Portfolio Optimization for Different Industries under Constraints

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Abstract. Portfolio optimization has indispensable role in nowadays financial market. In today's market with high volatility, portfolio optimization is particularly important. Investing in different industries seems to be a good way to diversify the risk. This paper study the asset allocation based on five different industries, i.e., technology, consumer goods, financial Services, healthcare and energy. To optimize the portfolio, Markowitz model is applied to predict the performance of the portfolio, meanwhile, four constraints which might appear in real world are used to help investors to determine the portfolio they prefer and deal with sophisticated situations. Maximum Sharpe Ratio, the minimum variance as well as the capital allocation line are calculated by Markowitz model and the minimum variance frontier under each constraint is also calculated by SolverTable. One conclusion the author gain is that SPX has high correlations with selected stocks except for NVDA but SPX is not a good choice for investors to balance risk and return. When constraints are added, the minimum variance frontier as well as the capital allocation line (Besides CAL4) behave not so good as before. These findings help investors to explore multiple optimization solutions in different situations, control risk exposures and avoid extreme portfolio allocations.

1 Introduction

In Harry Markowitz's paper "Portfolio Selection", Markowitz model is firstly published. Since then, how to balance risk and return has always been a hot topic. Markowitz model is particularly important for its function to optimize the portfolios and achieve highest return under certain risk level. By using Markowitz model, the investors construct efficient frontier that shows all efficient portfolios in a risk-return framework and helps to find maximum return when the amount of risk is given [1]. Investors choose portfolios they prefer. Adding constraints according to requirements provides potential better asset allocation which make portfolio management more acceptable.

It is known to all that there are a number of researching papers focusing on portfolio optimization, but they still don't cover all aspects of portfolios. For example, much research ignores the constraints may happen in the real world which may change their portfolio options significantly, one example is portfolio management made by Alexandre, Wai, and Bobby. In their research, they didn't add any constraints to Black-Litterman framework which they used

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to optimize the portfolio [2]. Additionally, many research papers focus on specific areas and ignore others. For instance, Wang selected 8 stocks in the AI industry, based on Markowitz portfolio theory, Wang constructed efficient frontiers which helps investors to choose portfolio according to their risk aversion degrees [3]. In research done by Suleman, Muhammad, Awais, and Aviral, they only choose stocks in manufacturing industry, and they found that there was a significant correlation between the oil assets and manufacturing firms' stocks [4]. What's more, researchers nowadays do most investigations related to portfolio management on the basis of the whole market, just like the portfolio optimization research done by João, Guilherme, and André. To know how to optimize portfolio more efficiently in changing environment, they used Dynamic Factor Models based on the whole US bond market [5]. Naccarato et al. selected 30 stocks which own the highest market value among all real European stocks is also an example [6]. It is concluded that though Markowitz model has developed for many years there are still a lot of problems for its intrinsic and external problems i.e., overreliance on historical data, focus on mean and variance only, over-concentration in certain assets and lack of dynamic adaptation.

This paper selected SPX and 5 outstanding stocks from different areas i.e., technology, consumer goods, financial Services, healthcare and energy to focus on asset allocation. Since relationships between correlations and portfolios is one of the aspects the paper studies, the author ignores the risk-free asset which is uncorrelated with the chosen risky assets [1]. Since many research papers just analyze the impacts brought by constraints based on theory just like William F. Sharpe in his paper 'Capital Asset Prices with and without Negative Holdings' that he analyzed how the forbiddance of short-selling affect whole portfolio with mathematical formulas without accurate statistics listed [7]. This paper used daily data for the past three years to calculate the annualized average return, annualized standard deviation and beta as well as the correlations between all stocks. Then the author adds constraints to Markowitz model to gain minimal variance portfolio, maximal Sharpe ratio portfolio, capital allocation line, minimal variance frontier, efficient frontier. In the end, the author analyzes the performance of these stocks under different constraints. This paper comprises five parts and here is the construction. Section 2 outlines the data used in this paper. Section 3 provides an overview of the methods. Section 4 demonstrates the results while Section 5 concludes the paper.

