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Portfolio Optimization of Stocks in Different Industries by Single-Index Model and Markowitz Model

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Abstract. In the financial sector, portfolio optimization is becoming more and more crucial. This article examines the portfolios of two industries, financial services and technology, in an effort to help prospective investors make decisions about their investments under a variety of constraints. The portfolio with the lowest volatility and the highest Sharpe ratio, and the constraint for the smallest value variance are the three practical restrictions that are paired with the Markowitz model and the Sharpe-single Index model approach to establish the ideal stock portfolios. The outcome demonstrates that, First, a risk-return portfolio can benefit from investing in the S&P index due to its strong correlation with listed companies; second, investing in the S&P index is preferred to hedge risk if short selling is prohibited; and third, it is advantageous to include the S&P index in the investment portfolio. This holds immense importance for the investigation of the most efficient distribution of monetary resources within the financial industry.

1 Introduction

Since the Mean-Variance model was introduced by Markowitz in 1952, portfolio optimization models and portfolio theory have been receiving widespread attention. The risk and return of an asset should be assessed not only on its own merits but also on how it influences the risk and return of the portfolio as a whole, according to contemporary portfolio theory. The standard deviation, or variation of return, is used as a measure of risk since it can be easily calculated when assets are combined into portfolios [1]. Values that indicate the forecast for the future are often represented by historical variance and covariance of returns [2]. Furthermore, portfolio modelling is essential in a world where anybody can invest since it presumes that the goal of the investor is to optimize the expected return of the portfolio at a certain degree of risk.

The fact that so few publications concentrate on stock portfolios in various niche industries is one of the primary issues identified after reviewing a significant number of portfolio optimization research. The majority of today's portfolio management research is grounded in either the Mean-Variance model's theoretical underpinnings or the market as a whole. For example, Sadeghi et al. used Markov models in conjunction with hierarchical analysis (AHP), the Technique of Ordering Similarity Preferences for Ideal Solutions

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(TOPSIS), and other methods to model optimal stock portfolio selection [3]. In addition, Mohammadi et al. proposed a DEA approach based on Markowitz modeling of business unit segmentation to improve the lack of fast computation in securities analysis [4]. Moreover, Uberti suggested a refinement of the Markowitz model that combines the traditional mean-variance allocation paradigm with skewness and kurtosis [5]. In Çaliskan's study, the daily updated prices of seventeen firms listed on the ISE between 2003 and 2009 were among the data used to evaluate the Black Litterman model and the Markowitz model with respect to firm-specific risk, and beta factor [6]. Ortiz uses a cutting-edge computational method to make the Markowitz model's ideal weights more volatile [7].

The asset allocation of five exemplary businesses from the financial services and technology industries is the main topic of this essay. The precise method of investigation is as follows. First, for closing price analysis, five equities from the financial services and technology industries during the previous 20 years have been chosen; second, the initial data is filtered to exclude monthly prices; third, Excel is utilized to find the five stocks' correlation. as well as the annualized mean return, annualized standard deviation, beta, alpha, and residual standard deviation of the Sharpe single-index model; fourth, three different constraints are added to construct a portfolio consisting of a minimal variance portfolios, maximum Sharpe ratio portfolios, capital allocation lines, minimum variance bounds, efficient bounds, and inefficient bounds for the Markowitz model and the index model; fifth, These five equities are evaluated based on their performance using the Excel charting feature under various restrictions within the identical model as well as The way these stocks have performed within the same limitations under alternative models. This paper can help prospective investors make investment decisions under various constraints by analyzing the equities of financial services and technology companies in a portfolio.

The paper is organized as follows. While Section 3 provides the methodology, Section 2 displays the data. The conclusion is found in Section 5, whereas Section 4 discusses the empirical findings.

2 Data

This article uses data from Yahoo Finance. The SPX500 and five exemplary companies from the financial services and technology sectors are chosen for this article. i.e., closing prices for ADBE, IBM, SAP, BAC, and C between May 11, 2001, and May 12, 2021. The annualized average return (AAR) and standard deviation (ASD) of Sharpe's mono-index model, as well as the beta, alpha and residual standard deviation (RSD) are calculated as presented in Table 1. The matrix of correlation coefficients for each stock is also calculated as displayed in Table 2.

