SHS Web of Conferences **208**, 04017 (2024) https://doi.org/10.1051/shsconf/202420804017
DSM 2024

Valuation of Convertible Bonds Based on the Black-Scholes Model

Yige Wang[1*]

[1]Department of Mathematics, University of Liverpool, Liverpool, England

Abstract. The objective of this paper is to estimate the value of convertible bonds using the Black-Scholes model. The paper commences with a comprehensive analysis of the theoretical composition of convertible bonds, with particular emphasis on their dual nature, comprising both a debt component and an embedded option component. Subsequently, the option aspect is valued using the Black-Scholes model. By integrating real market data, the study calculates the pure bond value and the option value for CITIC Convertible Bonds, thus determining the total bond value. The findings indicate that market prices for convertible bonds are below their theoretical values, suggesting potential arbitrage opportunities. In conclusion, the paper examines the key factors that influence the theoretical valuation of convertible bonds, including coupon rates, market interest rates, stock price volatility, and the risk-free rate. These factors play a critical role in determining the value of convertible bonds. The findings of this research are of value to both investors and issuers in their assessment of the value of convertible bonds.

1 Introduction

Convertible bonds are a type of financial instrument that blend both debt and option features. These bonds are issued by companies under legal agreements, giving bondholders the right to exchange the bonds for company stock within a specific time period under agreed-upon conditions [1]. As a hybrid financial product, convertible bonds combine the debt characteristics of bonds with the option-like characteristics of options. From the perspective of the issuer, convertible bonds provide an efficient way to raise capital at a relatively low cost, while limiting the dilution of equity caused by issuing new shares [2]. For investors, the debt component guarantees the safety of the principal, and the option component offers the opportunity to convert the bond into shares if the stock price increases, allowing for potential capital gains.

The total value of a convertible bond consists of two core components: the liability component and the option component. The debt component is similar to that of a traditional bond and reflects the inherent bond value, while the option component represents the value of the embedded call option. The value of a convertible bond can therefore be viewed as a combination of its debt value and its option value. Together, these two components determine the overall valuation of the bond. In the following sections, we will break down the

Corresponding author : sgywa181@liverpol.ac.uk

calculation process step by step based on the model and provide a detailed analysis of the various factors that influence the valuation of convertible bonds.

2 Convertible bonds

2.1 The pure bond value of convertible bonds

The formula is as follows:

$$B = \sum_{t=0}^{n} \frac{I}{(1+r)^{t+k}} + \frac{P}{(1+r)^{n+k}} \tag{1}$$

Here, B represents the pure bond value, I refers to the interest on the bond for each period, and P stands for the issue price of the bond. The term r represents the market risk-free interest rate, while n denotes the number of full years remaining from the current date until the maturity date. The variable k represents the time remaining (in years) from the current date until the next interest payment date, where $0 < k < 1$. Thus, $n + k$ indicates the total remaining time from the current date until the bond's maturity.

2.2 The option value of convertible bonds

The option component of a convertible bond functions in a manner analogous to a call option on a stock, comprising two principal elements: intrinsic value and time value [3]. The term 'intrinsic value' is used to describe the exercisable value of the option, which is determined by the prevailing market conditions at the time. It represents the intrinsic value of the option contract itself, with the objective of maximising its worth. In contrast, the time value of the option can be defined as the value in excess of its intrinsic value, paid by the holder of the convertible bond for the option.

The calculation formula is:

$$C_0 = MAX(S - X, 0) \tag{2}$$

Where C_0 is the option value, S is the stock's market price, and X is the conversion price. This formula indicates that when the market price exceeds the conversion price, the holder can profit through conversion; otherwise, the intrinsic value is zero.

The time value reflects the potential for excess returns within a certain future period, depending largely on market volatility and the future trend of the stock price. Although time value offers the potential for additional gains, it carries uncertainty.

The total option value is determined by both intrinsic and time value. In this paper, the option value of convertible bonds will be evaluated using the Black-Scholes model and other option pricing models, and a comparative analysis will be conducted with actual market prices to explore the differences between theoretical models and market performance

3 Black-Scholes Model

The Black-Scholes model is primarily employed for the pricing of European options. The model is founded upon a number of fundamental assumptions. Firstly, the stock price is assumed to follow a geometric Brownian motion, whereby the stock price is distributed lognormally over discrete time intervals, with constant volatility. Secondly, the market is free of arbitrage, whereby there are no opportunities to earn risk-free profits by simultaneously purchasing and selling options or stocks. Furthermore, the model assumes that no dividends are paid by the underlying stock during the option's lifetime. Fourthly, there are no transaction costs associated with the purchase and sale of options and stocks, and the market is characterised by perfect liquidity [4]. Furthermore, the risk-free interest rate is assumed to remain constant throughout the option's validity period. Finally, the option is of the European type, which means that it can only be exercised on the maturity date and not at any earlier date. These assumptions provide the theoretical foundation for the mathematical derivation of the Black-Scholes model and its option pricing.

