Constrained Portfolio Optimization: Markowitz Model and Index Model

Heran Wang[1*]

[1] School of Mathematics, Shandong University, Jinan, China

Abstract. Portfolio optimization is a crucial aspect of contemporary finance, facilitating the efficient balancing of risk and return for investors. In light of the intricate nature of actual investments, constrained portfolio optimization has assumed greater significance as a means of reflecting the practical realities of investment scenarios. Based on historical data, this study employs the Markowitz and Index models to optimize portfolios utilizing 24 years (2000-2024) of daily total return from 10 stocks across five sectors as well as the S&P 500 index. Several financial metrics are calculated, including annualized return, standard deviation, alpha, and beta. Five distinct constraints are applied to determine the minimum variance frontier, efficient frontier, and maximum Sharpe ratio for both models. The results indicate that an effective investment portfolio can reduce investment risk. Furthermore, constrained optimization yields portfolios with balanced risk and return characteristics, offering valuable insights for investors seeking practical strategies to manage risk within real-world limitations.

1 Introduction

It is widely acknowledged that Harry Markowitz, the inventor of modern portfolio theory, is the originator of the famous quote, "Diversification is the only free lunch in investing." Investors seek to maximize investment returns while minimizing investment risks. Consequently, investment necessitates the formulation of an efficacious strategy, wherein the portfolio optimization model assumes a pivotal role in determining the optimal solution. Nevertheless, it is imperative to recognize that risk and return cannot be simultaneously satisfied. Instead, a dynamic equilibrium exists between these two factors, and different investors exhibit disparate tolerance for risk. Consequently, it is of paramount importance to establish a personalized portfolio since the investor's risk and return preferences serve as a pivotal determinant in the construction of the portfolio optimization [1].

It is evident that modern portfolio theory has been extensively utilized in practical applications since its inception. Additionally, numerous extensions and developments have been made on the foundation of existing models. In recent years, numerous researchers have made significant advancements in the field. For example, Putra and Dana conducted a portfolio analysis on 28 stocks of the Indonesian Stock Exchange in conjunction with the LQ45 index and compared the performance of the two models [2]. Based on machine learning,

* Corresponding author: 202000130006@mail.sdu.edu.cn

Chen and Zhang constructed a novel portfolio optimization approach in a hybrid model, aiming for stock prediction and portfolio selection [3]. Copula-based approaches were considered in Zhi et al.'s analysis of inventory financing with monthly collateral returns on seven raw materials in the metal industry [4]. Nevertheless, apart from a multitude of recent studies, there has been a paucity of research focusing on constrained portfolio optimization for assets within particular fields.

The objective of this paper is to examine asset allocation issues by using the two models across five distinct industries. To this end, five illustrative constraints have been identified to reflect authentic investment scenarios. The initial step is to select ten stocks from five different industries. Additionally, the S&P 500 index (SPX) and the Federal Funds Effective Rate (FEDL01) were included. The data set comprised monthly closing prices over the past 24 years. Second, annualized average return, annualized standard deviation, alpha, beta, residual standard deviation, and correlation coefficients between each asset are calculated. Subsequently, the two models are established, and Excel Solver is utilized to ascertain the Minimum Variance Frontier, Minimum Risk Portfolio, Maximum Sharpe Ratio, Capital Allocation Line, Efficient Frontier, and Minimum Return Frontier of the investment portfolio under five distinct constraints. Ultimately, optimal weights of assets under the two models are enumerated, and the returns and risks under each constraint are evaluated. The objective of this article is to elucidate the fundamental tenets of two investment portfolio models for the benefit of the readership. Furthermore, it seeks to provide a pragmatic investment exemplar that may serve as a point of reference for investors.

2 Data

All asset data were sourced from Yahoo! Finance. A total of 6,183 data points were selected for the closing price of the day of each asset from January 3, 2000, to July 30, 2024. In order to minimize the correlation between assets, a portfolio comprising ten stocks from five different industries was constructed, including Financial Services, Technology, Consumer Defensive, and Consumer Cyclical. In constructing the single index model, the S&P 500 was employed as a means of operationalizing the single factor model, whereby the market index is employed as a proxy for the common factor. Additionally, it provides a significant amount of historical data that can be used to estimate systematic risk. In this paper, the Federal Funds Effective Rate is employed as the risk-free rate. Information of included assets are listed in Table 1.

