

The Application of Investment Portfolio in Practice

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Abstract. Research in portfolio optimization has become increasingly crucial in financial studies. While extensive research exists in this field, there remains a gap in examining specific industry sectors alongside real-world investment constraints. The research methodology involves selecting the SPX500 index and ten representative companies from these sectors to construct a correlation coefficient matrix. Using the Mean-Variance Model, the study calculates key metrics, including the maximum Sharpe Ratio, minimum variance, and capital allocation line. A solver table is then employed to determine the minimum variance frontier under various constraints. The findings reveal two key insights: firstly, the SPX500 demonstrates a strong correlation with the selected companies, suggesting its effectiveness as a portfolio component for balancing risk and return; secondly, both the minimum variance frontier and capital allocation line show reduced performance when additional constraints are introduced. These results provide valuable guidance for investors with varying risk preferences in making informed investment decisions. Furthermore, the study highlights the trade-offs involved when incorporating investment constraints into portfolio decisions.

1 Introduction

In the contemporary financial markets, portfolio selection and optimization continue to represent a significant challenge for investors. The increasing intricacy of market dynamics and the interconnection among disparate sectors underscores the imperative for a comprehensive understanding of effective investment allocation strategies across a diverse array of stocks. In the current era, characterized by heightened global economic uncertainty and market volatility, the construction of optimal portfolios has become imperative for investors seeking to maximize returns while effectively managing risk. This study employs Markowitz's Modern Portfolio Theory to analyze ten stocks, with the objective of identifying the efficient frontier and the optimal portfolio weights [1-3]. The theoretical foundation for the analysis is the Markowitz Mean-Variance Model, which posits that investors should consider both expected returns and the variance of these returns when making portfolio decisions [4,5]. While this model offers valuable insights into balancing risk and return, practical investment decisions often face various constraints that need to be carefully considered. These constraints include position limits, trading costs, and sector exposure

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requirements, which can significantly affect portfolio performance and may lead to solutions that deviate from theoretical optimums. The research employs the Markowitz model to assess the optimal portfolio allocation across a selected group of stocks, considering real-world constraints [6]. The study examines how these constraints influence portfolio efficiency and asset allocation, with a particular focus on the trade-off between risk and return under different constraint scenarios. The analysis also pays particular attention to sector diversification, as the selected stocks represent three distinct market sectors: technology, financial services, and consumer goods. The objective of the analysis is to furnish investors with pragmatic insights, helping them to strike a balance between theoretical optimization and practical investment constraints. By examining how different constraints affect portfolio composition and performance, the research aims to assist investors in making more informed decisions about their investment allocations across these sectors. Moreover, this study contributes to the broader understanding of how investment constraints influence portfolio optimization outcomes in actual market conditions.

2 Data

Yahoo Finance and Investing are world-class platforms for providing accurate industry information. This paper utilizes data from Yahoo Finance (<https://finance.yahoo.com/>) and cross-validates it with data from Investing (<https://cn.investing.com/equities/>). To ensure the reliability of the data and to capture long-term market trends, this study analyzes data from February 28, 2003, to March 6, 2023, spanning approximately two decades. Over this extended period, these ten stocks have demonstrated resilient performance, reflecting their strong market positions and business fundamentals. Successful investors often demonstrate patience, waiting for market confirmation of their investment thesis. Indeed, the long-term performance of these selected stocks validates their underlying strength. To facilitate effective portfolio allocation and risk management, the stocks selected in this study represent diverse sectors with varying market characteristics, enabling comprehensive analysis of portfolio optimization strategies.

Table 1. Descriptive statistics of the selected assets.

	Average Return	StDev	Beta	Annualized alpha	Residual StDev	ESG Total Score
SPX	9.60%	14.80%	1.000	0.000	0.00%	—
WFC	9.36%	29.01%	1.165	-0.018	23.33%	13.45
LUV	9.15%	31.41%	1.071	-0.011	27.12%	16.89
PGR	15.77%	20.91%	0.695	0.091	18.20%	9.06
LSTR	15.22%	23.28%	0.871	0.069	19.38%	9.97
CSCO	10.11%	26.13%	1.167	-0.011	19.60%	15.47
TD CN	12.13%	17.15%	0.709	0.053	13.57%	12.43
PG	8.70%	15.19%	0.461	0.043	13.57%	16.23
MSFT	15.35%	22.11%	0.951	0.062	17.04%	17.00
KO	8.39%	16.09%	0.567	0.030	13.72%	15.36
MCD	17.76%	16.84%	0.591	0.121	14.39%	13.74

