Research and Empirical Analysis of Optimal Virtual Currency Investment Portfolio Based on Markowitz Investment Model

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Abstract. Virtual currency assets are an important component of the international investment market. This study applies the Markowitz theory model to the data of the four main virtual currency assets in the market, Bitcoin, Tether, Ethereum, and BNB, in the past year. The Markowitz theory model is used to quantitatively analyze these four virtual currency assets and obtain the short-term data distribution of the investment portfolio when considering investing in these four virtual currency assets at the same time. In the short term, this study recommends investors who are concerned about returns to short Tether, Ethereum, and BNB virtual currencies and invest in Bitcoin and BNB virtual currencies. It is recommended that investors who are concerned about both returns and risks to short Tether, Ethereum, and BNB virtual currencies and invest in Bitcoin or BNB virtual currencies. Risk assets. This study only focuses on short-term data from the past year and provides recommendations. With the development of virtual currencies, market conditions may vary.

1 Introduction

The Markowitz theory model is a theoretical model developed based on Markowitz's 1952 article "Portfolio Selection - Effective Diversification of Investments" published in the Financial Journal [1]. Brealey pointed out in his article "Harry M. Markowitz's Contributions to Financial Economics" that Markowitz's theory has made significant contributions to the field of portfolio investment theory [2]. The Markowitz theory uses the covariance between investment assets to conduct risk analysis of investment portfolios, providing investors with a more accurate theoretical basis for investment. The Markowitz theory model focuses on both the return and risk of investment portfolios, which can better analyze portfolio optimization problems.

Virtual currency emerged in 2008. Wang et al. pointed out that as a new type of asset, it is not closely related to other assets in the market and is suitable for portfolio investment together with other assets to reduce the overall risk of investment portfolios [3]. Dibrova also indicates that virtual currency is an asset with great potential for development [4]. Virtual currency currently holds a very important position in the investment market. As of August 31, 2024, the total market value of virtual currencies in the market exceeds two

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trillion US dollars, making it an important component of the investment market [5]. Sauer pointed out in "Virtual Currencies, the Money Market, and Monetary Policy" that the popularity of virtual currencies is due to their resistance to authoritative monetary policies and their ability to serve as substitutes when some currencies encounter problems [6]. Antipova pointed out that the returns of virtual currencies fluctuate greatly, and investing in virtual currencies is like using a double-edged sword. Virtual currencies can provide investors with the expected returns, but also expose them to significant risks [7]. Therefore, analyzing virtual currency investment portfolios and providing more scientific theoretical guidance is important for investors.

2 Methodology and Data

2.1 The basic model: Markowitz model

The Markowitz model is a theoretical model derived from the Markowitz theory in Harry Markowitz's 1952 paper "Portfolio Selection". This model applies theoretical knowledge from probability and analysis, and plays an important role in solving problems such as optimal portfolio analysis. It can quantitatively analyze investment portfolios. The Markowitz model mainly focuses on portfolio data, including expected returns and portfolio risks. The Markowitz theory also suggests that the more projects a portfolio contains, the lower the risk of the portfolio.

In this study, Markowitz theory and models will be used to calculate the investment proportion of the required highest return investment portfolio, lowest risk investment portfolio, and maximum risk compensation investment portfolio, and provide investment recommendations.

2.2 Data analysis

Establishing a preliminary Markowitz theory data model. This paper will apply R programming language to analyze and study the problem.

2.2.1 Expected return

The theoretical formula for the expected return of an investment portfolio is as follows:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i) \tag{1}$$

In formula (1), $E(R_p)$ is the expected return of the investment portfolio, w_i is the investment proportion of the i-th asset, and $E(R_i)$ is the expected return of the i-th asset.

2.2.2 Risk

The risk theory formula for investment portfolios is as follows:

$$\partial^2 = \sum_{i=1}^{n} w_i^2 \partial_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \partial_{ij} \tag{2}$$

In formula (2), ∂^2 is the variance of the investment portfolio, ∂_i^2 is the variance of the i-th asset, w_i is the investment proportion of the i-th asset, and ∂_{ij} is the covariance between asset *i* and asset *j*

2.2.3 Sharpe ratio

The theoretical formula for the Sharpe ratio, the risk compensation ratio of an investment portfolio, is as follows:

$$Sharpe\ ratio = \frac{E(R_p) - E(R_f)}{\partial_p} \tag{3}$$

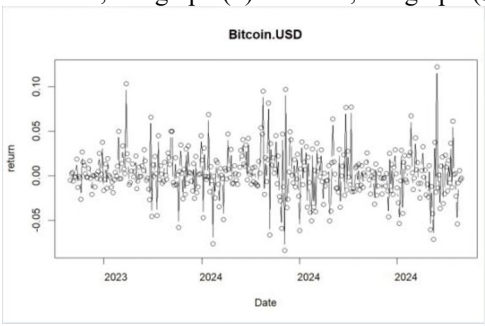
In this formula, $E(R_p)$ is the expected return of the investment portfolio, $E(R_f)$ (Assuming the annualized interest rate of R_f is 0.02) is the expect return of the risk free asset, ∂_p is the standard deviation of the investment portfolio.