Therefore, the formula is:

$$C = SN(d_1) - Ee^{-Rt}N(d_2) \tag{3}$$

$$d_1 = \frac{\left[\ln\left(\frac{S}{E}\right) + \left(R + \frac{1}{2}\sigma^2\right)t\right]}{\sqrt{\sigma^2 t}} \tag{4}$$

$$d_2 = d_1 - \sqrt{\sigma^2 t} \tag{5}$$

In the Black-Scholes model, C represents the European call option price, S is the current market price of the stock, E is the strike price, R represents the risk-free interest rate, and t is the time until the option's expiration. The functions $N(d_1)$ and $N(d_2)$ denote the cumulative distribution functions of the standard normal distribution, which are used to calculate the price of the option.

4 Experimental evidence

This experiment takes the CITIC Convertible Bond (113021) as an example.

4.1 The pure bond value of convertible bonds

The convertible bond in this case is a convertible bond issued on December 27, 2021, with an issuance size of 40 billion RMB, issued at face value, where each convertible bond has a face value of 100 RMB. The bond has a maturity of six years (from March 4, 2019, to March 3, 2025). It follows a progressive interest rate model: 0.3%, 0.8%, 1.5%, 2.3%, 3.2%, and 4.0% for years 1 through 6, respectively. The interest is paid annually. The data used in this paper is sourced from Eastmoney.

The current six-year bank loan interest rate of 4.9% is set as the annual risk-free rate, and the continuously compounded annual interest rate r equals ln(1+4.9%) = 4.78%. Ignoring the provisions for bond redemption and repurchase, the value of the bond at the time of issuance as a regular bond is calculated as follows: B = 85.35.

4.2 The option value of convertible bonds

4.2.1 Calculation of the variance of stock's continuous returns

Stock price volatility measures the uncertainty in stock returns. Volatility is influenced by factors such as a company's investment and financial strategies, industry characteristics, and economic cycles, making future volatility difficult to predict. Historical volatility is often used as a reference to estimate future volatility [5].

In this study, the stock price volatility is calculated based on the closing prices of the bank's stock over 60 consecutive trading days prior to the issuance of the convertible bond.

The daily volatility of the bank's stock, σ_1, is calculated to be 0.57%. This leads to an annualized volatility of the stock, $\sigma = 0.57\% * \sqrt{245} = 8.89\%$ (assuming there are 245 trading days in a year).

4.2.2. Calculation of the value of a call option

The stock conversion price of the convertible bond is 5.77 RMB per share, with a risk-free interest rate (RRR) of 4.78%. At the time of the convertible bond issuance, the company's stock price was 5.91 RMB, and the bond's maturity period t is 6 years. Substituting these parameters into the model, the value of the call option per share for the convertible bond is calculated as follows.

$$d_1 = \frac{\left[\ln\left(\frac{5.91}{5.77}\right)+\left(4.78\%+\frac{1}{2}*\left(8.89\%^2\right)*6\right)\right]}{8.89\%\sqrt{6}} = 1.535 \tag{6}$$

$$d_2 = 1.535 - 8.89\%\sqrt{6} = 1.317 \tag{7}$$

By substituting these values into the standard normal distribution for calculation, the result obtained is C = 1.618.

Each convertible bond with a face value of 100 RMB can be converted into 17.33 shares of the bank's common stock, and thus: C = 17.33*1.618 = 28.08

4.2.3. Pricing of convertible bonds

The value of the convertible bond = Pure bond value + Option value = 85.35 + 28.02 = 113.37 RMB.

The calculated value of the convertible bond is higher than its issuance value and current present value. The market's current pricing of this convertible bond is lower than its theoretical value, indicating that the bond may be undervalued in the market, presenting potential arbitrage opportunities [4].

5 Influencing factors

The initial issuance price of convertible bonds tends to be lower than their theoretical value, creating a noticeable discrepancy. The following section explores the potential reasons behind this difference and offers relevant explanations.

The valuation model employed in this study is predicated on a set of idealised assumptions that may not fully align with the nuances of the real-world context. To illustrate, the model assumes the underlying asset is a European option that does not pay dividends, whereas in practice, convertible bonds are linked to stocks that frequently do pay dividends. Moreover, bondholders are afforded the flexibility to convert the bonds into stock at any juncture during the bond's tenure. This implies that the embedded option may function as either an American or European option [6].

