Research on Optimal Portfolio Based on Markowitz Model-Taking the U.S. Stock Market as an Example

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Abstract. With the booming development of financial market, financial products are increasing, and the risk is also increasing, leading to the simple pursuit of maximizing returns can no longer meet the needs of investors, so how to choose the appropriate assets to build investment portfolios and weigh their risks and returns to achieve the optimal allocation of assets and maximize the investment returns has become the object of investors' attention. Markowitz's portfolio theory is sought after by investors. This paper takes Markowitz mean-variance model as the theoretical framework, on the basis of which the actual data of five stocks and SPX between May 2001 and May 2021 are used, and EXCEL is used to conduct empirical analysis to obtain the weights, minimum standard deviation, and yield curves of each stock, and ultimately the efficient frontier as well as the optimal asset allocation. The research in this paper verifies that Markowitz's model is still instructive for investors to construct portfolio models in today's market.

1 Introduction

With the development of the global financial market, more and more investors begin to buy stocks, funds and other financial products. More and more people realize that they cannot just focus on the return but also consider its risk after losing money in investment [1]. Therefore, it is very important to choose appropriate assets to construct an effective investment portfolio to maximize investment returns [2]. How to construct a suitable investment portfolio has become a widely discussed topic in the academic community in recent years.

In 1952, the famous American economist Markowitz cited the mathematical and statistical methods to the research of portfolio selection, for the first time from the perspective of the quantitative relationship between risk and return to analyze in detail the selection of the optimal asset portfolio, and put forward the famous mean-variance model [3]. This is a sign of the birth of modern portfolio management theory [4]. It also promotes the current asset management business towards scientization [5]. In recent years, there are many related researches. For example, Wang based on Markowitz mean-variance theory, using four kinds of energy futures as the underlying assets and MATLAB software to calculate, and verified

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that the Markowitz mean-variance theory can be applied to the portfolio of energy futures of a small number [6]. Gao, Huang, and Guo used the Markowitz model as a theoretical framework and selected three representative stocks in China's Chinese medicine stock market for case studies, which enriched the case studies on the application of the Markowitz model [7]. Some scholars even break the assumption that a relevant parameter is determined in the previous research, such as Li elected four stocks from the CSMAR database and established a Markowitz portfolio model with simultaneous changes in mean and variance, resulting in changes in the effective frontier curve and enriching traditional portfolio theory [8]. However, there is still a lack of research on constructing portfolios and selecting appropriate assets among different investment objects. Therefore, this paper tries to explore how portfolios maximize investment returns based on the Markowitz model.

Specifically, this paper takes the Markowitz model as the theoretical framework, based on which the daily closing prices and risk-free rate data of five stocks and SPX from May 2001 to May 2021 are extracted, and planning solution in EXCEL is applied to obtain the weights, minimum standard deviations, and yield curves of each stock, and an efficient frontier is obtained. The possible marginal contribution of this paper is to validate that the Markowitz model still works in today's market environment.

2 Data

In this paper, five stocks and SPX are selected as case studies through the Vantage database, and daily closing price data and risk-free rate data are extracted from May 2001 to May 2021, the five stocks are ADBE, IBM, AAPL, NVDA, and PG. These data are processed to find the average annual return, annual standard deviation, and correlation coefficients among the stocks for each of them separately. The results are shown in Table 1 and Table 2 below.

Table 1. Average annual returns and annual standard deviations

	SPX	ADBE	IBM	AAPL	NVDA	PG
AAR	7.542%	19.578%	4.754%	34.016%	32.802%	9.437%
ASD	14.850%	31.790%	23.181%	34.452%	55.774%	14.587%

Table 2. Correlation Coefficients

	SPX	ADBE	IBM	AAPL	NVDA	PG
SPX	1.000	0.665	0.649	0.542	0.527	0.412
ADBE	0.665	1.000	0.455	0.536	0.499	0.263
IBM	0.649	0.455	1.000	0.353	0.447	0.172
AAPL	0.542	0.536	0.353	1.000	0.475	0.197
NVDA	0.527	0.499	0.447	0.475	1.000	0.060
PG	0.412	0.263	0.172	0.197	0.060	1.000