2 Data

Based on the fact that Yahoo finance provide access to extensive historical data and it also has the advantage of real-time data updates, author get data in this paper from Yahoo Finance (https://finance.yahoo.com/). Five distinguished companies from different sectors and SPX500 are chosen to be analyzed. i.e., NVDA, PG, JPM, JNJ, NEE for closing price from September 11, 2021, to August 26, 2024. To have a closer look at recent market trends, data in the article is calculated based on the daily statistics within three years. The paper transfer the data into annualized average return (AVR) , annualized standard deviation (ASD) , sharp ratio and the β. All values are included in Table 1. The matrix of correlation coefficient for each stock is also calculated, as shown in Table 2.

Table 1. Descriptive statistics of the selected stocks

	SPX	NVDA	PG	JPM	JNJ	NEE
AVR	-0.073%	0.188%	-0.083%	-0.055%	-0.106%	-0.098%
ASD	25.002%	45.020%	37.902%	29.108%	39.279%	40.129%
Sharp	-0.293%	0.418%	-0.219%	-0.190%	-0.271%	-0.243%
β	1.000	0.469	1.350	0.883	1.395	1.246

Table 2. Correlation matrix

	SPX	NVDA	PG	JPM	JNJ	NEE
SPX	100.000%	26.063%	89.070%	75.854%	88.784%	77.642%
NVDA	26.063%	100.000%	5.276%	0.239%	1.190%	5.107%
PG	89.070%	5.276%	100.000%	72.930%	89.002%	76.733%
JPM	75.854%	0.239%	72.930%	100.000%	76.235%	59.449%
JNJ	88.784%	1.190%	89.002%	76.235%	100.000%	74.502%
NEE	77.642%	5.107%	76.733%	59.449%	74.502%	100.000%

It is concluded that all the five stocks have relatively high volatility which means that they are expected to have potential to earn more money than expected but the truth is that they all have actual return lower than expected except for NVDA. Although current situation is not so good as we have thought, β greater than 1 offer investors a positive aspect that as the market go up, it brings higher return. The low correlations between NVDA and others make NVDA a good choice to diversify the portfolio.

3 Method

In this paper, Markowitz model is selected to manage risk and return of the portfolio which is made up of SPX and five distinguished stocks. This model is chosen for reasons, Markowitz model is the main idea which used to build up the optimal portfolio in order to achieve the aim of maximize the return and minimize the risk according to Kamil, Anton, Fei, Chin, Kok and Lee [8].

The model was firstly put forward by Harry Markowitz whose groundbreaking work Portfolio Selection formed the foundation of 'Modern Portfolio Theory' (MPT). The theory was latterly expanded upon by William Sharpe for the Capital Asset Pricing Model [9]. One important concept in Markowitz model is that when an investor is equally ambiguous about all assets, then the optimal portfolio corresponds to Markowitz's fully diversified portfolio [10]. And it is important to know the model is based on complicated language and intricate mathematical expressions. Here is the precondition of Markowitz model.

$$\sum_{i=1}^{n} Weight_i = 1 \tag{1}$$

Where $Weight_i$ represents how much of $asset_i$ accounts for in a portfolio. And the return of portfolio is calculated as the formula demonstrates.

$$portfolio\ return = \sum_{i=1}^{n} Weight_i * asset\ return_i \tag{2}$$

Where the average annual return is denoted by $asset\ return_i$, then the standard deviation,

$$portfolio\ StDev = \sqrt{\sum_{i=1}^{n} Weight_i^2 StDev_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} Weight_i Weight_j StDev_i\, StDev_j\, Cov(i,j)} \tag{3}$$

Where standard deviation of $asset_i$ is denoted by $asset\ StDev_i$, and with $cov(i,j)$ denoting the covariance between $asset_i$ and $asset_j$ which is calculated as correlation efficient as the paper has mentioned. Additionally, the Sharpe ratio which offers an assessment method to investors to optimize their portfolio under certain risk level,

$$SharpeRatio = \frac{E(R_p) - R_f}{\sigma_p} \tag{4}$$

Where $E(R_p)$ represents the expected return, R_f represents the risk-free return, and σ_p represents the standard deviation of the portfolio.

4 Result

In this paper, the author uses four different constraints which may happen in real transactions to measure the portfolio and determine how to maximize the portfolio. The minimum variance portfolio, the maximize Sharpe Ratio portfolio, the minimum variance frontier and the capital allocation line are calculated to tell them from each other. Here are the results in details.