The two models are predicated on the assumption that the data will follow a normal distribution. Following an examination of the data, it was determined that the daily price distribution of assets does not align with the characteristics of a normal distribution. To mitigate the impact, daily data was integrated into monthly. Following integration, each asset had a total of 295 data points. Subsequently, Annualized Average Return (AAR), Annualized Standard Deviation (ASD), Beta, Alpha, and Residual Standard Deviation (RSD) of each asset are calculated, as demonstrated in Table 2.

Table 1. Stocks Information

Ticker	Full Name	Sector	Industry
SPX	S&P 500 Index		
WFC	Wells Fargo & Company	Financial Services	Banks-Diversified
TD	The Toronto-Dominion Bank	Financial Services	Banks-Diversified
PGR	The Progressive Corporation	Financial Services	Insurance-Property & Casualty
CSCO	Cisco Systems, Inc.	Technology	Communication Equipment
MSFT	Microsoft Corporation	Technology	Software-Infrastructure
KO	The Coca-Cola Company	Consumer Defensive	Beverages-Non-Alcoholic
PG	The Procter & Gamble Company	Consumer Defensive	Household & Personal Products
LSTR	Landstar System, Inc.	Industrials	Integrated Freight & Logistics
LUV	Southwest Airlines Co.	Industrials	Airlines
MCD	McDonald's Corporation	Consumer Cyclical	Restaurants
FED01	U.S. Federal Funds Effective Rate		

Table 2. Statistics of Assets

	SPX	WFC	TD	PGR	CSCO	MSFT
AAR	4.96%	6.79%	7.20%	16.30%	3.03%	10.77%
ASD	15.40%	28.73%	23.45%	24.38%	32.49%	27.85%
Beta	1.00	0.98	1.02	0.64	1.33	1.10
Alpha	0.00	0.02	0.02	0.13	-0.04	0.05
RSD	0.00%	24.46%	17.42%	22.32%	25.28%	22.06%
	KO	PG	LSTR	LUV	MCD	
AAR	3.08%	4.50%	15.31%	7.25%	8.03%	
ASD	17.15%	17.56%	24.27%	32.71%	19.25%	
Beta	0.48	0.30	0.73	1.07	0.65	
Alpha	0.01	0.03	0.12	0.02	0.05	
RSD	15.47%	16.95%	21.53%	28.27%	16.46%	

3 Methodology

Prior to the publication of modern portfolio theory, as detailed in Markowitz's article "Portfolio Selection," investors primarily concentrated on the risk and return of single asset. However, Markowitz utilizes diversification as an alternative approach, which involves selecting a portfolio in aggregate, rather than constructing distinct assets. The fundamental premise of the Markowitz Model is that asset returns over a specified time frame are a random variable. Consequently, the mathematical expectation and variance can be employed as measures of anticipated returns and risks. Let us assume that the weight and Annualized Average Return of each asset are $\omega_i, r_i, i = 1,2, \dots, n$. Then the expected return and standard deviation on an investment portfolio can be expressed as follows [5].

$$r_M = \sum_{i=1}^{n} \omega_i \, r_i \tag{1}$$

$$E[r_M] = E[\sum_{i=1}^{n} \omega_i \, r_i] = \sum_{i=1}^{n} \omega_i \, E[r_i] \tag{2}$$

$$\sigma_M^2 = Var[r_M] = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\omega_j Cov(r_i, r_j) = \sum_{i=1}^{n}\sum_{j=1}^{n}(\sigma_i\omega_i)\rho_{ij}(\sigma_j\omega_j) \quad (3)$$

The Markowitz Model is predicated on several assumptions pertaining to conduct of investors along with the functioning of financial market [6]. It necessitates a substantial quantity of data estimation to construct the covariance matrix; consequently, the model is unable to offer a prediction method for asset risk premium. However, the Index Model can address these two shortcomings. The Index Model disperses risk into systemic risk and corporate risk, while providing an estimate of asset risk premium. In contrast to the Markowitz Model, the core idea of the Index Model is multiple linear regression of asset's excess returns to market excess returns, which is manifested as below.