Table 1. Specific details for the chosen stocks

	SPX	ADBE	IBM	SAP	BAC	C
AAR	7.542%	19.578%	4.754%	11.996%	11.101%	1.032%
ASD	14.850%	31.790%	23.181%	33.909%	39.338%	42.470%
β	1.000	1.423	1.014	1.483	1.595	2.007
α	0.000%	8.847%	-2.892%	0.811%	-0.933%	-14.103%
RSD	0.000%	23.753%	17.627%	25.784%	31.403%	30.260%

Table 2. Correlation matrix

correlations	SPX	ADBE	IBM	SAP	BAC	C
SPX	100.0%	66.5%	64.9%	64.9%	60.2%	70.2%
ADBE	66.5%	100.0%	45.5%	53.4%	42.3%	46.3%
IBM	64.9%	45.5%	100.0%	58.5%	31.3%	42.0%
SAP	64.9%	53.4%	58.5%	100.0%	33.1%	43.4%
BAC	60.2%	42.3%	31.3%	33.1%	100.0%	82.6%
C	70.2%	46.3%	42.0%	43.4%	82.6%	100.0%

As can be seen from these two tables, the annualized mean of ADBE is highest, whereas the annualized mean of C is lowest. At the same time, the S&P 500 has the smallest annualized standard deviation relative to the largest annualized standard deviation for C. Only ADBE and SAP have alpha values above 0. In addition, BAC has the smallest correlation coefficient with IBM, while BAC has the largest correlation coefficient with C.

3 Methodology

This paper utilizes the Single Index Model and the Markowitz Model for portfolio management of a selected S&P 500 index and five representative stocks.

3.1 Markowitz model

Mean-variance analysis, or modern portfolio theory (MPT), is a mathematical framework that, for a given amount of risk, optimizes expected return. It can be used to assemble an asset portfolio. It is a development and formalization of the theory behind investment diversification, which states that having a range of financial assets rather than just one kind lowers risk. In a 1952 essay, economist Markowitz introduced MPT [1]. In the framework of proportionate reinsurance, Finetti developed the mean-variance analysis approach with a stronger assumption in 1940 [8]. Furthermore, the "critical line method" was created by Markowitz and is a general process for quadratic programming that can manage upper and lower holdings limits in addition to additional linear restrictions. Furthermore, inside this framework, the methodology offers a way to ascertain the full collection of effective portfolios [9].

$$\sum_i w_i = 1 \tag{1}$$

Where w_i represents for the stock's weight in the portfolio

The MM model portfolio's anticipated return p is calculated as follows.

$$\mathrm{R_P} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{w_i r_i} \tag{2}$$

The portfolio's standard deviation p is shown below.

$$\sigma_\mathrm{P} = \sqrt{\sum_{\mathrm{i=1}}^{\mathrm{n}} \sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{w_i w_j cov(R_i, R_j)}} \tag{3}$$

Where i denotes the entire amount of assets, w_i is the percentage of assets i in the portfolio, and $cov(R_i, R_j)$ is the covariance between the returns on asset i and asset j.

3.2 Single index model

In 1963, Sharpe developed the single index model (SIM) [10]. Sharpe discovered that an index of market, like the S&P 500 Index, can be used to more easily find the correlation between each pair of stock returns to ascertain the covariance, or how two variables differ, or the correlation, or the relationship between two variables, as the Markowitz model suggests. As a result, less computations are required during a portfolio analysis exercise. The portfolio of the IM model's anticipated return p is shown below.

$$R_P = \sum_{i=1}^{n} w_i r_i \tag{4}$$

The portfolio Standard Deviation p is calculated as follows.

$$\sigma_P = \sqrt{(\sum_{i=1}^{n} w_i \beta_i)^2 \sigma_M^2 + \sum_{i=1}^{n} w_i^2 \sigma_\varepsilon^2} \tag{5}$$

Where r_i stands for the asset's anticipated rate of return, w_i stands for the portfolio's proportion of asset i, n is the total quantity of assets i, β_i is an asset's risk factor, σ_M is the systematic risk and σ_ε is the unsystematic risk.