Note: ASD: Annual Standard Deviation AAR: Average Annual Returns

From the above data, it shows that AAPL has the highest average annual return while the average annual return of SPX is lower than that of other assets. Looking at the standard deviation again, NVDA has the highest annual standard deviation while the annual standard deviation of PG is lower than that of other assets. Moreover, PG's average annual return is slightly higher than SPX, but its standard deviation is smaller than SPX. However, IBM's performance is not as good as it should be, with a lower average annual return than SPX and a much higher standard deviation than SPX.

3 Model

The mean-variance theory is based on the following assumptions: (1) the securities market is efficient, i.e., the price of securities instantly and accurately responds to market information, and there is no variability in the investment information available to all traders; (2) transaction costs and taxes are absent in the market, and every investor is free to enter and exit the market without restriction; (3) investors are rational. All investors are risk-averse; (4) Investors make judgments based only on the return and risk of their portfolios, and investors understand the returns of different assets; (5) The returns of all assets are uncertain, and there is correlation between the returns of different assets [9].

In this paper, based on the above assumptions, in order to construct the optimal portfolio, first calculate the weight of each asset to find out the optimal risk portfolio ratio, and then use Markowitz's theory of the expected return of the portfolio, the calculation of risk and the effective boundary theory, to construct the mean-variance model [10].

Given a certain period, the return on assets is the difference between the end-of-period price and the beginning-of-period price, which is expressed by the formula.

$$r_{it} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \times 100\% \tag{1}$$

In Equation (1), r_{it} is the return of the ith asset in period t; $P_{i,t}$, $P_{i,t-1}$ denote the end-of-period price of the ith asset in period t and period t-1, respectively, t=1, 2.... T. The mean of the return is chosen to represent the expected future return of an asset, and the variance is chosen to represent the risk level of an asset (See Equation (2) and Equation (3))

$$\overline{r_i} = E(r_i) = \frac{1}{T}\sum_{t=1}^{T} r_{it} \tag{2}$$

$$\sigma_i^2 = var(r_i) = \frac{1}{T}\sum_{i=1}^{T} (r_{it} - u_i)^2 \tag{3}$$

If a risk portfolio p is assumed to consist of N assets, then the return u_p and the risk σ_p^2 of this risk portfolio are denoted as follows.

$$u_p = E(r_p) = E(\sum_{i=1}^{N} x_i r_i) = \sum_{i=1}^{N} x_i\, E(r_i) = \sum_{i=1}^{N} x_i u_i \tag{4}$$

$$\sigma_p^2 = var(r_p) = var(\sum_{i=1}^{N} x_i r_i) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \rho_{ij} \sigma_i \sigma_j \tag{5}$$

$$\sigma_{ij} = \frac{1}{T}\sum_{t=1}^{T} (r_{it} - u_i)\,(r_{jt} - u_j) \tag{6}$$

In Equation (5) and Equation (6), x_i denotes the proportion of the top-of-planning assets occupied in the total asset portfolio p, σ_{ij} is the covariance between the return of the ith asset and the return of the jth asset, and ρ_{ij} is the correlation coefficient between the return of the ith asset and the return of the jth asset, which takes the value in the range of [-1,1]. It can be seen that the total return of an asset portfolio is actually obtained by weighting the returns of each asset, while the risk of the portfolio depends on the risk of each asset, its respective weight and the covariance of the returns of the alternative assets.

4 Results

4.1 Constraint 1: No special constraint limitations

The model without constraints can be contrasted with other models with constraints to reflect the effect of constraints on the portfolio. The results are presented below in Table 3.