$$R_i = \alpha_i + \beta_i R_M + e_i \quad (4)$$

Where $R_i = r_i - r_f$, $R_M = r_M - r_f$, r_f is the return on the risk-free asset. Regression coefficient α represents the asset's anticipated excess return in the absence of market excess return. β_i denotes the asset's responsiveness to the index, signifying the range of fluctuations in asset i relative to the market index. The residual e_i, is a measure of firm-specific surprises in security returns, with a mean of zero. The apparent form is represented by the following expression.

$$E[R_i] = \alpha_i + \beta_i E[R_M] \quad (5)$$

$$E[R_I] = \sum_{i=1}^{n} \omega_i\, \alpha_i + \sum_{i=1}^{n} \omega_i\, \beta_i E[R_M] \quad (6)$$

$$\sigma_i^2 = Var[R_i] = Var[\beta_i R_M + e_i] = \beta_i^2\sigma_M^2 + \sigma_{e_i}^2 \quad (7)$$

$$\sigma_I^2 = Var\left[\sum_{i=1}^{n} \omega_i\, \beta_i R_M + \sum_{i=1}^{n} \omega_i\, e_i\right] = \sum_{i=1}^{n} \omega_i^2\beta_i^2\sigma_M^2 + \sum_{i=1}^{n} \omega_i^2\sigma_{e_i}^2 \quad (8)$$

It is evident that the Index Model is sufficient for calculating Alpha, Beta, and Residual Standard Deviation, thereby obviating the necessity for covariance estimation.

Once the model is built, the expected returns and standard deviations of portfolios under different allocations can be calculated and plotted on a graph with standard deviation on the x-axis and returns on the y-axis. In light of a given expected return on a portfolio, it is possible to identify a minimum acceptable level of risk. The curve that is formed by plotting these points is known as the Minimum Variance Frontier.

$$\min_{r_M = const} \sigma_M \quad (9)$$

This curve allows investors to understand the risk-return opportunities that they are facing, and it demonstrates that all individual assets are positioned to the right of the curve, indicating that an effective investment portfolio can serve to diversify risks.

The point on the curve situated furthest to the left exhibits the lowest standard deviation, which corresponds to the Minimal Risk Portfolio point.

$$min\, \sigma_M \quad (10)$$

The curve above this point represents the optimal risk-return combination, which is referred to as the efficient frontier.

$$\max_{\sigma_M = const} r_M \quad (11)$$

The curve depicted below represents the minimum value of return that can be achieved under the same level of risk, which is referred to as the Minimal Return Frontier:

$$\min_{\sigma_M = const} r_M \quad (12)$$

The aforementioned curve demonstrates that all points above it exhibit higher returns. Therefore, this curve is not an optimal investment strategy for investors.

The Sharpe measure is also referred to as the Sharpe Ratio (SR). It is a metric for evaluating the efficiency of a portfolio or the skills of an investor [7]. The formula is as follows.

$$SR = \frac{E[r_M - r_f]}{\sigma_M} \tag{13}$$

Where $r_M - r_f$ is the risk premium of a portfolio. Individuals frequently seek portfolios with a high Sharpe ratio, as it signifies that the same level of risk is assumed but with the potential for greater returns [8].

4 Results

The reality is more intricate than theoretical models allow. Furthermore, the weight of each asset cannot be any arbitrary value, as investors have limited assets and risk tolerance, and risk preference varies from one individual to another. When constructing a model, some constraints can be added to simulate real investment situations.

4.1 Constraint 1

Firstly, in order to ascertain the feasible range of an ordinary investment portfolio, no restrictions are imposed, thus rendering the problem free. In this case, the portfolio tables for the minimal risk portfolio (Min RP) and the maximal Sharpe ratio (Max SR) are calculated, with the expected returns and standard deviations as follows in Table 3-5 and Fig. 1.