Constraint 1. In this constraint, there is no limit which means that the investors can freely explore the portfolio and have a general sight of the portfolio.

The author has presented the related results of portfolio under constraint 1 in Table 3 and Fig. 1.

Table 3. Weights of each asset when constraint 1 is added

	SPX	NVDA	PG	JPM	JNJ	NEE	Returns	StDev	Sharpe
MinVar	144.77%	1.52%	-30.65%	31.98%	-43.43%	-4.20%	-0.05%	4.16%	-0.0109
MaxSharpe	-64.97%	76.04%	23.71%	87.81%	-26.46%	3.87%	0.15%	12.03%	0.0122

Notes: Minimum variance is denoted by MinVar; standard deviation is denoted by StDev and maximum Sharpe Ratio is denoted by MaxSharpe.

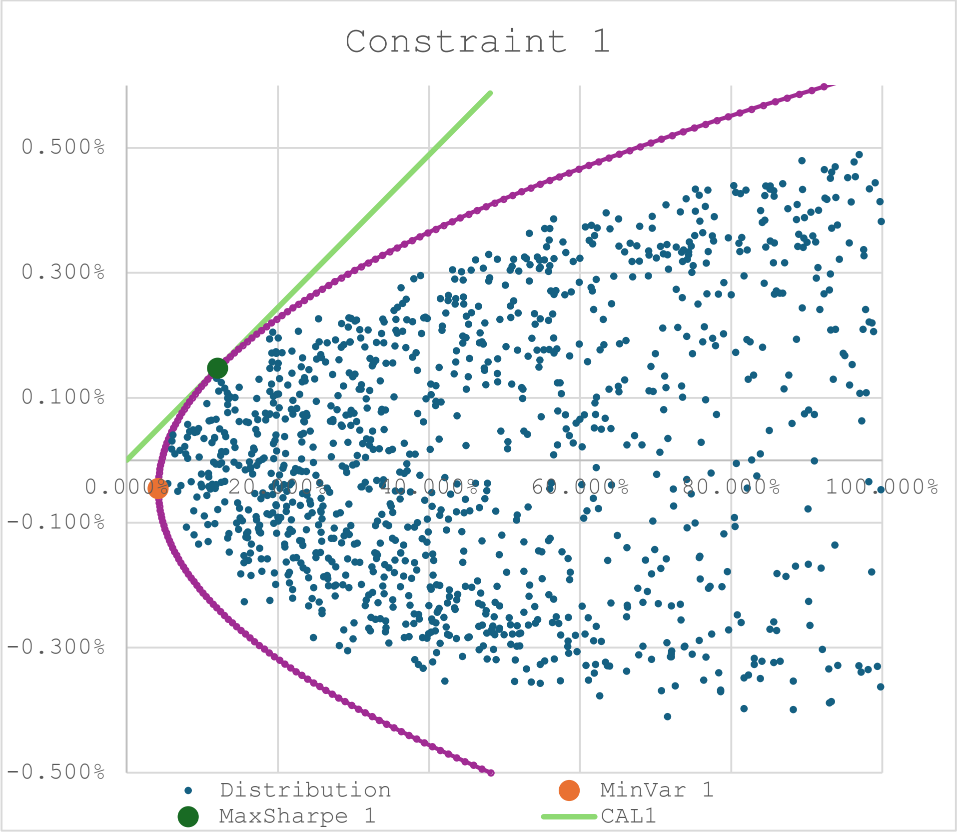

Fig. 1. Minimum variance frontier as well as capital allocation line when constraint 1 is added

To maximize Sharpe ratio, JPM has the maximum weight, SPX has the minimum weight. In this scenario high expected returns with low volatility, and low correlations with other assets is preferred (JPM), SPX behaves badly.

Constraint 2. The investors can ignore the risk brought by market through this constraint and find out the effect of SPX. It also helps the portfolio more global since SPX is highly related to the American market.

$$Weight_{\mathrm{SPX}} = 0 \tag{5}$$

The author has presented the related results of portfolio under constraint 2 in Table 4 and Fig. 2.