As shown in Table 1, among these ten stocks, NVIDIA demonstrates the highest average return at 38.83%, though this comes with the highest standard deviation of 51.38%. In contrast, consumer staples stocks like Colgate-Palmolive and Johnson & Johnson show stable but modest returns (around 7-8%), with much lower risk factors shown by their standard deviations of approximately 15%. TD Bank and Goldman Sachs maintain relatively balanced risk-return profiles, with returns around 12% and moderate standard deviations. Among technology stocks, while NVIDIA shows exceptional performance, both Cisco and Intel demonstrate more moderate returns with similar risk levels of about 26% standard deviation.

3 Method

The crux of Markowitz's contributions lies in his assertion that investors behave in a rational manner, driven by their innate reluctance to assume elevated levels of risk without commensurate enhancement in expected returns. The process of investing in securities and other risk assets necessitates the consideration of two fundamental issues: expected return and risk [6-9]. The pressing questions that market investors must address are how to measure the risk and return of portfolio investment and how to balance these two indicators for asset allocation.

$$\vec{r} = \{r^1, r^2, \dots, r_n\}^T \quad (1)$$

$$\vec{m} = \{\mu^1, \mu^2, \dots, \mu_n\} \quad (2)$$

$$R = \vec{r}^T \vec{w}, M = \vec{\mu}^T \vec{w} \quad (3)$$

$$\Sigma = ((\vec{r} - \vec{m})^T (\vec{r} - \vec{m})) \quad (4)$$

$$Var = \vec{w}^T \Sigma \vec{w} \quad (5)$$

$$P = \begin{Bmatrix} \rho^{11} & \rho^{12} & \dots & \rho^{1n} \\ \rho^{21} & \rho^{22} & \dots & \rho^{2n} \\ \dots & \dots & \dots & \dots \\ \rho_n^1 & \rho_n^2 & \dots & \rho_{nn} \end{Bmatrix} \quad (6)$$

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (7)$$

$$\sigma_p = \sqrt{\vec{v}^T P \vec{v}^T} \quad (8)$$

$$r_p = \vec{w} \cdot \vec{\mu}^T \quad (9)$$

$$\sigma_p = \sqrt{(\sigma_M \beta_p)^2 + \sum_{i=1}^n w_i^2 \sigma^2(\varepsilon_i)} \quad (10)$$

To address these questions, fluctuating assets returns are defined by Equation (1), with their averages shown in Equation (2). For a set of portfolio weights, the portfolio fluctuating return and its average return are given by Equation (3). The variance-covariance matrix is defined by Equation (4), leading to the portfolio variance expression in Equation (5).

The Markowitz model is comprised of two key components: the mean-variance analysis method and the portfolio efficient frontier model. In the context of a developed stock market, the efficacy of Markowitz's portfolio theory in practice has been substantiated, leading to its extensive utilization in the domains of portfolio selection and asset allocation [10].

For practical implementation, the correlation coefficient matrix is defined by Equation (6). The MM portfolio return and standard deviation are given by Equations (7) and (8) respectively.

4 Result

An analysis of the correlation coefficient matrix between these ten stocks reveals several notable patterns. The highest correlations are observed between stocks within similar sectors, the financial sector stocks show relatively strong correlations with each other (GS and USB at 50.51%). Additionally, consumer staples stocks exhibit strong internal correlations, with PG, JNJ, and CL demonstrating correlation coefficients above 50% among themselves (PG-CL at 56.56%, JNJ-CL at 55.56%). Conversely, NVIDIA displays notably low correlations with consumer staples stocks, particularly with CL (6.70%) and PG (8.80%), suggesting a considerable diversification potential. Investors who were to combine positions in these minimally correlated stocks could potentially achieve effective risk hedging and reduce overall portfolio volatility. The technology sector stocks (NVDA, CSCO, INTC) generally show moderate correlations with each other but lower correlations with consumer staples, indicating potential diversification benefits across these sectors (see Table 2).

Table 2. Correlation coefficient matrix of the 10 assets.

