality suggests that knowing a mathematical concept means knowing why statements about what we know are true or false. Teleological rationality pertains to the awareness of selecting and employing suitable mathematical strategies, theorems, definitions, and/or principles to accomplish a goal in a mathematical task. Communicative rationality necessitates that students have the ability to articulate their mathematical performance clearly and acceptably to both the teacher and their peers in the classroom. The components of Habermas' construct of rationality have an intertwined structure. Studies have shown that there is a strong dynamic interaction among these three components that can be observed during mathematical activities [17-25].

4 Conclusion: A proposal for combining Duval's theory and Habermas' construct of rationality

According to Duval [8], students' performance in using representations in mathematical activities provides important clues about conceptual understanding. Therefore, students' performances in constructing and transforming representations should be analyzed to accurately determine their mathematical understanding. Transformation of representations depends on the coordination of various registers and the cognitive gap between different registers due to the difference in their contents can lead to some difficulties in the mathematical activities to reach the aim of the task [8]. Duval's theory could not adequately explain these difficulties. Hence, an additional theoretical tool is needed to identify students' difficulties in constructing and using representations and theoretically explain the reasons behind these difficulties. For this purpose, we think that Habermas' construct of rationality can be used together with Duval’s theory of registers of semiotic representations. Transforming between registers requires knowing the same mathematical object in the context of the content of at least two different registers and deciding whether the change between registers is fit for purpose in the task to reach the aim [8]. Duval's theory does not focus on whether the student consciously controls the change of the content between different registers of a mathematical object. Students' content knowledge in different representation registers of a mathematical object can be analyzed in terms of epistemic rationality. In such an analysis, the student is expected to justify the content knowledge in different representation registers and to defend the representation transformation providing mathematically reliable reasons. Representation register transformation in a mathematical activity requires the ability to choose the register necessary to achieve the goal in the mathematical task [8]. Duval's theory is not focused on explaining whether the student conducts purposeful coordination when transforming between different registers. Whether the coordination between registers is appropriate for the purpose of mathematical tasks can be explained in the context of teleological rationality. A student can know the content of the different registers of a mathematical object and can make transformations within the same register or between different registers. However, it is also important that the student communicates these transformations in a way that is understandable and acceptable to the teacher and other students in the mathematics classroom. When Duval’s theory of registers of semiotic representations is combined with Habermas' construct of rationality, students' performance in communicating through representations can be analyzed in a detailed manner.

Duval's theory also provides an additional perspective to Habermas' construct of rationality. It is possible to enrich the criteria of the components of Habermas' construct of rationality, which focuses on interaction in mathematical activities and is used to determine whether students' behaviors in this process are correct, appropriate, and approvable, by considering what is expected of students in the context of constructing and transforming representations. Epistemic rationality involves students modeling a mathematical concept or situation and expressing their knowledge using representations. Therefore, it can be argued that there is a critical relationship between epistemic rationality and Duval’s theory of registers of semiotic representations. Teleological rationality involves students' goal-based steps and helps to determine whether the student's acts are intentional to reach the aim of the mathematical task. Therefore, determining the student's use of appropriate representations, deliberate construction and/or transformation provides a systematic analysis in the context of teleological rationality. Communicative rationality is associated with the student's presentation of a comprehensible and acceptable product. Duval’s theory of registers of semiotic representations allows the researcher to determine whether the student uses flexible representations and explains transformations within and/or between representation registers of a mathematical object. Therefore, it can be argued that Duval’s theory of registers of semiotic representations contributes to more precise and accurate results by extending the criteria for the components of rationality theory, thus contributing to analyses based on Habermas' construct of rationality.

The results obtained by explaining and comparing the two theoretical frameworks suggest that if these theories are used together in the analysis of interaction in mathematics classrooms, students' performance in using and transforming representations can be analyzed in more detail and accurately. In this study, it is suggested that Duval’s theory of registers of semiotic representations and Habermas' construct of rationality should be taken into consideration and these theories should be used together to analyze students' representation construction and transformation behaviors during interaction in mathematical activities. The results to be obtained in experimental studies to be conducted in this context are very important and necessary for the evaluation of the proposal for the use of these two theories together.

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