Portfolio Optimization under ESG Constraints: Markowitz Model vs. Index Model

Mengjie Wu[1*]

[1]Business, Economics and Law, The University of Queensland, Brisbane, QLD 4072, Australia

Abstract. This study addresses how to optimize portfolios under ESG constraints and short position constraints and explores the trade-off between sustainable investing and financial performance. This study is significant as it provides some new ideas and insights for improving sustainable finance by providing insights into how ESG factors affect portfolio construction and risk management to drive the development of sustainable finance. In this study, the Markowitz Model (MM) and Index Model (IM) are used to construct the optimal portfolio of 10 stocks in the US market, and the constraints of banning short selling and ESG score restrictions are added. The results show that MM is more effective in integrating ESG constraints, which can improve the portfolio ESG score and reduce the exposure to high carbon emission industries, and is more suitable for investors who prioritize sustainability, while IM is more suitable for investors who adopt passive investment strategy. The study highlights the importance of incorporating ESG factors into portfolio optimization, providing some valuable ideas for institutions and investors in balancing financial returns with sustainable practices.

1 Introduction

The integration of environmental, social and governance (ESG) factors in investment decisions has received wider attention in recent years as awareness of sustainability and corporate social responsibility has increased. However, striking a balance between ESG and financial performance remains a critical issue for institutions and investors. In addition, ESG investing has a strong track record in terms of risk management and long-term returns, especially during economic downturns or bear markets, and ESG funds tend to be more resilient [1]. This study discusses the problem of optimizing portfolios under ESG constraints and short selling constraints, analyzing these constraints as well as portfolio construction and risk management under different models. The reason for studying this issue is that sustainable finance is becoming more and more important and has gradually become an essential consideration for company valuation and investment, as well as investors are paying more and more attention to the impact of non-financial factors (ESG) on company financial data, which also provides a reference for investors.

In the field of research on the integration of sustainable finance and ESG, a large literature has explored the impact of ESG factors on portfolio performance. The mean-variance model

---

* Corresponding author: S4814074@uq.edu.au

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

proposed by Markowitz laid the foundation of modern portfolio theory, emphasizing the importance of reducing risk through diversification [2]. Sharpe's Capital Asset Pricing Model (CAPM) further introduces the concept of systematic risk and provides a framework for understanding asset returns [3]. These classical theories provide a theoretical basis for subsequent research on the application of ESG factors in investment portfolios. Fama and French show that non-financial factors, such as firm size and value, have a significant impact on asset reporting, which provides implications for the introduction of ESG factors [4]. Elton et al. further pointed out that ESG score can be used as an important indicator to measure a company's long-term performance, especially that companies with excellent performance in environmental and social responsibility tend to have stronger financial resilience [5]. However, these studies have mainly focused on the impact of ESG factors on a single asset or sector and have rarely addressed specific approaches to portfolio optimization under ESG constraints. For example, Bodi et al. explores how to incorporate ESG outcomes into a mean-variance model [6]. In addition, some scholars pointed out that ESG constraints may limit portfolio diversification, thus increasing risks. However, most of these studies do not deeply explore the performance differences of different portfolio models under ESG constraints. The existing literature provides an important theoretical foundation for ESG investing, but there are still shortcomings in the empirical research on portfolio optimization under ESG constraints, especially the research on how to optimize portfolios under specific constraints, such as no short positions and ESG score restrictions. This study aims to fill this gap by comparing the performance of MM and IM under ESG constraints through empirical analysis.

In this study, MM and IM are used to construct the optimal portfolio with 10 US stocks, and no short position and ESG score limit constraints are added for portfolio optimization. The study analyzed ten years of data from Yahoo Finance, Capital IQ and Investing.com to evaluate the performance of portfolios by constructing minimum variance portfolios, maximum Sharpe ratio portfolios, etc. The study compares the effectiveness of the two models in integrating ESG constraints and evaluates their impact on risk-adjusted returns, and this study aims to provide some suggested guidance for investors seeking to balance financial performance with sustainable investment practices.

2 Methodology and data processing

This study applies MM and IM to construct an optimal portfolio consisting of 10 stocks, while exploring the impact on the portfolio under the following two constraints.