3 Result

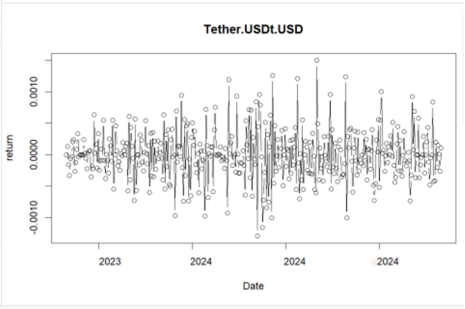
3.1 Analysis results of each virtual currency assets

3.1.1The historical return changes of each virtual currency asset

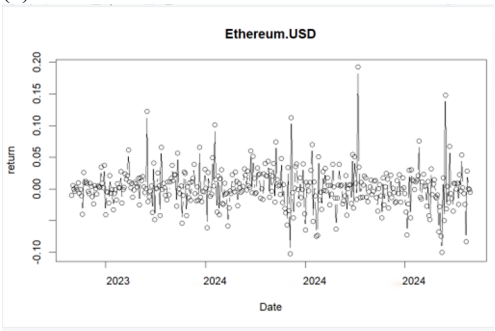
The sequence from the first figure to the fourth figure is as follows: Subgraph (1) of Figure 1 ---Bitcoin, Subgraph (2)---Tether, Subgraph (3) ---Ethereum, Subgraph (4) ---BNB.



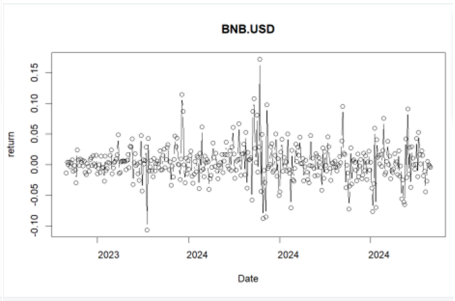
(1) Historical Return Chart of Bitcoin



(2) Historical Return Chart of Tether



(3) Historical Return Chart of Ethereum



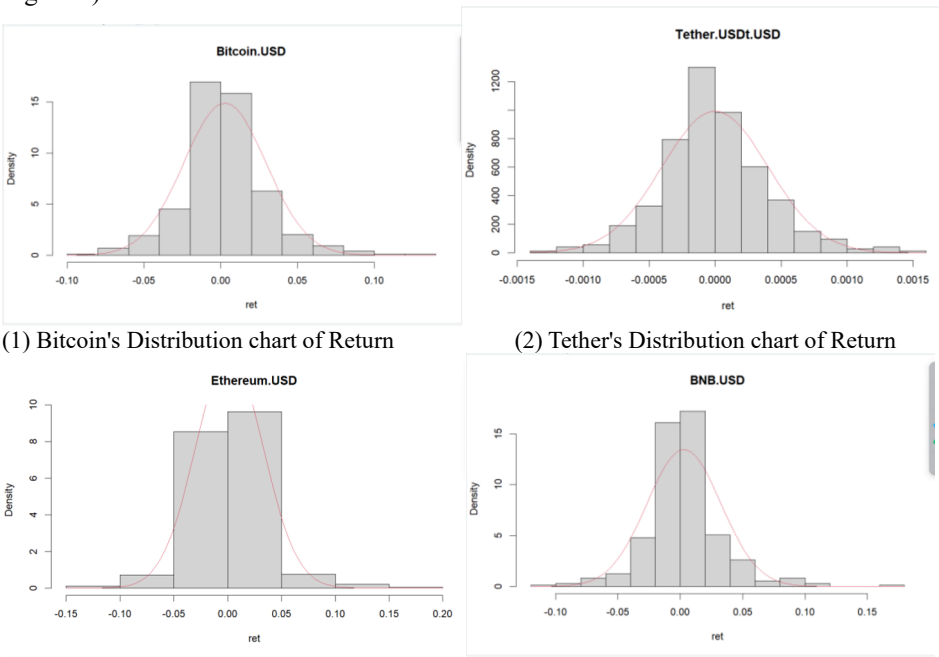
(4) Historical Return Chart of BNB

Fig. 1. Historical Return Chart.