Table 4. Weights of each asset when constraint 2 is added

	SPX	NVDA	PG	JPM	JNJ	NEE	Returns	StDev	Sharpe
MinVar	0.00%	28.26%	0.41%	71.18%	-13.18%	13.33%	0.01%	5.85%	0.0025
MaxSharpe	0.00%	65.26%	10.09%	70.19%	-40.95%	-4.58%	0.12%	10.47%	0.0118

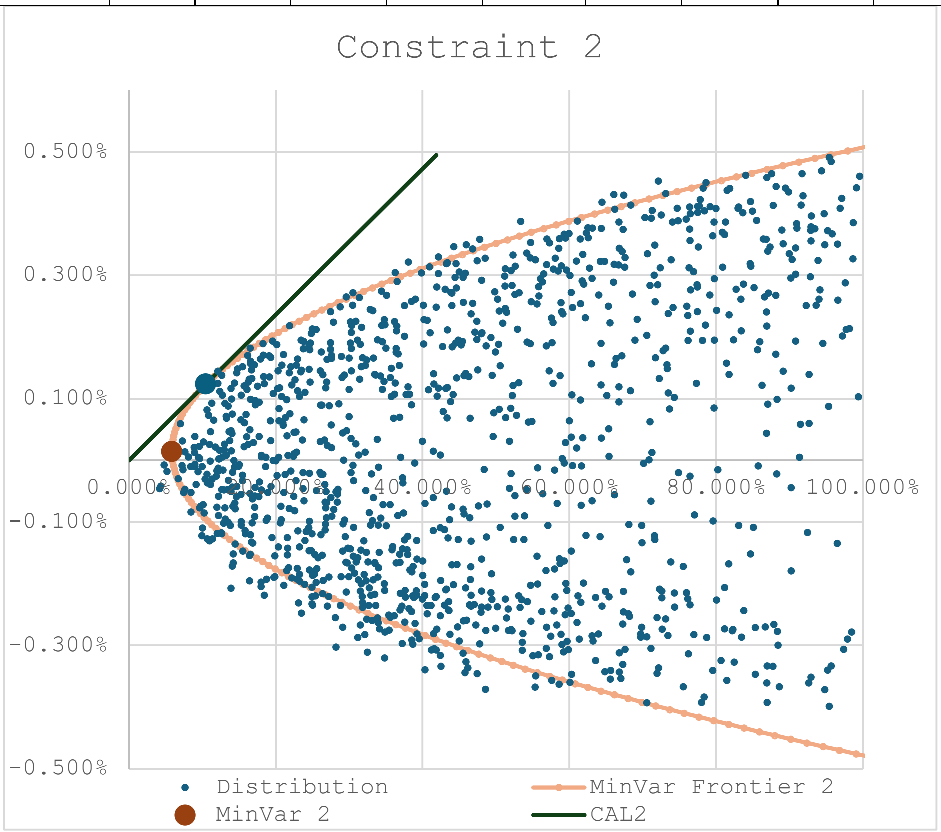

Fig. 2. Minimum variance frontier as well as capital allocation line when constraint 2 is added

To maximize Sharpe ratio, JPM has the maximum weight, JNJ has the minimum weight. It is quite similar to the result of constraint 1 since the only change for the constraint is that SPX is ignored. The stock which behaves only better than SPX(JNJ), take place of the SPX.

Constraint 3. Short selling is not allowed in this case. In real world, many investors are restricted by policies and laws for many reasons 1. Stabilize the market if the confidence of investors is destroyed by overweight long-selling, 2. To maintain the fairness of the market if someone spread false negative information and so on. So, it is common for investors to take the constraint into account.

$$w_i \geq 0, for\ \forall i \quad (6)$$

The author has presented the related results of portfolio under constraint 3 in Table 5 and Fig. 3.