2.1 Markowitz model

However, these studies have mainly focused on the impact of ESG factors on a single asset or sector and have rarely addressed specific approaches to portfolio optimization under ESG constraints. For example, Bodi et al. explores how to incorporate ESG outcomes into a mean-variance model [3]. The model measures risk using the variance (or standard deviation) of portfolio returns and considers the benefits of diversification. The theory assumes an investor's portfolio consisting of assets in N. The expected return of the portfolio is given by equation (1):

$$E(R_p) = \sum_{i=1}^{N} W_i \, E(R_i) \tag{1}$$

Where W_i represents weights of asset i in the portfolio; $E(R_i)$ represents expected return of asset *i*. The portfolio variance (risk measure) is given by Equation (2):

$$\sigma_{ij}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j \sigma_{ij} \tag{2}$$

Where σ_{ij}^2 represents covariance between assets *i* and *j*. The efficient frontier of MM represents the set of portfolios that provide the highest expected return for a given level of risk, and the optimal portfolio is usually determined by the Sharpe ratio, which is defined as equation (3)[7]:

$$\text{Sharpe Ratio} = \frac{E(R_P) - R_f}{\sigma_p} \tag{3}$$

Where R_f is the risk-free rate. In MM, investors are assumed to be rational and risk-averse; investors make decisions based on expected returns and variance; meanwhile, asset returns follow the general distribution; and there are no transaction costs or taxes.

2.2 Index model

Introduced by William Sharpe in 1963, the IM simplifies portfolio optimization by linking individual stock returns to the market index, rather than calculating all pairwise covariances between stocks, which makes the portfolio more computationally efficient [3,7].
The return of an individual stock is modeled as Equation (4):

$$R_i = \alpha_i + \beta_i R_m + e_i \tag{4}$$

Where R_i is return of stock *i*; R_m is return of the market index (e.g., S&P500); α_i is stock-specific component (not related to the market); β_i is stock's sensitivity to the market (systematic risk); e_i is firm-specific(idiosyncratic) risk, assumed to have zero mean and independent across stocks.
The portfolio variance under the IM is shown in Equation (5):

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum w_i^2 \sigma_{e_i}^2 \tag{5}$$

Where β_p is the beta sum of the weighted average (portfolio beta); σ_m^2 is variance of the market return; $\sigma_{e_i}^2$ is firm-specific variance.
Constraint 1:
This optimization is designed to mimic the typical constraints found in the US mutual fund industry: A US open-end mutual fund is not allowed to have any short positions, and the constraints are given in equation (6) [8]:

$$Wi \geq 0, \text{ for } \forall i; \tag{6}$$

Constraint 2:
Additional ESG restrictions are imposed under constraint 1 to control the ESG score of the portfolio so that it cannot exceed 90% of the benchmark ESG score shown in equation (7):

$$\sum_{i=1}^{10} (E_i + S_i + G_i) w_i \leq 0.9 \times \sum_{i=1}^{10} (E_i + S_i + G_i) w_i' \tag{7}$$

Where E_i, S_i, G_i represent the environmental, social and corporate governance scores of the ith stock, respectively; w_i is the weight of the ith stock in the portfolio under the constraint; w_i' is the weight of the ith stock in the unconstrained portfolio.

2.3 Data source

The data used in the study came from Yahoo! Finance, Capital IQ and investing.com. The time dimension of data collected is from December 31, 2014 to December 31, 2024. The selection of stocks is screened through the screening function of capital IQ. Firstly, the market where the stocks are located is selected. The 10 stocks are all listed in the United States and are all listed in New York or Nasdaq. At the same time, the stock price change of the past five years is set in the range of -50% to 100%, which is to select the companies with extreme stock price volatility. The industry screening is carried out, and the portfolio needs to include companies with high ESG scores and companies with low ESG scores. Generally, companies in technology, consumer and renewable energy will have high ESG scores, while companies in the mining and heavy industry will have low ESG scores. Through the above steps, some eligible companies can be selected. Companies with complete stock price data in the past 10 years and whose ESG score can be queried are selected, and finally 10 companies are selected as the research objects, as shown in the following table 1:

Table 1. Company profile.

Stock	Company name	Industry
MSFT	Microsoft Corporation	Technology
AAPL	Apple Inc.	Technology
KO	The Coca-Cola Co.	Consumer Defensive
UNP	Union Pacific Corp.	Industrial
PG	Procter&Gamble Co.	Consumer Defensive
XOM	Exxon Mobil Co.	Energy
V	Visa Inc.	Financial Service
JPM	JP Morgan Chase & Co.	Financial Service
JNJ	Johnson&Johnson	Healthcare
CVX	Chevron Corp.	Energy

Since the 10 stocks selected are all constituent stocks of S&P 500, the historical data of S&P 500 can be selected as the benchmark of the whole market, and the one-month federal funds rate can be used as the risk-free interest rate.