Table 3. Weights of each asset under constraint 1

	SPX	ADBE	IBM	AAPL	NVDA	PG	Return	StDev	Sharp
minVar	0.5895	-0.0910	0.0604	-0.0035	-0.0281	0.4727	0.0637	0.1200	0.5312
maxsharp	-0.5804	0.1016	-0.1801	0.5938	0.1354	0.9297	0.3017	0.2611	1.1557

When looking for the minimum standard deviation, it can be seen that SPX has the largest weighting and ADBE, AAPL and NVDA are all shorted, while when looking for the maximum Sharpe ratio, it shows that AAPL has the largest weighting and SPX and IBM are also shorted.

The following Fig. 1 shows that the curves for the maximum and minimum returns are adequately fitted to the minimum standard deviation curve.

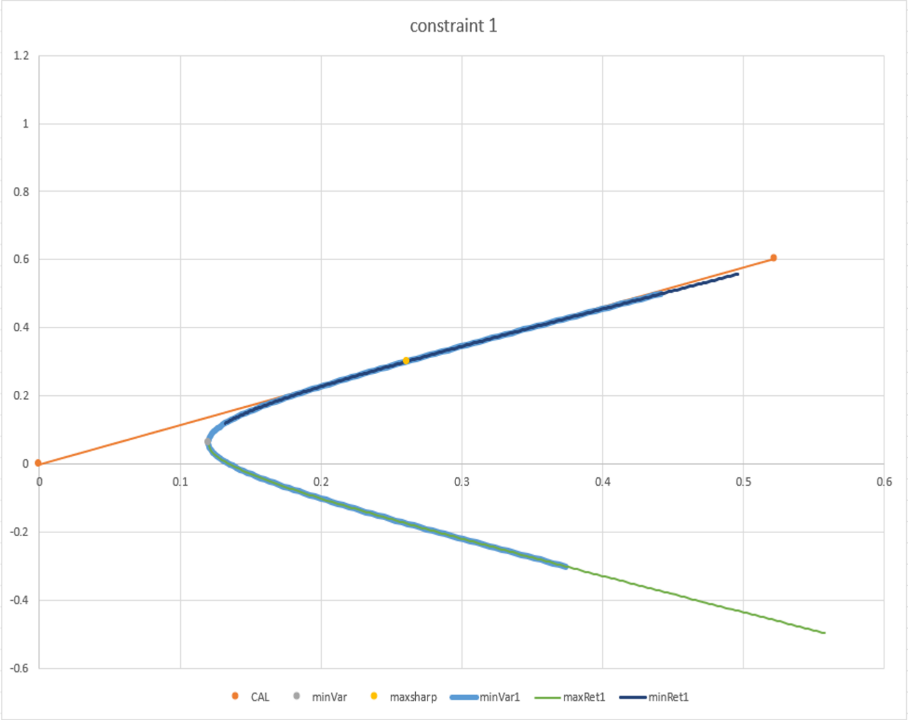

Fig. 1. Effective Frontier and CAL for Constraint 1

4.2 Constraint 2: The first term is weighted at 0

Index investment is considered as a passive investment and when its weight is fixed to 0, the effect of active investment can be tested. The results are given below in Table 4.

Table 4. Weights of each asset under constraint 2

	SPX	ADBE	IBM	AAPL	NVDA	PG	Return	StDev	Sharp
minVar	0.0000	-0.0133	0.2389	0.0407	-0.0082	0.7420	0.0899	0.1318	0.6821
maxsharp	0.0000	0.0196	-0.2700	0.4767	0.0980	0.6758	0.2491	0.2194	1.1353

From the above table, it can be observed that in finding the minimum standard deviation, PG has the highest weights, while ADBE and NVDA are shorted. In finding the maximum Sharpe ratio similarly it can be found that PG also has the highest weights while IBM has a shorting situation. Like constraint 1, the yield curve of constraint 2 is also fully fitted to the minimum standard deviation curve (See Fig. 2).