Table 5. Weights of each asset when constraint 3 is added

	SPX	NVDA	PG	JPM	JNJ	NEE	Returns	StDev	Sharpe
MinVar	49.40%	19.35%	0.00%	31.25%	0.00%	0.00%	-0.02%	5.38%	-0.0032
MaxSharpe	0.00%	68.91%	0.00%	31.09%	0.00%	0.00%	0.11%	10.46%	0.0108

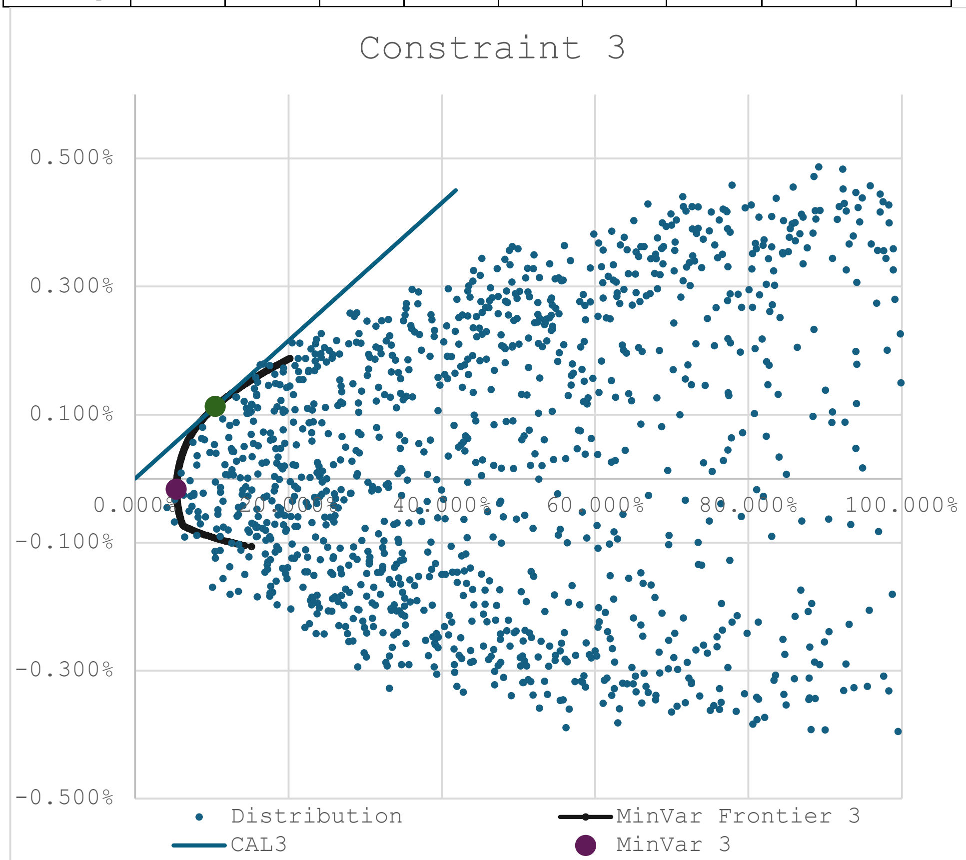

Fig. 3. Minimum variance frontier as well as capital allocation line when constraint 3 is added

To maximize Sharpe ratio, NVDA has the maximum weight, SPX PG JNJ and NEE have the minimum weight (0%). Since short selling is forbidden, stocks behave badly such as SPX, PG, JNJ and NEE cannot have negative weights in the portfolio, they are no longer in investor's consideration.

Constraint 4. In this case, the absolute values of all the stocks cannot greater than one which avoids the overconcentration of investment, and the financial risks brought by overleveraging.

$$|w_i| \leq 1, for\ \forall i \quad (7)$$

The author has presented the related results of portfolio under constraint 4 in Table 6 and Fig. 4.

Table 6. Weights of each asset when constraint 4 is added

	SPX	NVDA	PG	JPM	JNJ	NEE	Returns	StDev	Sharpe
MinVar	100.00%	9.79%	-21.04%	44.10%	-34.07%	1.22%	-0.03%	4.32%	-0.0062
MaxSharpe	-64.97%	76.04%	23.71%	87.81%	-26.46%	3.87%	0.15%	12.03%	0.0122