2.4 Data processing

In order to minimize noise, reduce data volatility and make the analysis more stable, it is necessary to convert daily prices to monthly data. Daily prices are converted to monthly prices using some functions in Excel, and the return per share is calculated by dividing the end-of-period price by the beginning-of-period price, minus one. Excess returns are calculated by subtracting the S&P 500 return for the corresponding date from each stock's return. The calculation of the remaining return is shown in equation (8):

$$\mathrm{R} = \mathrm{R}' - (\beta * \mathrm{Rm} + \alpha) \tag{8}$$

Where R is residual return; R' is actual return; Rm is market return; α is monthly alpha.
Turning to the calculation of some key data used by MM and IM, the average rate of return is the average rate of return calculated for the asset over the observed period as shown in equation (9):

$$\mathrm{E(R)} = \frac{1}{\mathrm{T}}\sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{R_t} \tag{9}$$

Where E(R) is the average return on an asset (usually monthly); T is Total number of periods (months); R_t is Return on assets in period t.
Standard deviation is used to measure the volatility (risk) of asset returns as shown in Equation (10):

$$\sigma=\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t-E(R))^2} \tag{10}$$

Where σ is Standard deviation of the asset (risk); Rt is returned on assets in period t; E(R) is average rate of return.
Beta measures the sensitivity of an asset to market movements as shown in Equation (11):

$$\beta=\frac{COV(R_i,R_m)}{\sigma_m^2} \tag{11}$$

Where β is the sensitivity of the asset to the market index; $COV(R_i, R_m)$ is Covariance between asset i and market m; σ_m^2 is Variance of market return.
Alpha measures the excess return of an asset, that is, the extra gain of an asset relative to its market performance，it is shown in Equation (12):

$$\alpha=E(R_i)-(R_f+\beta(E(R_m)-R_f)) \tag{12}$$

Where α is Excess earnings; R_f is Risk-free interest rate; $E(R_i)$ is Expected rate of return on assets; β is beta coefficient of the asset; $E(R_m)$ is the expected rate of return of the market.
The residual standard deviation measures the volatility of the part of the asset return that cannot be explained by the market，it is shown in Equation (13):

$$\sigma_{residual}=\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(\varepsilon_t)^2} \tag{13}$$

Where $\sigma_{residual}$ is Standard deviation of residuals; ε_t is Residual of regression.
Use the function in excel to calculate the correlation coefficient between 10 stocks and S&P500. Table 2 shows some basic data that will be used in the subsequent optimization, as well as table 3 shows the correlation between each company's pairwise. The results are as follows table 2 and table 3.

Table 2. Basic company data.

	SPX	MSFT	AAPL	KO	UNP	PG	XOM	V	JPM	JNJ	CVX
Average Return	0.10	0.23	0.24	0.04	0.08	0.06	0.03	0.16	0.15	0.03	0.05
StDev	0.15	0.23	0.28	0.16	0.23	0.16	0.27	0.2	0.24	0.16	0.27
Beta	1	-0.22	1.22	0.56	1.06	0.43	0.93	0.94	1.14	0.57	1.09
Annualized Alpha	0	0.26	0.12	-0.02	-0.03	0.01	-0.06	0.07	0.04	-0.03	-0.06
Residual StDev	0	0.23	0.20	0.13	0.17	0.15	0.23	0.14	0.17	0.13	0.22
ESG Score		14.2	16.8	24.1	20	25	43.7	15.4	27.2	20	38.3

Table 3. Correlation.

Correlation	SPX	MSFT	AAPL	KO	UNP	PG	XOM	V	JPM	JNJ	CVX
SPX	1	-0.15	0.68	0.54	0.7	0.41	0.53	0.72	0.71	0.55	0.61
MSFT	-0.15	1	0.02	-0.10	-0.09	-0.04	-0.25	-0.08	-0.07	-0.14	-0.27
AAPL	0.68	0.02	1	0.29	0.37	0.28	0.19	0.52	0.29	0.34	0.20
KO	0.54	-0.10	0.29	1	0.43	0.61	0.36	0.53	0.36	0.56	0.38
UNP	0.70	-0.09	0.37	0.43	1	0.32	0.37	0.43	0.51	0.41	0.47
PG	0.40	-0.04	0.28	0.61	0.32	1	0.17	0.33	0.19	0.47	0.24
XOM	0.53	-0.25	0.19	0.36	0.37	0.17	1	0.45	0.52	0.4	0.86
V	0.72	-0.08	0.52	0.53	0.43	0.33	0.45	1	0.50	0.44	0.48
JPM	0.71	-0.07	0.29	0.36	0.51	0.19	0.52	0.50	1	0.38	0.58
JNJ	0.55	-0.14	0.34	0.56	0.41	0.47	0.4	0.44	0.38	1	0.41
CVX	0.61	-0.28	0.20	0.38	0.47	0.24	0.86	0.48	0.58	0.41	1