Correlation	SPX	NVDA	CSCO	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL
SPX	100.00%	51.85%	66.10%	57.18%	71.13%	62.13%	61.16%	62.97%	44.92%	53.96%	46.65%
NVDA	51.85%	100.00%	41.40%	41.67%	32.78%	19.06%	31.27%	18.44%	8.80%	10.37%	6.70%
CSCO	66.10%	41.40%	100.00%	53.73%	48.00%	41.78%	40.67%	43.87%	31.86%	29.55%	26.34%
INTC	57.18%	41.67%	53.73%	100.00%	39.88%	34.21%	40.70%	36.20%	19.18%	32.57%	16.70%
GS	71.13%	32.78%	48.00%	39.88%	100.00%	50.51%	48.09%	43.05%	19.88%	29.52%	23.09%
USB	62.13%	19.06%	41.78%	34.21%	50.51%	100.00%	53.68%	53.14%	32.54%	22.69%	25.41%
TD CN	61.16%	31.27%	40.67%	40.70%	48.09%	53.68%	100.00%	43.80%	23.75%	27.23%	23.06%
ALL	62.97%	18.44%	43.87%	36.20%	43.05%	53.14%	43.80%	100.00%	37.42%	49.48%	39.38%
PG	44.92%	8.80%	31.86%	19.18%	19.88%	32.54%	23.75%	37.42%	100.00%	52.46%	56.56%
JNJ	53.96%	10.37%	29.55%	32.57%	29.52%	22.69%	27.23%	49.48%	52.46%	100.00%	55.56%
CL	46.65%	6.70%	26.34%	16.70%	23.09%	25.41%	23.06%	39.38%	56.56%	55.56%	100.00%

The following exposition elucidates the purpose of establishing three portfolio constraints:

Constraint 1: An unconstrained portfolio scenario permits complete flexibility in asset allocation without any restrictions on position sizes or directions. As shown in Table 3, this theoretical framework serves as a baseline for comparing more realistic investment scenarios and helps understand the maximum potential efficiency of the portfolio. The result is shown in Figure 1.

Table 3. Weights of each asset under constraint 1.

	MinVar.	MaxSharpe
NVDA	0.06%	20.59%
CSCO	-0.06%	-7.44%

INTC	3.25%	-18.14%
GS	-2.73%	-0.36%
USB	4.58%	-5.34%
TD CN	29.70%	44.86%
ALL	-9.62%	2.84%
PG	22.87%	30.41%
JNJ	31.25%	23.42%
CL	20.69%	8.16%
Return	8.83%	16.43%
StDev.	11.28%	15.40%
Sharpe	0.78	1.07

Notes: MinVar. is used to represent the minimum variance; MaxSharpe is used to represent the maximum Sharpe Ratio.