The bond component of a convertible bond is subject to a number of influencing factors. Firstly, an elevated coupon rate serves to enhance the value of the bond, as bondholders receive a greater income in the form of interest [7]. Secondly, alterations in market interest rates exert a considerable influence on the value of bonds. In general, an increase in interest rates results in a reduction in bond prices. Bonds with longer maturities are subject to greater uncertainty regarding future cash flows, which can result in increased fluctuations in value [8]. The credit rating of the issuer is another significant factor. Bonds issued by companies with higher credit ratings are generally considered to be more valuable. In addition, the erosion of future cash flows caused by inflation can result in a reduction in the bond's value.

Similarly, the option component is subject to the influence of a number of elements. A rise in the underlying stock price will boost the intrinsic value of the option, while a decrease in the conversion price will increase its value by allowing bondholders to convert at a more favorable rate [6]. Furthermore, elevated stock price volatility serves to enhance the option's time value, given that it increases the probability of price fluctuations [9]. An analogous effect is observed with an increase in the risk-free interest rate, which reduces the present value of future cash flows and thus increases the overall value of the option [2]. An increase in the time to maturity results in a corresponding increase in the value of the option, as bondholders are afforded a longer period within which to potentially benefit from rising stock prices. Furthermore, dividend policies exert an influence. Stocks that pay high dividends render the option less attractive, as the bondholders are not in receipt of those dividends [10].

The selection of a risk-free interest rate is another crucial aspect to be taken into account [6]. The risk-free rate is a measure of the return that investors can expect from a risk-free asset, which is often represented by government bond yields. In this study, the six-year bank loan rate was employed as a proxy for the risk-free rate. An increase in this rate has the effect of enhancing the value of the option, which is achieved by reducing the present value of the future cash flows [11].

It should be noted that additional factors, such as the overall financial market environment and the internal conditions of the issuing company, have the potential to cause market prices to deviate from theoretical values [12]. Nevertheless, the theoretical model remains a valuable tool for both issuers and investors alike, providing a benchmark for the assessment of the value of convertible bonds within the financial market.

6 Conclusion

The analysis has shown that even though the actual issuance price may vary from the theoretical model's value for several reasons, this does not invalidate the importance of the theoretical price. By applying the theoretical model, it is possible to assess the value of convertible bonds with precision, offering useful guidance for decisions regarding whether to hold or sell the bonds. A solid combination of theoretical analysis and practical conditions is crucial for ensuring the sustainable development of the bond.

Looking forward, as artificial intelligence and big data technologies continue to evolve, models that can capture market fluctuations and investor sentiment are likely to improve. These advancements will provide more accurate tools for assessing financial instruments like convertible bonds. For investors, a full integration of theoretical models with real-time market data will help identify more investment opportunities and lead to more consistent returns.

References

1. Z. L. Zheng, & H. Lin, Research on the valuation of Chinese convertible bonds, Doctoral dissertation (2004)
2. M. Zhang, Theoretical and case study on the valuation of convertible bonds, Doctoral dissertation (2001)
3. Z. L. Zheng, & H. Lin, Optimal decision for issuing convertible bonds by companies. Journal of Finance and Economics Issues. (2004) (in Chinese)
4. Hu, Research on the valuation model of Black-Scholes options. Journal of Guiyang University: Natural Science Edition, (2), 13-18 (2010) (in Chinese)
5. Z. Q. Li, Valuation and trading strategy of convertible bonds based on the Black-Scholes option model. Business Accounting, (06), 32-38 (2023) (in Chinese)
6. G. Z. Cheng, Valuation of convertible bonds based on the Black-Scholes model: A case study of a listed company. Business Observation, 21, 45-48 (2024) (in Chinese)
7. J. H. Li, & Y. N.Li, Research on the valuation of Chinese convertible bonds based on the Black-Scholes model. Business and Management, 12, 6-13 (2023) (in Chinese)

8. L. Wei, Research on the valuation of convertible bonds: A case study of Shanghai Electric convertible bonds. National Business Conditions, 28, 108-110 (2016) (in Chinese)
9. L. R. Tang, Optimization research on the valuation of convertible bonds based on the B-S model. Shanghai : East China Normal University (2022) (in Chinese)
10. M. Y. Xiong & J. Zhu, Research on the valuation methods of Chinese convertible bonds. China Valuation, (03), 5-10 (2024) (in Chinese)
11. M. J. Brennan, & E. S. Schwartz, Analyzing convertible bonds. Journal of Financial and Quantitative analysis, 15(4), 907-929. (1980).
12. J. A. Batten, K. L. H. Khaw, & M. R. Young, Pricing convertible bonds. Journal of Banking & Finance, 92, 216-236. (2018).