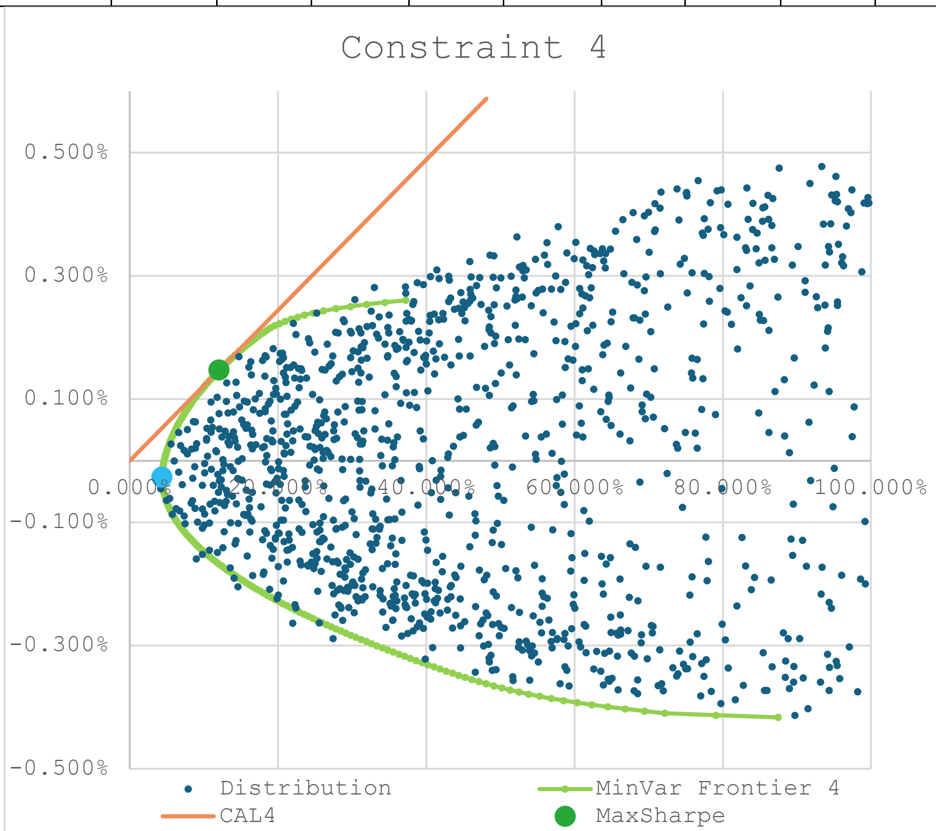

Fig. 4. Minimum variance frontier as well as capital allocation line when constraint 4 is added

To maximize Sharpe ratio, JPM has the maximum weight, SPX has the minimum weight. The weights allocation is same as constraint 1 that the result of constraint 1 already satisfy the condition of constraint 4.

The constraints limit the extreme allocation of portfolio weights, which reduces the potential to leverage or engage in substantial short selling, thereby lowering potential returns and risks. This leads to an overall shift of the efficient frontier to the left and downward, indicating that both risk and return decrease. At the same level of risk, adding constraints results in lower achievable returns.

To minimize the variance SPX always owns the most weights and JPM second to SPX, and when SPX is not allowed to exist the JPX owns the most. It seems reasonable that SPX has the lowest ASD and JPM owns the second lowest (Table 1), what's more, they have relatively low correlation. The efficient frontier contracts due to the imposition of the constraint and constraint 3 has the greatest impact. Additionally, all the constraints have very close CAL.

It can be observed from Fig. 5 that constraint 3 and constraint 4 have great impacts on the efficient frontier. For constraint 3, The investors have no chance to hedge the risks of high-

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volatility assets, and this is why it is limited in a much smaller StDev range. For constraint 4, investors cannot get higher return since the weights of stocks which have high returns are restricted to a certain level. It is interesting to know that CAL1 and CAL2 overlap, it is also proved by the values of MaxSharpe in Table 3 and Table 6. When there is no constraint, all weights of stocks to meet the max sharpe ratio already meet the constraint that the absolute values of all the stocks cannot greater than one (See Fig. 6)