Table 3. Portfolios Using Markowitz Model under Constraint 1

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	40.3%	-1.8%	-4.6%	8.2%	-5.5%	2.7%
Max SR	-92.4%	1.8%	4.0%	59.2%	-12.5%	39.7%
	KO	PG	LSTR	LUV	MCD	
Min RP	14.6%	28.5%	11.6%	-2.7%	8.7%	
Max SR	-30.1%	28.4%	61.2%	0.4%	40.4%	

Table 4. Portfolios Using Index Model under Constraint 1

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	21.5%	0.4%	-0.8%	8.9%	-6.3%	-2.6%
Max SR	-90.5%	6.1%	13.3%	49.8%	-10.5%	20.5%
	KO	PG	LSTR	LUV	MCD	
Min RP	26.6%	30.0%	7.2%	-1.0%	16.0%	
Max SR	5.4%	19.9%	47.7%	4.6%	33.6%	

Table 5. Results under Constraint 1

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Min RP	7.0%	12.1%	0.58	6.5%	11.1%	0.59
Max SR	22.4%	21.6%	1.04	18.3%	18.6%	0.98

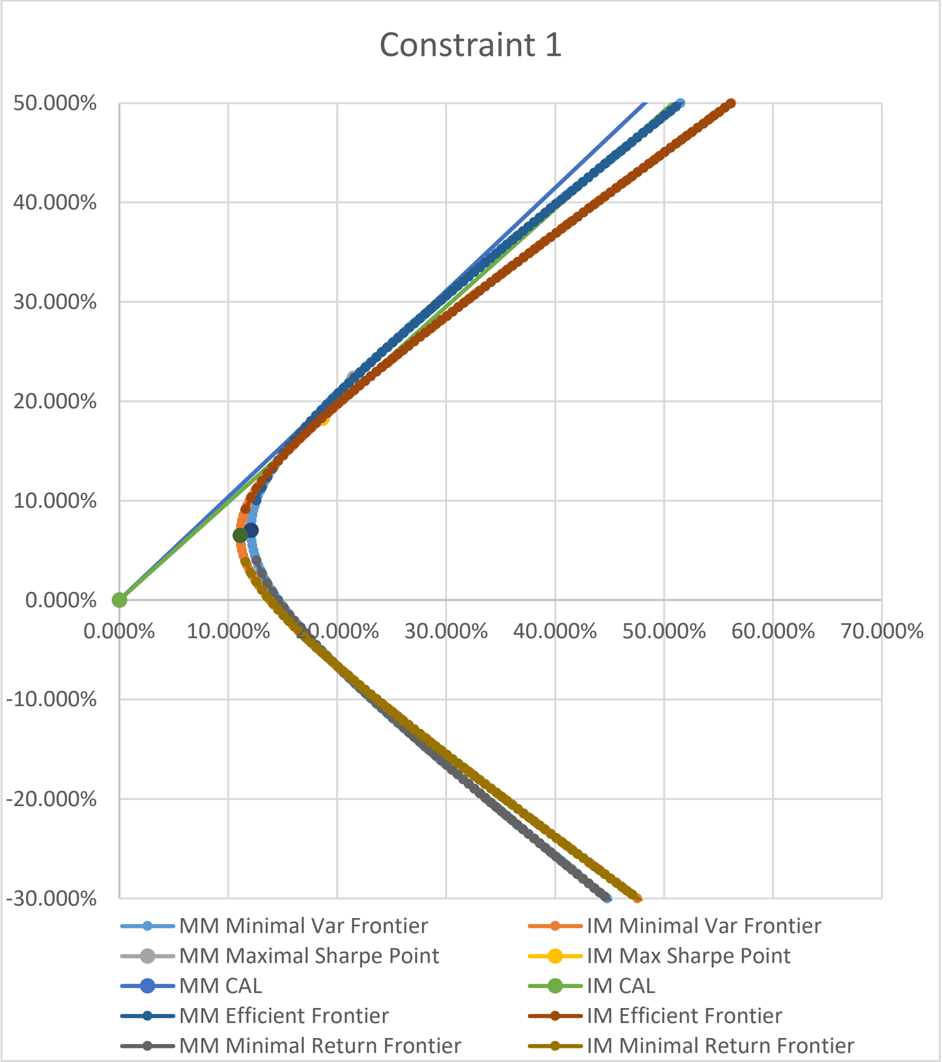

Fig. 1. Results under Constraint 1

Furthermore, to enhance the verisimilitude of the research content, this article has selected four additional constraints on asset weights to simulate authentic trading scenarios. These constraints are commonly encountered in real-world trading environments. This article then proceeds to examine the results and the distinctions between the two models under these four constraints.