From Subgraph (1), (2), (3), and (4), it can be seen that the historical return data of these four virtual currencies are relatively stable from a macro perspective, with the reported data roughly converging towards a horizontal line. Among them, Tether's historical return data fluctuates more over time, while Ethereum and BNB's historical return data fluctuate less over time and have more stable returns.

3.1.2 Distribution analysis of each virtual currency asset

The historical return distribution chart of each virtual currency asset is as follows (see Figure 2):



(1) Bitcoin's Distribution chart of Return (2) Tether's Distribution chart of Return

(3) Ethereum's Distribution chart of Return (4) BNB's Distribution chart of Return

Fig. 2. Distribution chart.

Figures 2(1) to 2(4) reflect the historical return distribution of four virtual currency assets, where the red lines in Figures 2(1) to 2(4) are the distribution curves of the standard normal distribution. From the four figures, it can be seen that the historical return distribution of these four virtual currency assets generally conforms to the law of normal distribution.

Table 1. Empirical rule verification table for each virtual currency asset

Virtual currency asset	Expected return	Standard deviation	Within 1 SD	Within 2 SDs	Within 3 SDs
Bitcoin	0.0026	0.0268	0.7677	0.9398	0.9863
Tether	0	0.0004	0.7377	0.9426	0.9863
Ethereum	0.0016	0.0313	0.7814	0.9398	0.9808
BNB	0.0029	0.0296	0.7923	0.9426	0.9808

From Figure 2(1) to2(4) and table 1, it can be observed that the distribution of each virtual currency asset is roughly normal, and it can be roughly seen that the distribution of Figure 2(3), which is Ethereum, is more concentrated and has a higher kurtosis, while the distribution of Figure 2(2), which is Tether, is more scattered and has a lower kurtosis. This corresponds to the data results in section 3.1.1.

3.1.3 Basic statistical data of each virtual currency asset

Table 2. Basic statistical data table of each virtual currency asset

Virtual currency assets	Expected return	Standard deviation	Variance	Median	Minimum value	Skewness	Kurtosis	Sharpe Ratio
Bitcoin USD	0.0026	0.0268	0.0007	-0.0009	-0.08343	0.5053	5.4774	0.0348
Tether USDt USD	0.0000	0.0004	0.0000	0.0000	-0.0013	0.2845	4.1270	-4.1348
Ethereum USD	0.0016	0.0313	0.0010	-0.0014	-0.1024	0.8925	8.8355	-0.0008
BNB USD	0.0029	0.0296	0.0009	-0.0014	-0.1070	0.7511	7.6889	0.0414

From the table 2, it can be seen that in terms of expected returns for individual assets, Tether has the lowest expected return value, while BNB has the highest expected return value. In terms of risk for individual assets, BNB has the highest risk, Tether has the lowest risk, and BNB has the highest risk compensation for individual assets.

The skewness of all four virtual currency assets is positive, so it can be concluded that the historical reporting distribution of all four virtual currency assets is skewed to the right, but their skewness is relatively small. This also corresponds to the result of 1.2, and their distribution roughly follows the normal distribution pattern. In terms of kurtosis of individual assets, BNB has the highest kurtosis, the most concentrated data, and the most stable returns.

Therefore, if investing only in a single virtual currency without considering risk-free assets, BNB has the greatest investment potential. If investors are extremely sensitive to risk, Tether is the best investment target. However, in real investment, risk-free assets have a significant impact on market investment, as Ganzach and Wohl pointed out, the interest rate of risk-free assets has a certain impact on the market demand for risky assets [8]. And investing in more assets can effectively reduce investment risks. According to the Markowitz model, it can be observed that the specific risk of an investment portfolio decreases as the portfolio increases [1]. Therefore, the above results cannot provide effective recommendations for virtual currency investment

3.1.4 Correlation and Independence Testing between Each Virtual Currency Asset

From Table 3 and Table 4, it can be seen that the relationship coefficients between the four virtual currency assets are not close to zero, which means that the four virtual currency assets are not independent of each other and have a certain correlation, that is, the historical return data of each virtual currency is related to the historical return data of other virtual currencies.