3.3 Comparison objective

The Sharpe Ratio, which is also employed as an index to determine the attribute of an asset, is one of the three conventional signs that can take return and risk into account at the same time. The indicator calculates the extra return per unit of risk. The formula that follows can be applied to compute it.

$$SharpeRatio = \frac{E(R_p) - R_f}{\sigma_p} \tag{6}$$

Where $E(R_p)$ is the portfolio's anticipated return, R_f is the risk-free rate, and σ_p is the portfolio's standard deviation.

Frontier of Minimal Risk or Variance:

$$\begin{cases} \sigma(w) \rightarrow min \\ subject\ to: r(w) = const \end{cases} \tag{7}$$

Frontier of Minimal Return:

$$\begin{cases} r(w) \rightarrow min \\ subject\ to: \sigma(w) = const \end{cases} \tag{8}$$

Efficient Frontier: All the points on the minimal variance boundaries that are above the minimal variance portfolio can be considered the optimal portfolio since they provide the best risk and return.

$$\begin{cases} r(w) \rightarrow max \\ subject\ to: \sigma(w) = const \end{cases} \tag{9}$$

Global Minimal Risk Portfolio:

$$\sigma(w) \rightarrow min \tag{10}$$

Efficient Risky Portfolio: the intersection between the CAL and the efficient frontier with the lowest variance and the maximum return, it has Maximal Sharpe Ratio as follows.

$$\frac{r(w)}{\sigma(w)} \rightarrow max \tag{11}$$

3.4 Constrains

Constrain 1. To illustrate the overall scope of permitted portfolios and the efficient frontier in particular appears when there are no restrictions, consider a "free" problem that has no additional optimization constraints.

Constrain 2. This extra optimization restriction is meant to mimic the common restrictions seen in the US mutual fund market: no short positions may be held by US open-ended mutual funds, See Section 12(a)(3) of the Investment Company Act of 1940 for further information (http://www.law.cornell.edu/uscode/text/15/80a-12).

$$w_i \geq 0 \text{ , for } \forall i \tag{12}$$

Constrain 3. Finally, investor may want to examine when incorporating the broad index to our portfolio has a beneficial or detrimental impact. In light of that, investors consider accounting one more optimization limitation.

$$w_1 = 0 \tag{13}$$

This implies that the S&P 500 has a weight of zero.

4 Results

In this study, portfolios are designed under three limitations using Markowitz model and Single index model, and calculated minimum variance bounds, capital allocation lines, Portfolios with greatest Sharpe ratios and minimum variance to discover the distinction.

As Fig. 1 and Fig. 2 show, the unconstrained portfolio outperforms the limited portfolio on the highest Sharpe portfolio, the lowest variance portfolio and the minimal variance frontier, because unconstrained portfolios have higher returns than constrained portfolios for the same variance. In addition, the frontier of lowest variance for constraint 2 is located to the right of constraint 1's, as seen both in Fig. 1 and Fig. 2. , this means that constraint 2 is more risky for the same rate of return. Because certain bad assets cannot be shorted by investors, this results in investors having to take on greater risk stemming from distressed assets. It can also be seen from Fig. 1 and Fig. 2 that the curve for constraint 3 has been located to the right of the constraint curve 1, which indicates that it is advantageous to include the S&P index in the portfolio which will deliver greater returns with the same variance. If short sales are prohibited, it is preferable to make an S&P index investment as a risk hedge, this is because when short selling is prohibited, both models show that full investment in the S&P index leads to high returns with minimal risk, as shown by Tables 3 and 4. Additionally, as Tables 3 and 4 demonstrate for Constraint 3, the MM and SIM models yield comparable results for these two points. Both models show that the return of constraint 3 is higher than constraint 1 at the same time the risk of constraint 3 is higher than constraint 1 when the portfolio has the minimum variance and the maximum Sharpe ratio, this means that the portfolio without the index will have higher returns than the portfolio with the index, but it will also take more risk.