4.2 Constraint 2

This constraint is designed to emulate the Regulation T guidelines set forth by the Financial Industry Regulatory Authority (FINRA). In essence, under the Federal Reserve Board Regulation T (Reg T), brokers are permitted to extend loans to customers up to 50% of the total purchase price of a margin equity security for new purchases [9]. The objective of this constraint is to encourage diversification, which may potentially reduce both risk and return.

$$\sum_{i=1}^{11} |\omega_i| \leq 2 \tag{14}$$

Related results are shown below in Tables 6-8 and Fig. 2.

Table 6. Portfolios Using Markowitz Model under Constraint 2

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	40.3%	-1.8%	-4.6%	8.2%	-5.5%	2.7%
Max SR	-29.9%	-0.1%	-0.1%	43.8%	-11.6%	21.9%
	KO	PG	LSTR	LUV	MCD	
Min RP	14.6%	28.5%	11.6%	-2.7%	8.7%	
Max SR	-7.8%	17.0%	45.1%	-0.5%	22.1%	

Table 7. Portfolios Using Index Model under Constraint 2

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	21.5%	0.4%	-0.8%	8.9%	-6.3%	-2.6%
Max SR	-39.1%	2.0%	4.7%	42.3%	-10.9%	13.2%
	KO	PG	LSTR	LUV	MCD	
Min RP	26.6%	30.0%	7.2%	-1.0%	16.0%	
Max SR	2.9%	18.0%	39.6%	1.2%	26.1%	

Table 8. Results under Constraint 2

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Min RP	7.0%	12.1%	0.58	6.5%	11.1%	0.59
Max SR	16.8%	16.8%	1.00	15.7%	16.2%	0.97

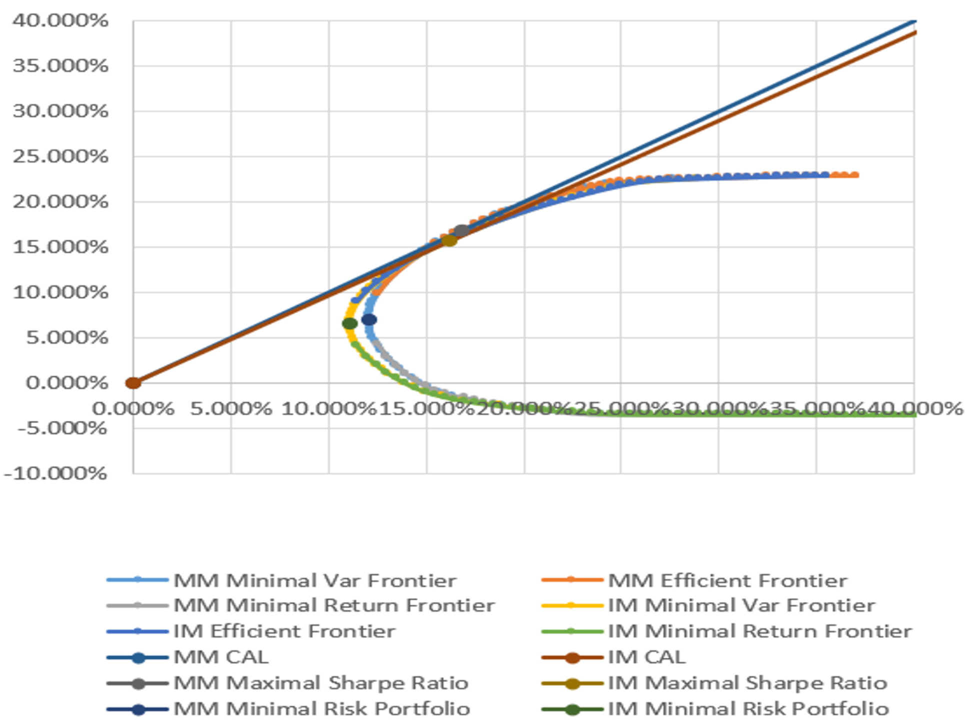

Fig. 2. Results under Constraint 2

4.3 Constraint 3

This supplementary constraint is intended to emulate arbitrary "box" constraints on weights, which may restrict extreme positions, thereby producing a conservative portfolio.

$$|\omega_i| \leq 1, \forall i \quad (15)$$

Related results are shown below in Tables 9-11 and Fig. 3.