Table 3. Covariance table between each virtual currency asset

	Bitcoin.USD	Tether.USDt.USD	Ethereum.USD	BNB.USD
Bitcoin.USD	10.3571	0.0483	9.6817	6.8323
Tether.USDt.USD	0.0483	0.0023	0.0405	0.0382
Ethereum.USD	9.6817	0.0405	14.1757	7.7611
BNB.USD	6.8323	0.0381	7.7610	12.6545

Table 4. Correlation table between each virtual currency asset

	Bitcoin.USD	Tether.USDt.USD	Ethereum.USD	BNB.USD
Bitcoin.USD	1.0000	0.3106	0.7990	0.5967
Tether.USDt.USD	0.3106	1.0000	0.2228	0.2219
Ethereum.USD	0.7990	0.2228	1.0000	0.5794
BNB.USD	0.5967	0.2219	0.5794	1.0000

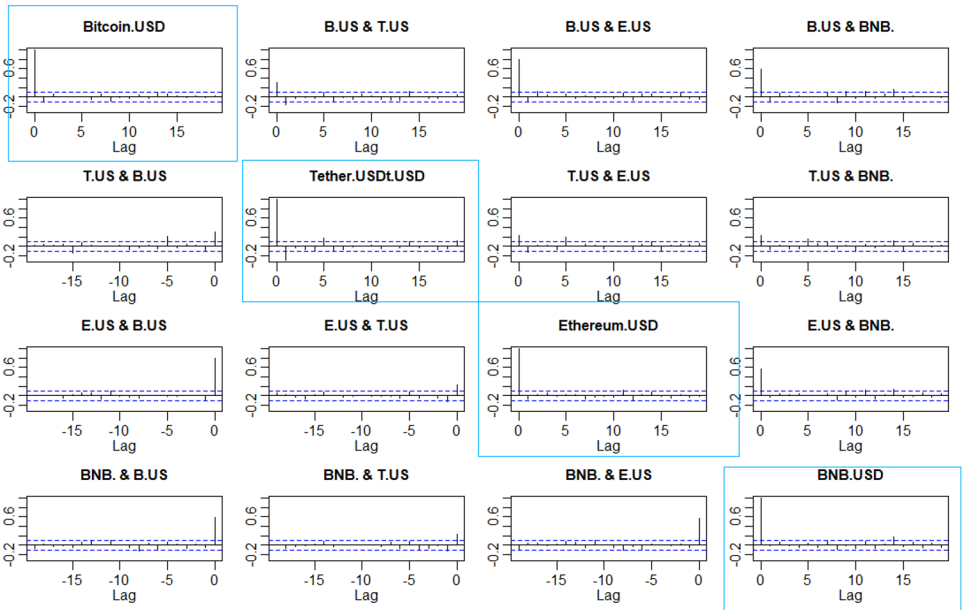


Fig. 3. Figure of independence test results for each virtual currency asset

From Figure 3, it can be seen that there are data beyond the specified range in the independence test chart of Tether and BNB. Therefore, it can be seen that the four virtual currencies are not completely independent of each other, which corresponds to the results of Table 3 and Table 4.

3.2 Virtual currency investment portfolio results

3.2.1 Portfolio establishment results

Using formulas (1) and (2), exhaustively combine four virtual currency assets and risk free assets according to different proportions, and label the expected return and standard deviation of each investment portfolio in the graph to obtain the scatter plot of the investment portfolio with respect to expected return and standard deviation.