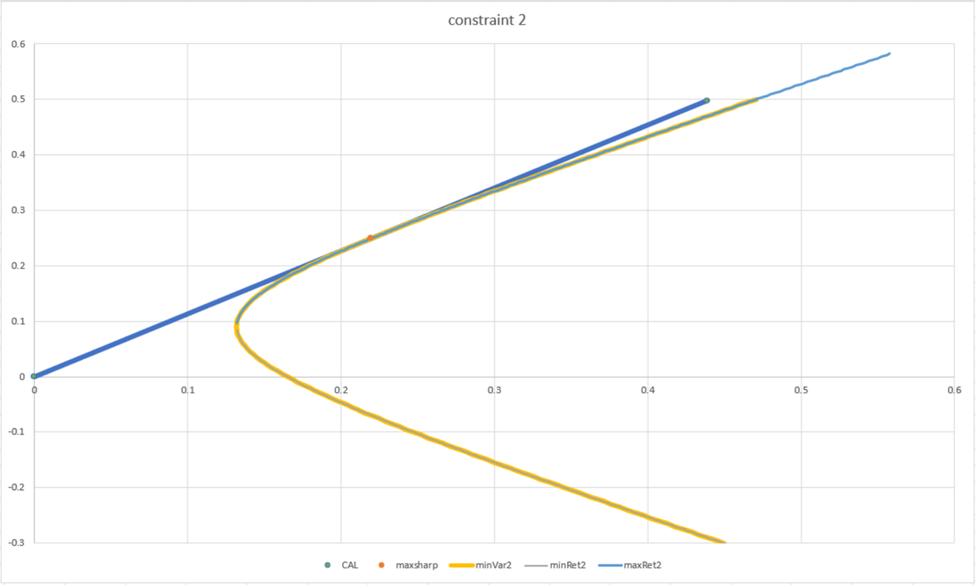

Fig. 2. Effective Frontier and CAL for Constraint 2

4.3 Constraint 3: Weights all greater than zero

In some regions, short selling is not allowed, so portfolios with weights all greater than zero are constructed to fulfill the requirement. The results are given below in Table 5.

Table 5. Weights of each asset under constraint 3

	SPX	ADBE	IBM	AAPL	NVDA	PG	Return	StDev	Sharp
minVar	0.4305	0.0000	0.0478	0.0000	0.0000	0.5217	0.0840	0.1234	0.6806
maxsharp	0.0000	0.0000	0.0000	0.3963	0.0528	0.5509	0.2041	0.1854	1.1007

In finding the minimum standard deviation, it can be found that the weights of SPX and PG dominate, while in finding the maximum Sharpe ratio, the weights of PG and APPL dominate.

As can be seen in the Fig. 3, the yield curve and the minimum standard deviation curve are partially adequately fitted, but as the standard deviation rises, the minimum yield curve rises while the maximum yield curve slowly declines, but eventually intersects at a point.