3 Empirical analysis

Tables 4 and Table 5 present the portfolio distributions for both the Minimum Variance Strategy and the Maximum Sharpe Ratio Strategy. Portfolio performance varies with different constraints, relative to MM and IM in the Minimum Variance and Maximum Sharpe Ratio strategy for portfolio performance.

Table 4. MM optimization results.

(1)		MSFT	AAPL	KO	UNP	PG	XOM	V	JPM	JNJ	CVX
	MiniVariance	28.40%	0.07%	13.00%	2.11%	20.48%	7.45%	4.18%	2.07%	21.86%	0.37%
	MaxSharpe	49.25%	21.30%	-20.60%	-6.32%	18.98%	-2.52%	32.56%	18.45%	-12.89%	1.78%
Cons1.	MiniVariance	28.37%	0.05%	12.98%	2.17%	20.52%	7.72%	4.21%	2.12%	21.86%	0.00%
	MaxSharpe	45.81%	18.17%	0.00%	0.00%	2.84%	0.00%	21.71%	11.47%	0.00%	0.00%
Cons2.	MiniVariance	73.83%	0.00%	0.00%	0.00%	0.00%	0.00%	26.17%	0.00%	0.00%	0.00%
	MaxSharpe	73.83%	0.00%	0.00%	0.00%	0.00%	0.00%	26.17%	0.00%	0.00%	0.00%
(2)	Markowitz Model					—					
		Return	StDev	Sharpe	ESG						
	MiniVariance	10.30%	10.48%	0.98	21.70						
	MaxSharpe	24.16%	16.06%	1.50	16.13						
Cons1.	MiniVariance	10.30%	10.48%	0.98	21.70						
	MaxSharpe	20.45%	13.95%	1.47	16.73						
Cons2.	MiniVariance	21.42%	17.03%	1.26	14.51						
	MaxSharpe	21.42%	17.03%	1.26	14.51						

Table 5. IM optimization results.

(1)		MSFT	AAPL	KO	UNP	PG	XOM	V	JPM	JNJ	CVX
	MiniVariance	21.98%	-2.93%	24.73%	0.40%	25.45%	2.28%	5.95%	-1.70%	24.05%	-0.22%
	MaxSharpe	56.27%	34.88%	-10.21%	-9.24%	9.08%	-10.75%	42.59%	15.98%	-15.34%	-13.28%
Cons1.	MiniVariance	22.29%	0.00%	23.80%	0.00%	24.92%	1.67%	4.23%	0.00%	23.07%	0.00%
	MaxSharpe	46.91%	21.87%	0.00%	0.00%	3.02%	0.00%	23.79%	4.42%	0.00%	0.00%
Cons2.	MiniVariance	28.06%	0.00%	15.38%	0.00%	16.22%	0.00%	15.78%	0.00%	24.56%	0.00%
	MaxSharpe	46.91%	21.87%	0.00%	0.00%	3.02%	0.00%	23.79%	4.42%	0.00%	0.00%
(2)	Index Model					—					
		Return	StDev	Sharpe	ESG						
	MiniVariance	8.22%	9.52%	0.86	21.21						
	MaxSharpe	28.89%	17.84%	1.62	9.87						
Cons1.	MiniVariance	8.85%	9.55%	0.93	21.13						
	MaxSharpe	20.91%	13.72%	1.52	15.95						
Cons2.	MiniVariance	11.25%	9.93%	1.13	19.09						
	MaxSharpe	20.91%	13.72%	1.52	15.95						

Under constraint 1, the weights of MSFT and PG in the investment portfolio are relatively high, indicating that these two assets have a significant impact on minimizing the portfolio risk in the portfolio. Under Constraint 2, the MLP tends to allocate to higher risk assets, such as MSFT, but this also means that increasing the weight of MSFT in the asset portfolio will also increase the risk of the portfolio, and the international investment portfolio will be allocated relatively evenly.