Table 9. Portfolios Using Markowitz Model under Constraint 3

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	40.3%	-1.8%	-4.6%	8.2%	-5.5%	2.7%
Max SR	-92.4%	1.8%	4.0%	59.2%	-12.5%	39.7%
	KO	PG	LSTR	LUV	MCD	
Min RP	14.6%	28.5%	11.6%	-2.7%	8.7%	
Max SR	-30.1%	28.4%	61.2%	0.4%	40.4%	

Table 10. Portfolios Using Index Model under Constraint 3

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	21.5%	0.4%	-0.8%	8.9%	-6.3%	-2.6%
Max SR	-90.5%	6.1%	13.3%	49.8%	-10.5%	20.5%
	KO	PG	LSTR	LUV	MCD	
Min RP	26.6%	30.0%	7.2%	-1.0%	16.0%	
Max SR	5.4%	19.9%	47.7%	4.6%	33.6%	

Table 11. Results under Constraint 3

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Min RP	7.0%	12.1%	0.58	6.5%	11.1%	0.59
Max SR	22.4%	21.6%	1.04	18.3%	18.6%	0.98

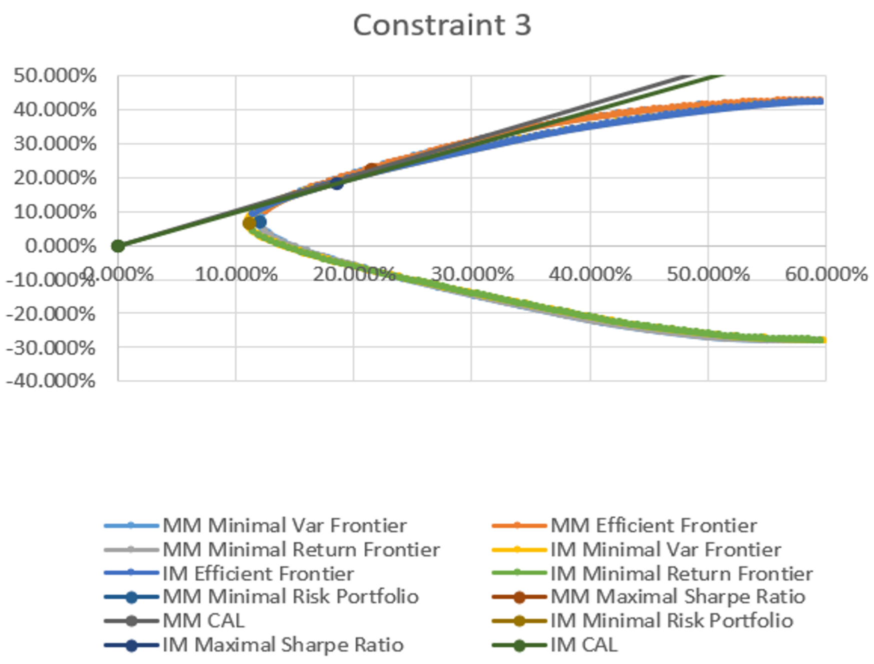

Fig. 3. Results under Constraint 3

4.4 Constraint 4

This constraint is expected to emulate the typical restrictions that exist in the US mutual fund industry: an open-ended mutual fund domiciled in the United States is not permitted to take a short position [10]. The restriction increases the exposure to market risk by limiting hedging.

$$\omega_i \geq 0, \forall i \quad (16)$$

Related results are shown below in Tables 12-14 and Fig. 4.

Table 12. Portfolios Using Markowitz Model under Constraint 4

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	23.9%	0.0%	0.0%	8.0%	0.0%	4.0%
Max SR	0.0%	0.0%	0.0%	39.2%	0.0%	8.3%
	KO	PG	LSTR	LUV	MCD	
Min RP	15.2%	29.5%	10.4%	0.0%	9.0%	
Max SR	0.0%	6.5%	36.5%	0.0%	9.5%	

Table 13. Portfolios Using Index Model under Constraint 4

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	8.6%	0.5%	0.0%	9.2%	0.0%	0.0%
Max SR	0.0%	0.0%	0.0%	37.2%	0.0%	3.6%
	KO	PG	LSTR	LUV	MCD	
Min RP	27.3%	30.8%	7.4%	0.0%	16.3%	
Max SR	0.0%	11.2%	33.3%	0.0%	14.7%	