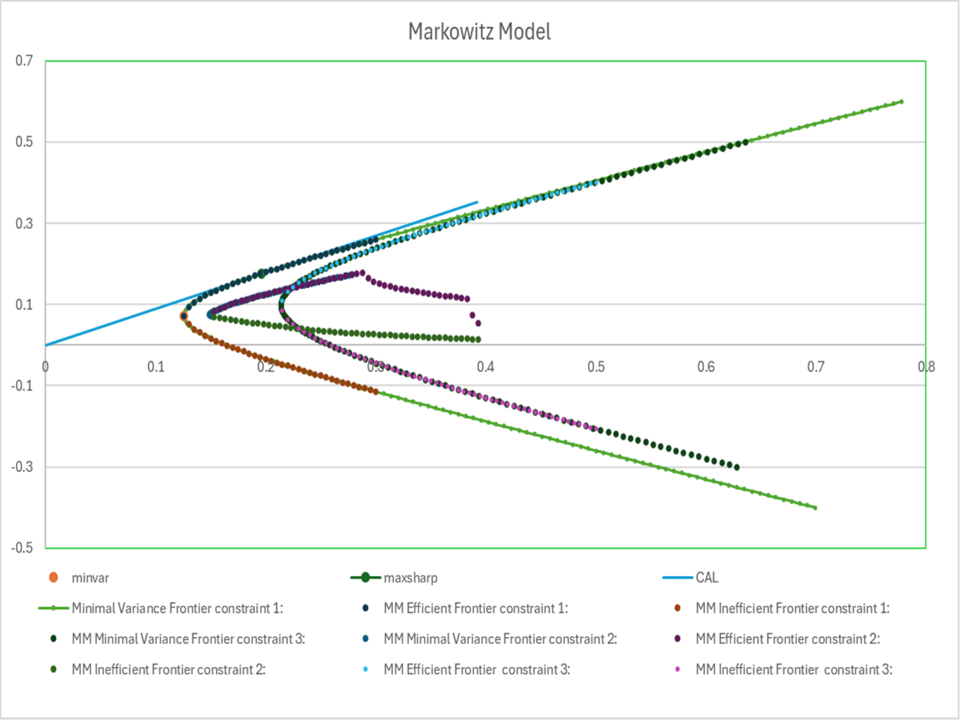

Fig. 1. Effective frontiers for the Markowitz model under various conditions

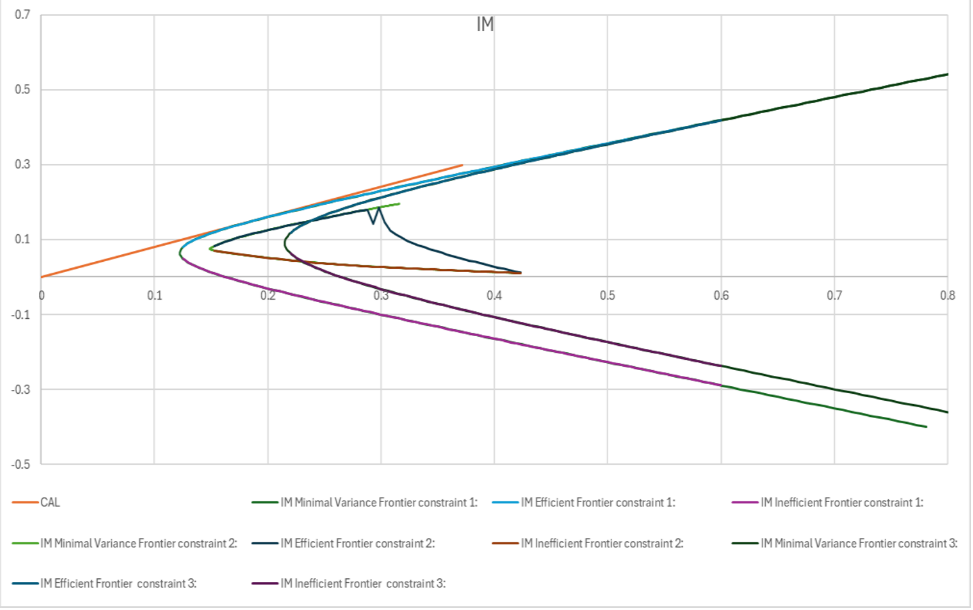

Fig. 2. Effective frontiers for the index model under various constraints

Table 3. Markowitz model portfolio performance

Markowitz Model (Constraint1):	Return	Standard Deviation		Sharpe	SPX500
MinVar	7.27%	12.60%		57.68%	134.70%
MaxSharpe	17.58%	19.60%		89.72%	110.60%
Markowitz Model (Constraint2):	Return	Standard Deviation		Sharpe	SPX500
MinVar	7.54%	14.85%		50.79%	100.00%
MaxSharpe	14.39%	22.84%		62.98%	43.13%
Markowitz Model (Constraint3):	Return	Standard Deviation		Sharpe	SPX500
MinVar	9.72%	21.41%		45.40%	0.00%
MaxSharpe	31.05%	38.27%		81.13%	0.00%
Markowitz Model (Constraint1):	ADBE	IBM	SAP	BAC	C
MinVar	-10.22%	2.99%	-10.87%	4.37%	-20.97%
MaxSharpe	31.92%	-21.72%	3.27%	38.53%	-62.59%
Markowitz Model (Constraint2):	ADBE	IBM	SAP	BAC	C
MinVar	0.00%	0.00%	0.00%	0.00%	0.00%
MaxSharpe	56.87%	0.00%	0.00%	0.00%	0.00%
Markowitz Model (Constraint3):	ADBE	IBM	SAP	BAC	C
MinVar	17.89%	72.75%	1.52%	24.82%	-16.97%
MaxSharpe	98.01%	-7.55%	27.26%	90.66%	-108.39%

Table 4. Portfolio performance of Index model

Index Model (Constraint1):			Return	Standard Deviation	Sharpe	SPX500
MinVar			6.47%	12.25%	52.87%	148.33%
MaxSharpe			14.88%	18.57%	80.17%	120.28%
Index Model (Constraint2):			Return	Standard Deviation	Sharpe	SPX500
MinVar			7.54%	14.85%	50.79%	100.00%
MaxSharpe			14.74%	23.37%	63.06%	37.37%
Index Model (Constraint3):			Return	Standard Deviation	Sharpe	SPX500
MinVar			9.03%	21.50%	41.98%	0.00%
MaxSharpe			26.56%	36.89%	72.01%	0.00%
Index Model (Constraint1):	ADBE	IBM	SAP		BAC	C
MinVar	-11.24%	-0.66%	-10.89%		-9.05%	-16.49%