The objective of the maximum Sharpe ratio is to find the optimal balance point between risk and return. Under Constraint 1, the medium-term investment portfolio will tend to place a greater proportion on high-risk, high-returning assets. These assets are intended to provide higher returns but also make the portfolio experience greater volatility. Under this constraint, the money manager has the same proportion of assets as the financial manager. Under Constraint 2, the results of the maximum sharpe ratio portfolio and the minimum variance MM portfolio are consistent, while the international portfolio increases the MSFT ratio and decreases the PG ratio compared to the minimum variance portfolio.

ESG score is a constraint for optimization. In the MM, ESG score tends to be higher, indicating that the MM takes sustainability into account while pursuing the maximization of returns, while the IM has relatively little change. It shows that the IM is not as good as the MM in controlling the non-systemic risk, and the focus is on the systemic risk of the market.

A minimum variance portfolio is suitable for risk-averse investors and can minimize portfolio volatility.

The Maximum Sharpe Ratio portfolio is suitable for investors seeking to achieve high risk-adjusted returns by optimizing asset allocation to improve the Sharpe Ratio. ESG restrictions have a significant impact on the investment portfolio, particularly in the Maximum Sharpe Ratio portfolio, which significantly reduces the proportion of high carbon emitting industries. MM better reflects the impact of ESG and can be directly integrated into ESG constraints for optimization.

In the empirical analysis, considering the impact of ESG factors on portfolio performance, the study finds that companies with higher ESG scores show stronger resilience during market fluctuations [9]. In addition, the application of ESG investment strategies can effectively reduce systemic risks and improve the risk-adjusted returns of portfolios [10].

4 Discussion and policy implications

As can be seen from the fig. 1 and fig. 2, with the increase of constraints, the figure will shrink more to the middle, indicating that under the influence of constraints, the diversification of asset allocation is limited and the space for choice is narrowed. ESG investing may drive capital flows to sustainable companies, affecting market dynamics and potential asset pricing anomalies [2]. The point distribution in the graph has some outliers, either because the historical data of the assets selected in the portfolio has abnormal fluctuations, or because the data cleaning part is not properly and thoroughly cleaned, or because the optimization process will converge to undesirable points due to local minimal or boundary restrictions.

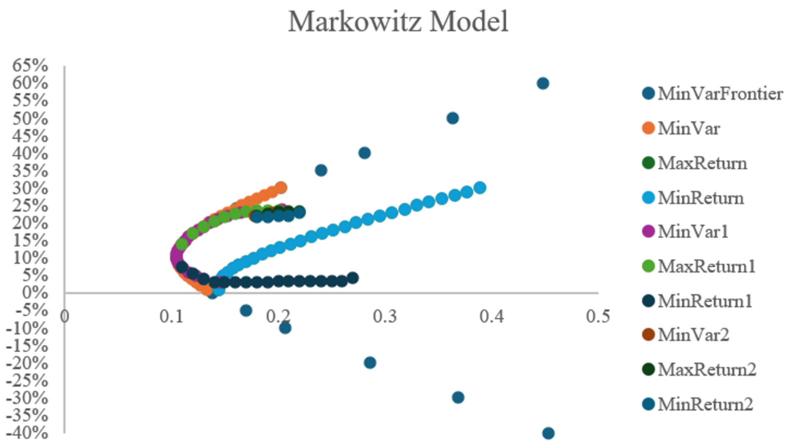

Fig.1. MM's summary of results.

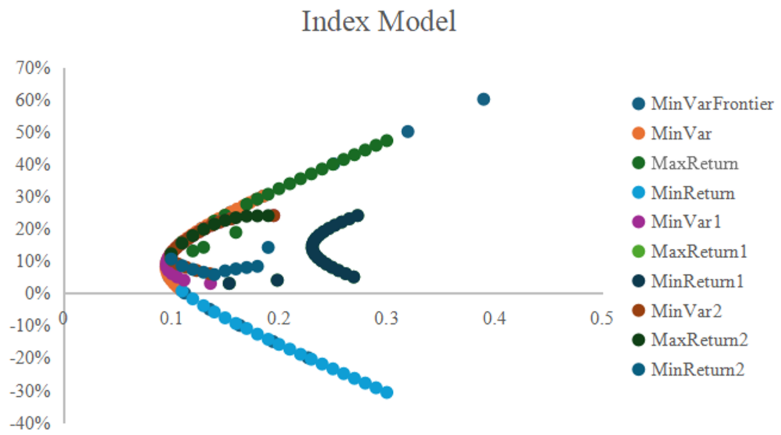

Fig. 2. IM's summary of results.