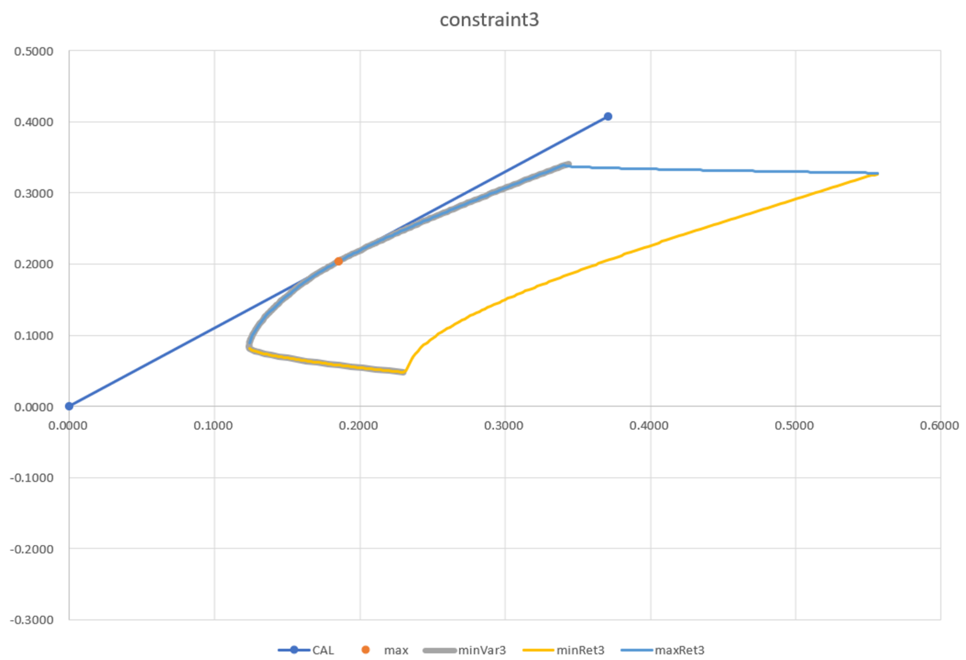

Fig. 3. Effective Frontier and CAL for Constraint 3

4.4 Constraint 4: The sum of the absolute values of all weights is less than 2

There are capital constraints on short selling and short buying in actual financial investments, and in this study, the maximum amount of borrowed and lent capital is limited to the investor's own funds, and portfolios are constructed on this basis. The results are as follows in Table 6.

Table 6. Weights of each asset under constraint 4

	SPX	ADBE	IBM	AAPL	NVDA	PG	Return	StDev	Sharp
minVar	0.5895	-0.0910	0.0604	-0.0035	-0.0281	0.4727	0.0637	0.1200	0.5312
maxsharp	-0.5804	0.1016	-0.1801	0.5938	0.1354	0.9297	0.3017	0.2611	1.1557

From the above data, it can be seen that when constructing the minimum standard deviation portfolio, SPX and PG weights account for the majority of the portfolio, while ADBE, AAPL, and NVDA all experience short selling. When constructing the maximum Sharpe ratio, PG still accounts for the majority, as well as AAPL weighting, while SPX and IBM appear to be short selling.

As can be seen in the Fig. 4, the maximum return curve and the minimum standard deviation curve are basically fitted, while the minimum yield curve is partially fitted, while the minimum standard deviation curve slowly declines, then rises rapidly, and finally declines slowly again.

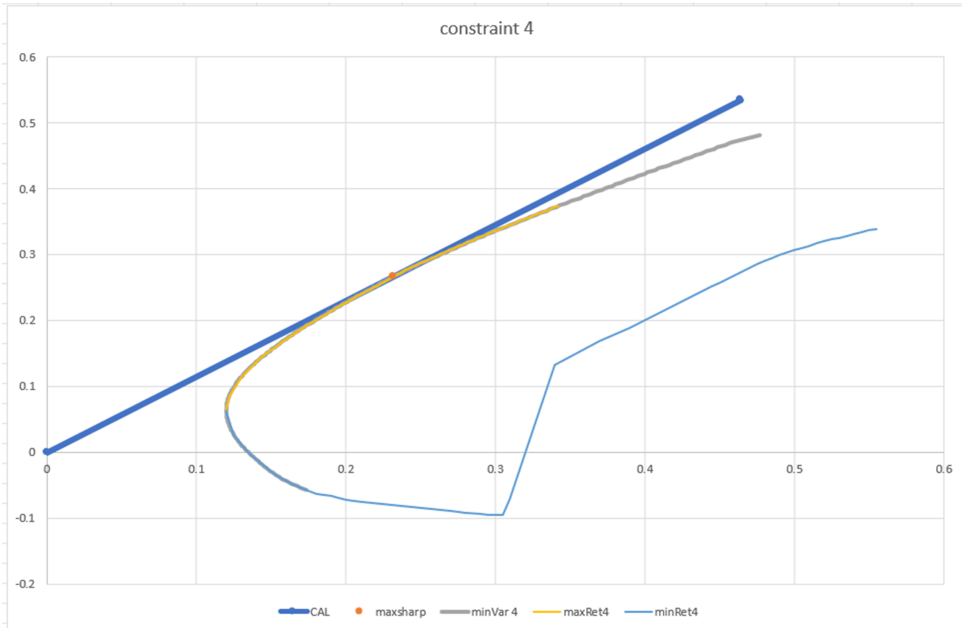

Fig. 4. Effective Frontier and CAL for Constraint 4

4.5 Constraint 5: Absolute value of weights is less than 1

Some regions have restrictions on leveraging, and this constraint aims to construct a model with low leverage to meet demand. The results are given below in Table 7.

