

Option Pricing Models: A Study of the Black-Scholes-Merton Model

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Abstract. Financial Derivatives refer to financial instruments whose value depends on or is derived from other underlying assets such as stocks, bonds, commodities, exchange rates, and interest rates. Examples include futures and options. This paper first introduces the early theories of option pricing. It then focuses on the Black-Scholes model, discussing the academic modifications and expansions made to this model due to its theoretical assumptions not aligning with reality. These adjustments aim to explore more efficient and practical methods for calculating option prices.

Keywords: Financial derivatives; Options; Asset pricing.

1. Introduction

Financial Derivatives refer to financial instruments whose value depends on or is derived from other underlying assets such as stocks, bonds, commodities, exchange rates, and interest rates [1]. Simply put, the value of a derivative is determined by the price fluctuations of its underlying asset or other factors. Common financial derivatives include futures, options, forward contracts, and swaps [1]. The main uses of derivatives include hedging to avoid or reduce market risks, such as hedging the risk of price fluctuations through futures or options, speculating on some derivatives to try to gain profits from price fluctuations, and taking advantage of the price differences between different derivatives or assets in the market to carry out low-risk arbitrage trading. Financial derivatives play an important role in the modern financial market, although they provide more investment and risk management tools, but because of their leverage effect, there are also large market risks.

The research of financial derivatives is a very active field in finance, involving many aspects such as theory, demonstration and application. With the continuous development of the derivatives market, scholars have conducted a lot of in-depth research on the nature, function, risk and pricing mechanism of derivatives. At present, the most famous pricing model in derivatives research is the option pricing model proposed by Black-Scholes (1973), which helps the financial market price European options based on stochastic process theory and risk-free arbitrage principle. Later, this model was extended to different types of derivatives, such as stock futures, bond options, etc. With the complexity of derivatives market, the traditional Black-Scholes model and futures pricing model face many challenges. Many

extended models have been proposed. At the same time, issues such as regulation, market stability, cross-border trading, and the impact of derivatives on the real economy have also become the focus of academic attention.

This paper will study the pricing of financial derivatives through the early theory of option pricing, the discussion of extended models. After the discussion, it summarizes the problems existing in the traditional asset pricing model and determines its future research direction.

2. Early Theories of Option Pricing: The Black-Scholes-Merton Model

In 1900, French mathematician Louis Bachelier derived an option pricing formula based on the assumption that asset prices follow Brownian motion with zero drift. However, this model assumed that asset prices were normally distributed, which violates the constraints of limited liability. Samuelson (1965) addressed this by assuming that asset prices follow geometric Brownian motion and priced warrants, accordingly, hypothesizing that the discounting of option payoffs satisfies a Martingale [2]. In 1969, Samuelson and Merton, without assuming that the discounting of option payoffs meets a Martingale condition, derived similar Martingale constraints implicitly by maximizing the utility function of investors. Simultaneously, Samuelson and Merton (1969) noted that option prices could be viewed as the discounted expectation, where the probability density function used for expectation need not be the actual probability density function. Given that representative investors are risk-averse, a risk-adjusted probability density function can be employed to calculate the expected future cash flows of the option and discount

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them, thereby determining the option price [3]. In conclusion, early option pricing formulas required the incorporation of the expected return on asset prices or the asset's risk premium (excess return rate). However, the non-stationary nature of risk premiums makes them difficult to estimate.

Fischer Black and Myron Scholes (1973), along with Merton (1973), were awarded the Nobel Prize in Economics for their contributions to pricing options under no-arbitrage hedging conditions. Black-Scholes-Merton (BSM) model is an early classical model in the field of financial derivatives pricing, which is used to estimate the theoretical price of European options. The goal of the BSM model is to calculate the fair price of an option by using known market data such as the price of the underlying asset, the strike price of the option, the expiration time, the risk-free interest rate and volatility. For European call options, the pricing formula is: $C = S_0 N(d_1) - Ke^{-rt} N(d_2)$. C is the current price of the option, S_0 is the current price of the stock, K is the Strike price of the option, r is the risk-free interest rate, T is the time between the expiration date of the option in years, $N(d)$ is the cumulative distribution function of the standard normal distribution representing the probability of a certain value. It needs to be obtained by looking up the probability table of $N(d)$ or excel calculation. d_1 and d_2 are the intermediate variables calculated by the following formula: $d_1 = \frac{\ln(S_0/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma T$, σ is the volatility of the underlying asset [4].

The Black-Scholes-Merton option pricing model is derived in two distinct methods. The first derivation method is based on the argument for economic hedging. Hedging refers to an investor holding a portfolio of stock assets and short-selling option assets to obtain a risk-free return. If the economic system meets these characteristics, a partial differential equation (PDE) exists that satisfies a series of boundary conditions when pricing options. The solution to a specific partial differential equation represents the first method, which yields the Black-Scholes option pricing formula. The second method for deriving the Black-Scholes option pricing formula is based on the martingale pricing theory, as proposed by Harrison and Kreps (1979) [5], and further developed by Harrison and Pliska (1981, 1983) [6].