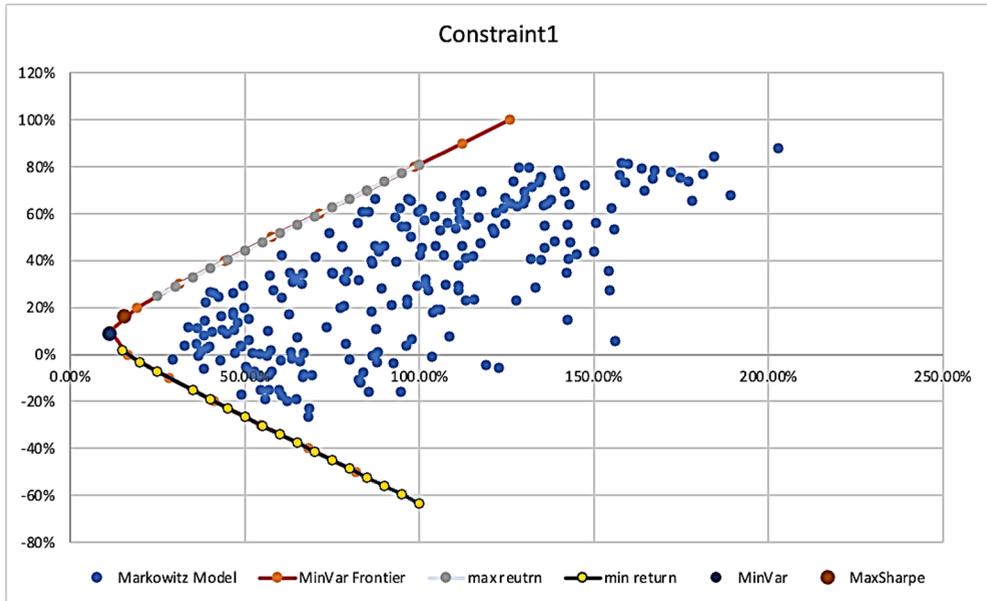


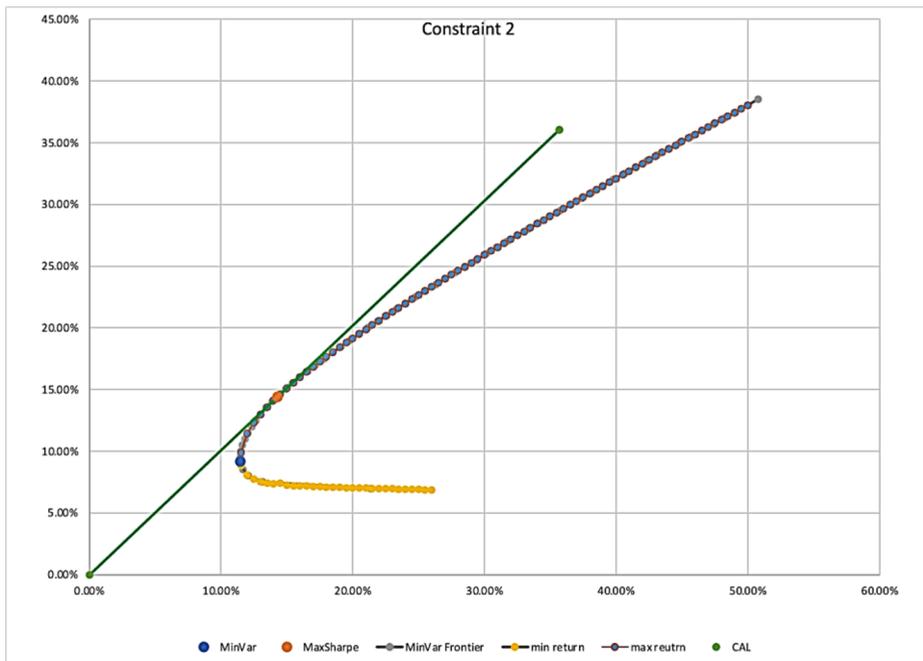
Fig. 1. Efficient and Inefficient Frontier of Markowitz Model under Constraint 1.

In Fig.1, the blue dots representing the Markowitz Model's diverse investment portfolios, the coloured lines track different investment approaches like the MinVar. Frontier, maximum return, and minimum return, visually demonstrating the trade-offs between risk and potential returns across various portfolio configurations.

Constraint 2: The long-only constraint is reflective of real-world market conditions in which numerous investors, particularly institutional funds and retail investors, are restricted from or elect not to engage in short-selling. As shown in Table 4, This constraint necessitates that all portfolio weights be non-negative ($w_i \geq 0$), a crucial element for practical portfolio management and risk control. The restriction is particularly pertinent in markets where short-selling mechanisms are limited or prohibited. The result is shown in Fig.2.

Table 4. Weights of each asset under constraint 2.

	MinVar.	MaxSharpe
NVDA	0.00%	16.11%
CSCO	0.00%	0.00%
INTC	2.09%	0.00%
GS	0.00%	0.00%
USB	0.01%	0.00%
TD CN	27.97%	32.01%
ALL	0.00%	0.00%
PG	23.87%	26.45%
JNJ	26.48%	16.34%
CL	19.58%	9.08%
Return	9.17%	14.41%
StDev.	11.45%	14.28%
Sharpe	0.8	1.01
ESG score	16.70228	15.98633

**Fig. 2.** Efficient and Inefficient Frontier of Markowitz Model under Constraint 2.

In Fig.2, the green CAL line represents the most efficient investment strategy with a steep, linear increase in returns, while other lines and points represent alternative portfolio configurations such as minimum and maximum return strategies. The visualization helps investors understand the trade-offs between risk and potential returns, illustrating how different investment methods perform under specific constraints.

Constraint 3: Termed the ESG-enhanced constraint, builds upon the long-only requirement by incorporating sustainability considerations into the portfolio construction process. This constraint reflects the growing importance of responsible investing in modern portfolio management. By adding ESG criteria to the investment process, Table 5 aim to explore how incorporating sustainability factors affects portfolio optimization outcomes while maintaining the practical benefits of a long-only approach. The result is shown in Figure 3.

Table 5. Weights of each asset under constraint 3.