Table 7. Weights of each asset under constraint 5

	SPX	ADBE	IBM	AAPL	NVDA	PG	Return	StDev	Sharp
minVar	0.5895	-0.0910	0.0604	-0.0035	-0.0281	0.4727	0.0637	0.1200	0.5312
maxsharp	-0.3431	0.0598	-0.1569	0.5185	0.1100	0.8117	0.2674	0.2321	1.1524

When constructing the minimum standard deviation portfolio, it can be found that SPX and PG are heavily weighted and ADBE, AAPL, and NVDA appear to be short. When constructing the maximum Sharpe ratio, a very high weighting of PG can be found. The maximum and minimum yield curves and the minimum standard deviation curve are adequately fitted as can be seen in the Fig. 5.

From the above five constraints it can be one found that when constructing the minimum standard deviation portfolio, the PG share is always very high, and the SPX share is equally high at the non-zero constraint. And when constructing the maximum Sharpe ratio portfolio, IBM has many short sales, and SPX has the same number of short sales at the non-zero constraint.

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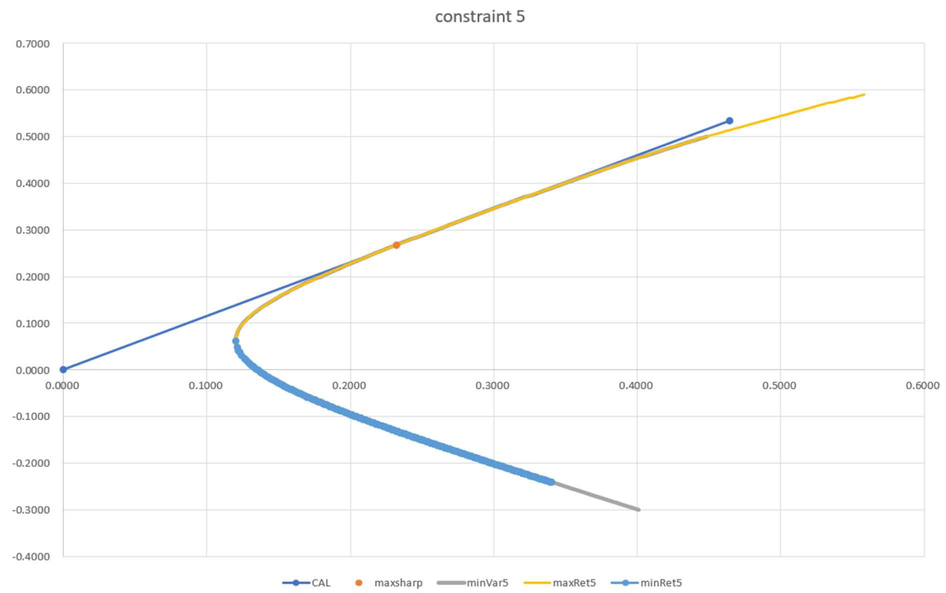

Fig. 5. Effective Frontier and CAL for Constraint 5

5 Conclusion

Along with the booming development of financial markets, investors have changed from maximizing returns only to weighing risks by constructing portfolios so as to achieve optimal asset allocation. Based on Markowitz mean-variance theory, this paper selects the closing price data and risk-free interest rate data of five stocks and SPX between May 2001 and May 2021, and uses EXCEL to plan and solve the weights, minimum standard deviation, and yield curves of each stock and SPX, and ultimately obtains the efficient frontier and the optimal asset allocation, which verifies that the Markowitz model still has a useful role to play in the present day. The Markowitz model is still useful today. Although the Markowitz model has a guiding role in constructing investment portfolios, considering that it utilizes the correlation between each asset, which leads to a large amount of computation when constructing a portfolio of multiple assets, the model can be improved from this aspect.

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