MaxSharpe	36.31%	-21.56%	2.83%		-2.19%	-35.67%
Index Model (Constraint2):	ADBE	IBM	SAP		BAC	C
MinVar	0.00%	0.00%	0.00%		0.00%	0.00%
MaxSharpe	58.11%	0.00%	4.52%		0.00%	0.00%
Index Model (Constraint3):	ADBE	IBM	SAP		BAC	C
MinVar	18.46%	66.66%	13.39%		6.16%	-4.68%
MaxSharpe	111.21%	-7.71%	33.58%		14.98%	-52.05%

Notes: The terms "minVar" and "maxSharpe" stand for the least and maximum Sharpe Ratio, respectively.

5 Conclusion

Currently, the majority of portfolio studies are founded on analyses of certain sectors or overall market conditions. This article does a portfolio study of two different industries, financial services and technology, in an effort to help potential investors make investment decisions under varied limitations. The portfolio with the least amount of volatility, the portfolio with the highest Sharpe ratio, and the least variance bound are all constructed utilizing the Markowitz model and the Sharpe-single Index model to locate the most outstanding portfolios for these equities. Two models are further enhanced with three practically relevant constraints in order to investigate the effects of real financial market factors on investment portfolios. This study concludes that, in the event that short sales are forbidden, investment in the S&P 500 can be utilized as a risk hedge. The S&P 500 is also a solid option for a risk-return portfolio.

On the other hand, there are drawbacks. For instance, the selection is narrow, with only a few representative companies from each of the two distinct industries. Furthermore, the fewer constraints for the two models are not very comprehensive.

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