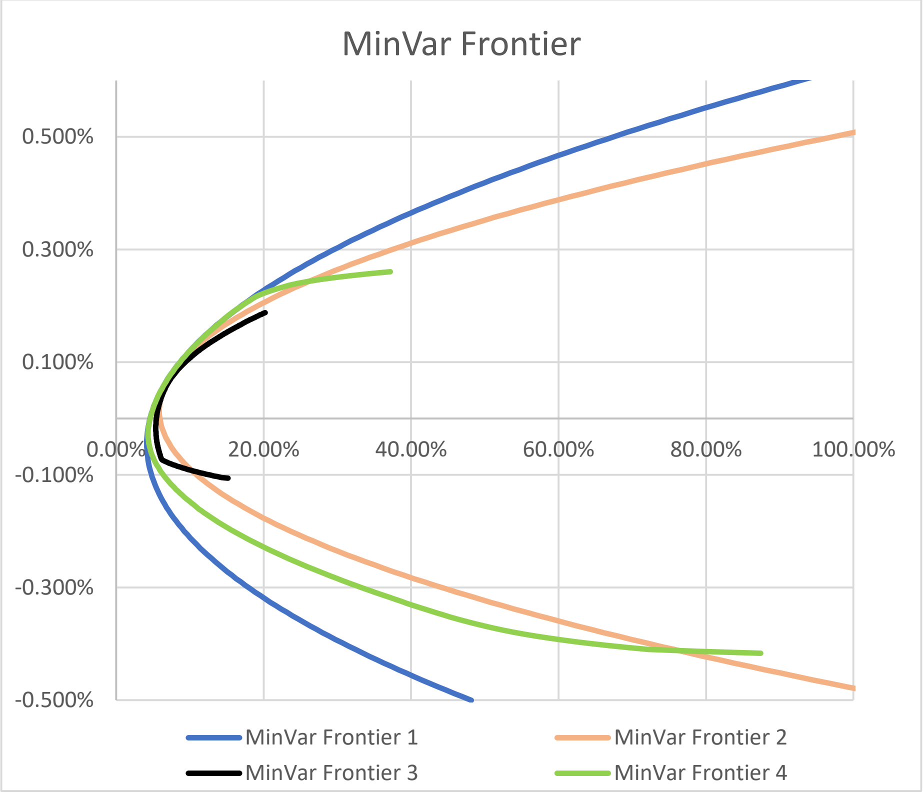

Fig. 5. Combine all frontier curves

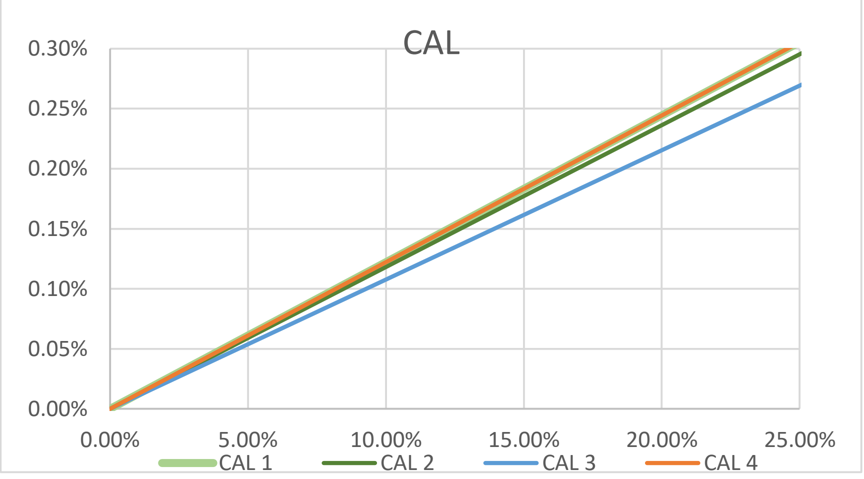

Fig. 6. Combine all CAL curves

5 Conclusions

In current world, investors face market with high volatility and overvaluation and a series of uncertainty. It is wise for investors to diversify their risks by investing in different areas. This paper focuses on technology, consumer goods, financial Services, healthcare and energy to help potential investors to determine their portfolios under different constraints. Correlation coefficients among the six assets in this paper are calculated by statistical analysis, the stocks chosen to have high correlations with each other besides NVDA. Minimum volatility portfolio, minimum variance frontier and maximum Sharpe ratio portfolio calculated by Markowitz model also show the investors have less chances to hedge the risks of high-volatility assets which means they gain lower returns when risks remain unchanged. This paper finds that JNJ is always in the short position if allowed, NVDA and JPM are always in the long position and have high weights. Technology and financial services may be promising prospects.

However, deficiencies also exist. These five stocks and not reflect their areas perfectly, there are still many industries this paper has not taken into account. What's more, Markowitz model relies too much on historical data an ignore dynamic nature of markets. In future study, the author will focus more on decrease the error brought by correlation efficiencies and dynamic markets.

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