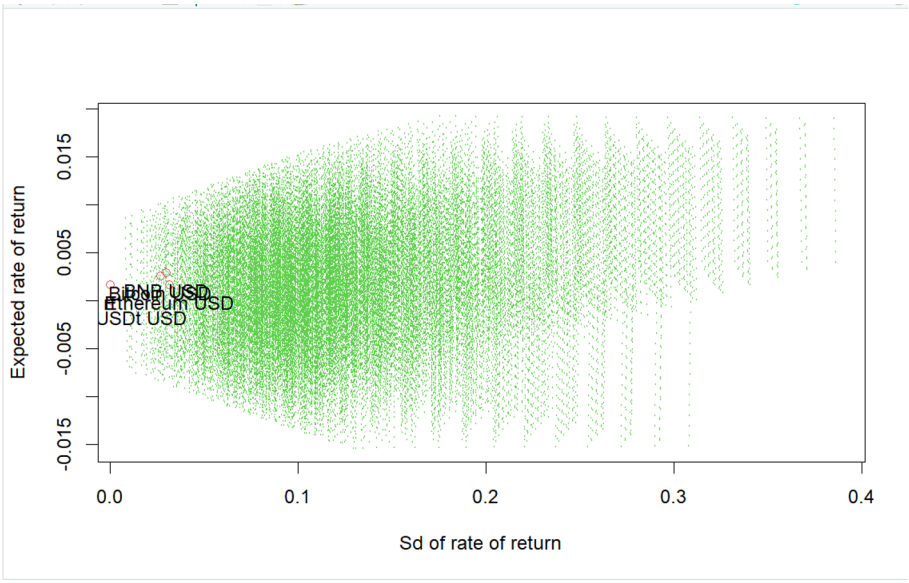


Fig. 4. Scatter distribution chart of investment portfolio

The green dots in scatter plot Figure 4 represent investment portfolios, which are obtained by combining four virtual currency assets and risk-free assets in different proportions, while the black dots represent investment portfolios obtained by investing only in a single virtual currency asset. From the graph, it can be seen that under the same expected return conditions, there is always a portfolio with lower risk than investing in a single virtual currency asset. Under fixed risk conditions, there are always investment portfolios that yield higher returns than investing in a single virtual currency asset.

Therefore, finding the optimal virtual currency investment portfolio can provide more useful advice for virtual currency investors.

3.2.2 Evaluation of portfolio investment proportion

By comparing the various data of the exhausted investment portfolio with those of other investment portfolios, we obtain the proportion of each asset in each investment portfolio under the following conditions: 1) focusing only on investment return rate, 2) focusing only on investment risk, and 3) paying attention to both investment return rate and investment risk.

Let w1, w2, w3, w4, and w5 be the proportions of Bitcoin, Tether, Ethereum, BNB, and risk-free assets in the investment portfolio, respectively (see Table 5).

Table 5. Optimal Investment Portfolio Data Table under Three Conditions

Investment portfolio	W1	W2	W3	W4	W5	Expected return	SD	Sharp ratio
Maximum expected return	5	-4	-4	5	-1	0.0193	0.1755	0.1002
Minimum risk	0	0	0	0	any	0.0017	0	NA
Maximum Sharpe Ratio	0.5	-4	-0.1429	-0.1429	4.7857	0.0074	0.0598	0.8238

When conducting an exhaustive analysis of the investment portfolio, we set the investment upper and lower limits for each asset to be 5 and -4, with an initial capital of 1.

In the distribution of the proportion of the maximum return investment portfolio, the proportion of Bitcoin and BNB investment portfolios has reached the upper limit, while the proportion of Tether and Ethereum investment portfolios has reached the lower limit. These data indicate that when establishing a maximum return investment portfolio, it is advisable to empty Tether and Ethereum as much as possible and invest funds in Bitcoin and BNB as much as possible to achieve the highest return.

For the lowest risk investment portfolio, due to the inclusion of risk-free assets, if we want to minimize risk, we can only invest in risk-free assets.

For the highest Sharpe ratio investment portfolio, this portfolio has the highest risk compensation, which means that investing in this portfolio can maximize our investment risk return. This is also the portfolio we are looking for that is most suitable for investors to invest in in the short term.

The Markowitz theoretical model has some flaws. According to Janková, compared with postmodern investment portfolio theory, the Markowitz theoretical model has certain shortcomings [9]. Guo also pointed out in his research "Review of Research on Markowitz Models in Portfolios" that the Markowitz theoretical model has certain flaws [10]. So the optimal investment portfolio in the study is not necessarily the "best".

4 Conclusion

By using the Markowitz theory model, investors can better analyze the recent short-term performance of virtual currencies. Based on the above results, it can be seen that the development of virtual currencies has not been very good recently, with low returns and a relatively stable overall historical return of the currency. However, in a shorter time period, there will be significant changes.

Based on the investment portfolio data formed by Bitcoin, Tether, Ethereum, BNB, and risk-free assets, if investors want to invest in these four virtual currencies, it is recommended that they short sell Tether, Ethereum, and BNB in the short term and invest in Bitcoin and risk-free assets to obtain higher risk returns. If investors prioritize returns over risks, it is recommended that they short Tether and Ethereum in the short term while investing in Bitcoin and BNB, two virtual currencies.

Due to programming limitations in R language and the flaws of the Markowitz theoretical model, the optimal investment portfolio results obtained from research may have slight deviations from the true optimal investment portfolio, which means that the data obtained from research may deviate from real data.

This study only focuses on data from August 31, 2023 to August 31, 2024, which is a short time interval and can only be used as a reference for short-term investments. It cannot conduct investment analysis in the long-term investment aspect

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