3. Modifications and Extensions of the Black-Scholes-Merton Model

After the publication of the Black-Scholes-Merton option pricing formula, scholars made revisions to the model. One such revision introduced a model for discontinuous stock returns, known as the jump diffusion model. This adjustment accounts for scenarios in real-world situations where the prices of the underlying assets exhibit jump behavior. In 1976, Merton first introduced the compound Poisson process to characterize "jump" risk and developed the normal jump diffusion model to correct the B-S model's difficulty in adapting to the leptokurtic, peak thick tail of actual stock data and the volatility "smile" characteristic. He also provided a pricing formula for European options under this stock model [7]. Subsequent developments include Mancini's independent Poisson processes jump diffusion model [8], Duffie's affine jump diffusion model [9], Chan's geometric Levy process model [10], and Kou's double exponential jump diffusion model [11]. Another modification considered interest rate risk. Merton (1973) [12], Cox (1975) [13], and Cox and Ross (1976) [14] were among the first to use stochastic volatility models to price options in a complete market, while Hull and White (1987) and Wiggins (1987) examined the pricing of European options under stochastic volatility conditions [15]. Amin and Jarrow (1991) developed a pricing model for foreign currency options under a stochastic interest rate framework [16]. Aim and Jarrow (1992) extended the interest rate derivatives pricing model proposed by Heath, Jarrow, and Morton (1992), applying the stochastic interest rate model to the unpriced equity of risky assets, and constructed both European and American option pricing models under the stochastic interest rate framework [17]. Heston (1993) applied the pricing method for options with stochastic volatility to the study of bond and currency options in 1993 [18]. Scott (1997) introduced stochastic volatility and stochastic interest rates under a jump diffusion model and used the Fourier inversion formula to derive closed-form solutions for stock option prices. An empirical study on S&P 500 index options revealed that interest rates and returns are negatively correlated, which is significant for accurate option pricing [19]. Amin and Bodurtha (1995), Ho, Stapleton, and Subrahmanyam (1997), Medvedev, and Scaillet (2010) also explored how the stochastic variability of interest rates affects the pricing of American options under a stochastic interest rate framework [20].

In addition, Liu (2009) proposed in 2009 the use of uncertain differential equations to model the variations in stock prices, introducing an uncertain stock model and deriving a formula for pricing European options under this model [21]. Subsequently, Chen (2011) explored the pricing of American options based on Liu's uncertain stock model, providing the corresponding formula for American options in 2011 [22]. Zhang and Liu (2014) developed a formula for pricing geometric Asian options under Liu's uncertain stock model in 2014 [23], while the pricing formula for arithmetic mean Asian options under the uncertain stock model was presented by Sun and Chen (2015) in 2015. Peng and Yao (2011) introduced an uncertain mean-reverting stock model in 2011 and obtained the relevant option pricing formula. Following this, Yao (2012) established a no-arbitrage theorem related to the uncertain mean-reverting stock model in 2012 [24]. Yu (2012) provided the option pricing formula under the jump diffusion uncertain stock model in 2012.

4. Conclusion

Black and Scholes (1973) and Merton (1973) pioneered the development of the Black-Scholes-Merton option pricing formula. However, the validity of this pricing model is predicated on certain assumptions. Firstly, it assumes that the market is perfectly competitive. In such a market, all traders are price takers, meaning that their transactions do not influence market prices. Yet, in real financial markets, there exist instances of price manipulation for profit, and no country's financial markets are completely competitive. 2) The market operates without friction. A frictionless market implies no transaction costs or trading constraints, such as no short selling restrictions. In the Black-Scholes model, it is assumed that the option price on an underlying asset can be replicated through combinations of the asset and risk-free bonds; therefore, the option price should equal the price of the replication portfolio. However, in reality, portfolio replication and dynamic adjustments involve transaction costs. Given that diffusion processes undergo infinite variations, both the costs of multiple transactions and continuous trading can lead to prohibitively high costs. Considering the existence of transaction costs, Leland (1985) explored the pricing of European options in scenarios involving transaction fees in 1985. 3) Asset prices follow a geometric Brownian motion. This means that the prices of underlying stocks adhere to a log-normal distribution with constant volatility. 4) The risk-free rate is constant. A fixed rate indicates that the risk-free interest rate does not vary over time. 5) There is no credit risk. The absence of credit risk implies that all traders will not default when buying and selling financial securities. Many studies have found these assumptions to be inconsistent with actual facts. Subsequent research has focused on relaxing these model assumptions and exploring more efficient and practical methods for calculating option prices.

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