Table 14. Results under Constraint 4

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Min RP	7.0%	12.2%	0.58	6.6%	11.2%	0.59
Max SR	13.9%	15.6%	0.89	13.2%	15.2%	0.87

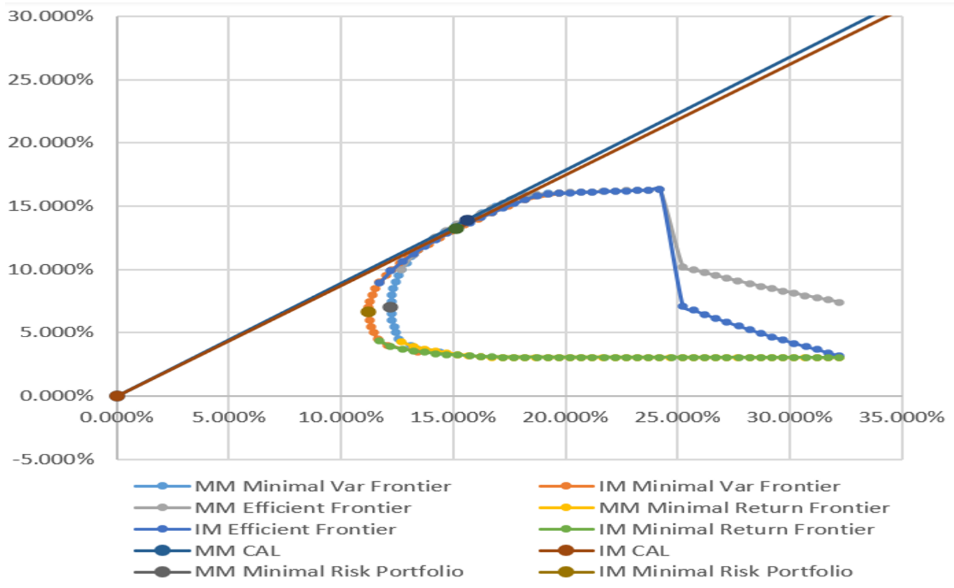

Fig. 4. Results under Constraint 4

4.5 Constraint 5

The objective of this constraint is to ascertain whether the incorporation of the broad index into the portfolio exerts a beneficial or detrimental influence. It highlights the performance of individual stocks, potentially increasing risk due to the lack of broad market exposure.

$$\omega_1 = 0 \tag{17}$$

Related results are shown below in Tables 15-17 and Fig. 5.

Table 15. Portfolios Using Markowitz Model under Constraint 5

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	0.0%	1.4%	1.3%	10.7%	-0.9%	10.3%
Max SR	0.0%	-4.9%	-8.4%	48.9%	-20.5%	21.0%
	KO	PG	LSTR	LUV	MCD	
Min RP	16.8%	32.5%	14.5%	-0.5%	13.8%	
Max SR	-29.7%	20.6%	50.2%	-4.2%	27.1%	

Table 16. Portfolios Using Index Model under Constraint 5

	SPX	WFC	TD	PGR	CSCO	MSFT
Min RP	0.0%	2.2%	2.7%	10.5%	-4.2%	-0.3%
Max SR	0.0%	-1.0%	-1.2%	41.1%	-18.2%	9.9%
	KO	PG	LSTR	LUV	MCD	
Min RP	29.3%	31.7%	9.0%	0.3%	18.8%	
Max SR	-3.1%	14.2%	37.9%	-1.1%	21.5%	

Table 17. Results under Constraint 5

	Markowitz Model			Index Model		
	Return	StDev	Sharpe	Return	StDev	Sharpe
Min RP	8.3%	12.4%	0.67	7.1%	11.2%	0.64
Max SR	18.2%	18.4%	0.99	15.1%	16.3%	0.93