	MinVar.	MaxSharpe
NVDA	0.00%	16.83%
CSCO	0.00%	0.00%
INTC	1.55%	0.00%
GS	0.00%	0.00%
USB	3.35%	0.00%
TD CN	48.99%	50.62%
ALL	0.02%	0.02%
PG	37.32%	33.03%
JNJ	0.00%	0.00%
CL	8.77%	0.00%
Return	10.25%	15.36%
StDev	12.52%	15.64%
Sharpe	0.82	0.98
ESG score	14.38769	14.38769

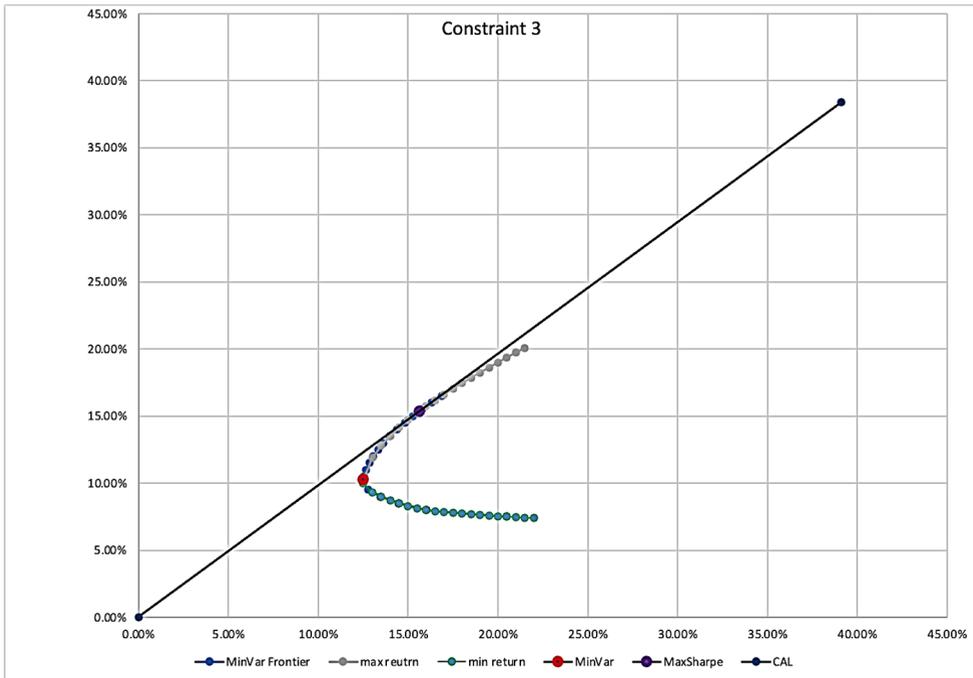


Fig.3. Efficient and Inefficient Frontier of Markowitz Model under Constraint 3.

In fig.3, the black diagonal line (CAL - Capital Allocation Line) represents the most efficient investment strategy, showing a consistent linear increase in returns. The scattered points and coloured lines represent different portfolio approaches like MinVar. Frontier, minimum return, MaxSharpe, and other investment configurations, demonstrating the complex trade-offs between risk and potential returns under this specific constraint.

5 Conclusion

Most of the extant portfolio research is based on the analysis of general market situations or specific industries. The purpose of this study is to perform a portfolio analysis of the retail, technology, financial service, and industrial sectors. This analysis will benefit potential investors when making investment decisions under different constraints. This paper utilizes statistical analysis to calculate the correlation coefficient among the 10 assets, and the index exhibits a high correlation coefficient with the other 10 assets. Subsequently, the Mean Variance Model is employed to optimize the portfolio and construct the minimum volatility portfolio, maximum Sharpe ratio portfolio, and minimum variance frontier. The study has identified that portfolios constrained by external factors exhibit higher volatility and lower returns compared to portfolios operating without constraints. It is therefore recommended that clients be made aware of this cost and carefully consider constraints that are not mandated by law. However, it should be noted that the study is not without its limitations. A notable constraint of the study is its exclusive focus on a limited number of representative companies across three distinct industries, which restricts its generalizability to real-world scenarios. Future research should address the current study's limitations by broadening the sample size across more industries, developing more sophisticated portfolio optimization models that better reflect real-world market complexities, and conducting a more comprehensive analysis of regulatory constraints. These improvements would provide more nuanced and representative insights into portfolio performance under various external factors,

ultimately offering more robust guidance for investors navigating complex market environments.

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