If the portfolio size is large and requires precise optimization, the MM method is recommended, as it fully takes into account the variance between assets; if managing index funds or adopting a passive investment strategy, the IM method can be chosen as it will be more computationally efficient. To pay attention to the impact of ESG constraints, the impact of the ESG score should be fully considered when optimizing the portfolio. If risk tolerance is low, a minimum variance portfolio is recommended as its volatility is relatively low. If investors are seeking high returns and can tolerate high risk, can choose the Maximum Sharpe Ratio portfolio to optimize the return per unit of risk. To optimize the ESG management system, more transparent and quantifiable ESG indicators could be developed, promoting sustainable investment, for example by providing tax breaks or other ESG-related incentives.

From a policy perspective, promoting ESG investment can help guide capital to sustainable enterprises and promote the transformation and upgrading of economic structure [11]. Therefore, regulators should establish clear ESG disclosure standards to encourage companies to improve ESG performance and ensure a more transparent information environment for investors [12].

5 Conclusion

This study explores, by applying the ESG management model and the risk management model, how to optimise investment portfolios under ESG constraints and without short position constraints and compares the performance of the two models in integrating ESG factors. The results show that the ESG model is more effective in integrating ESG factors and can significantly improve the ESG score and reduce exposure to high-carbon industries, while the integrated governance model has advantages in computational efficiency and systemic risk management. At the same time, ESG constraints have a significant impact on portfolio allocation and risk-adjusted returns, especially in the construction of the minimum variance portfolio and the maximum Sharpe ratio portfolio, ESG constraints can channel capital flow to more sustainable companies. These results highlight the trade-off between sustainable investment and financial performance and provide some references for investors in future investment choices.

However, there are also some limitations in this study. Second, there may be differences in the data sources and calculation methods of ESG scores, which affect their general applicability. In addition, there are shortcomings in selecting optimization methods and handling extreme values and cleaning data, resulting in the appearance of some extreme values in the results. Future research can expand the sample scope and cover more countries and industries to improve the wide applicability of the research. In addition, future research could explore more complex ESG constraints. Finally, with the continuous development of sustainability, future research can also combine new financial products such as green bonds to further optimize the framework of portfolio enrichment. Through continuous improvement of research methods, future research will provide a more comprehensive and in-depth reference for sustainable investment.

References

1. R. Joliet, Y. Titova, Equity SRI funds vacillate between ethics and money: An analysis of performance, volatility, and managerial skill. J Bus Ethics, **150**,181-195 (2018)
2. H. Markowitz, Portfolio selection. J Finance. **7**(1):77-91 (1952)
3. W. F. Sharpe, A simplified model for portfolio analysis. Manag. Sci. **9**(2), 277-293 (1963)
4. E. F. Fama, K. R. French, Common risk factors in the returns on stocks and bonds. J Financ Econ, **33**(1), 3-56 (1993)
5. Z. Bodie, A. Kane, A. J. Marcus. Investments. 13th ed. McGraw Hill, (2024)
6. E. J. Elton, M. J. Gruber, S. J. Brown, W. N. Goetzmann, Modern Portfolio Theory and Investment Analysis. 9th ed. Wiley, (2014)
7. W. F. Sharpe, Capital asset prices: A theory of market equilibrium under conditions of risk. J. Finance. **19** (3), 425-442 (1964).
8. Investment Company Act of 1940, Section 12(a) (3). Available at: https://www.law.cornell.edu/uscode/text/15/80a-12.
9. D. C. Broadstock, K. Chan, L. T. Cheng, X. Wang, The role of ESG performance during times of financial crisis: Evidence from COVID-19 in China. Finance Res. Lett. **38**,101716 (2021).
10. L. Becchetti, R. Ciciretti, A. Dalò, Stock market reaction to the announcement of the Dow Jones Sustainability Index. J. Bank Finance. **87**, 61-70 (2018).
11. J. He, H. Liu, R. Zhong, ESG disclosure and firm value: The role of corporate governance and information asymmetry. J. Bus. Ethics. **176**, 581-602 (2023).
12. United Nations Environment Programme Finance Initiative. Investor Duties and ESG Integration in China. (2018). Available at: unepfi.org.