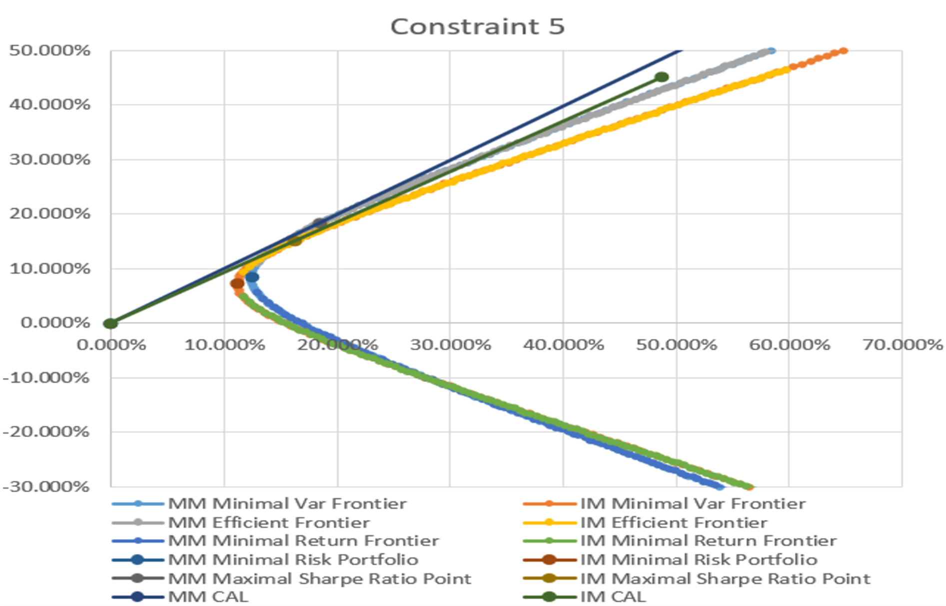

Fig. 5. Results under Constraint 5

4.6 Analysis of results

In the lowest risk scenario, the average return of the portfolio under the five constraints is 7%, which is relatively high, with the highest being 8.3% under Constraint 5. The average standard deviation is 11.7%, which is 12.2% lower than the average standard deviation of individual assets.

At its maximum value, the Sharpe ratio is 0.97, with the highest values observed for Constraint 1 and Constraint 3, which reach 1.04. The mean return increases to 17.4%, with an average standard deviation of 17.9%, a decline of 6% in comparison to the mean standard deviation of 23.9% for individual assets. This suggests that the investment portfolio is capable of diversifying risk to a relatively low level.

In the absence of constraints (i.e., Constraint 1), the efficient frontier of the two models exhibits the highest and most outward points, representing the theoretical maximum risk-return combinations. When constraints are introduced, the efficient frontier of each constraint is typically lower than that of the unconstrained case, demonstrating the trade-offs between practical limitations and theoretical optimality.

Constraint 4 exhibits the smallest range of returns and the greatest degree of restriction. In comparison to other restrictions, it presents lower risk under equal returns, rendering it more suitable for conservative investors and confirming its presence in mutual funds.

Constraints 1 and 3 yield identical portfolio outcomes, as all weights in Constraint 1 satisfy the range condition (i.e., the range [-1,1]) specified in Constraint 3.

Assets with higher annualized average returns, such as PGR and LSTR, are allocated a greater proportion of the portfolio. In contrast, assets with lower annualized average returns, such as CSCO and KO, are assigned lower weights in asset allocation. Since the annualized standard deviation of CSCO and LUV are among the top two, their weights are predominantly negative under all constraints. Results indicates that, assets with high returns and low risks should be prioritized in asset allocation, and their investment proportion should be augmented.

5 Conclusion

Portfolio optimization is a crucial tool for assisting investors in attaining a well-balanced risk-return profile. This study employed the Markowitz and Index models to analyze 24 years of data from a multitude of sectors and the S&P 500 index. By calculating pivotal financial metrics and applying five distinct constraints, optimal portfolio composition were able to be ascertained. The findings highlight that constrained optimization effectively strikes a balance between risk and return, offering practical guidance for investors confronted with tangible limitations.

Further research could investigate advanced machine learning techniques to enhance the efficacy of portfolio optimization models, particularly in the context of more complex real-world constraints. Additionally, expanding the data set to include emerging markets and alternative assets, such as cryptocurrencies, could facilitate a more nuanced understanding of the subject matter. Another promising avenue for further inquiry would be the investigation of dynamic portfolio optimization, where adjustments are made in accordance with prevailing market conditions, with the objective of enhancing adaptability and performance over time.

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